



**HAL**  
open science

## On the information that prospective teachers assume when modelling and its impact on the modelling outcomes

Jesús Montejo-Gámez, Elvira Fernández-Ahumada, Natividad  
Adamuz-Povedano

### ► To cite this version:

Jesús Montejo-Gámez, Elvira Fernández-Ahumada, Natividad Adamuz-Povedano. On the information that prospective teachers assume when modelling and its impact on the modelling outcomes. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03759043

**HAL Id: hal-03759043**

**<https://hal.science/hal-03759043v1>**

Submitted on 23 Aug 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# On the information that prospective teachers assume when modelling and its impact on the modelling outcomes

Jesús Montejo-Gómez<sup>1</sup>, Elvira Fernández-Ahumada<sup>2</sup> and Natividad Adamuz-Povedano<sup>2</sup>

<sup>1</sup>University of Granada, Education Sciences Faculty, Granada, Spain; [jmontejo@ugr.es](mailto:jmontejo@ugr.es)

<sup>2</sup>University of Cordoba, Education Faculty, Córdoba, Spain; [elvira@uco.es](mailto:elvira@uco.es); [nadamuz@uco.es](mailto:nadamuz@uco.es)

*This paper characterises the mathematical models that prospective elementary teachers develop to solve a modelling task, as well as its dependence on those information and assumptions that students use without justification when developing the model. For this purpose, a sample of 74 prospective elementary teachers (45 women) was chosen to estimate the distance a ship is from shore when it sees the lighthouse on the horizon. The use of an analysis tool specific to characterize models and the analysis of those statements that individuals use in their models without justification showed a wealth of models. The relationship between individuals' awareness of certain information and the validity of such information was also suggested. Several assumptions on the curvature of the earth's surface or on the height of the ship resulted to had an impact on the models analysed.*

*Keywords: Mathematical models, educational research, prospective elementary teachers.*

## Introduction

Nowadays, mathematical educators in general and researchers in mathematics education in particular acknowledge the benefits of modelling at all levels of education. In the scope of teacher education, particularly, several authors point out the potential of the modelling activity to provide prospective teachers with challenging experiences allowing to explore mathematics content, serve as examples of teaching and help to manage students' beliefs (see Fernández-Ahumada & Montejo-Gómez, 2019 and references therein for a discussion on the benefits of modelling in teacher education). This makes modelling a field of great interest to researchers in mathematics education and teacher educators. One of the most relevant and differentiating skills of mathematical modelling is linked to taking actions that facilitate the application of mathematical tools to obtain knowledge about real systems, which are usually complex. In other words, modelling implies simplifying or organising a real system, in order to bring it closer to a mathematic formulation. Within Ärlebäck and Albarracín (2019) activity-based approach, for instance, those actions devoted to facilitate the application of mathematics are encompassed in the "Modelling" sub-activity. Under a modelling-cycle approach, on the other hand, they are included in the Simplifying/Structuring transition.

## The role of assumptions in the transition of Simplifying/Structuring

Previous research has emphasised that difficulties in modelling are especially obvious when real-world situation has to be simplified (Dede, 2016; Kaiser et al., 2010), and there are studies describing the actions involved in the simplifying/structuring process of a real system (see e. g. Czocher, 2016; Maas, 2006; Montejo-Gómez & Fernández-Plaza, 2021). These include drawing/sketch of such real system, identifying and naming variables, operationalising relationships or patterns, introducing outside knowledge and estimating data, carrying out systematic experiments and using and formulating assumptions. In particular, there is consensus among the different authors on the

importance of the management of assumptions within the modelling activity. However, some studies suggest that much of the difficulty in the simplification process arises when making assumptions (Dede, 2016; Hidiroğlu et al., 2014 cited by Dede, 2016), and the focus of teaching is hardly focused on the formulation, use or discussion of the assumptions of a model. On the other hand, Wozniak (2012) highlighted the importance of prospective teachers being able to explicitly express the actions carried out during the modelling activity. Consequently, it is appropriate to help students to become aware of the information they apply when modelling.

Research on the impact of this information on the modelling activity is not very extensive. Fernández-Ahumada and Montejo-Gámez (2019) introduced the term premises to refer to mathematical statements used in a model without prior justification. The analysis of this type of statements allowed the detection of difficulties in the modelling activity. Other studies made emphasis on the formulation of the assumptions. At this respect, Wozniak (2012) found ‘mute praxeologies’ when analysing prospective elementary teachers’ activity: they used hypotheses, but generally did not formulate them. These studies suggest the usefulness of making visible the information that prospective teachers may assume when solving a modelling task and measuring the influence that this may have on the modelling outcomes. This leads to the following research question: what information do prospective elementary teachers assume as valid when approaching a modelling task and what is its impact on the models developed? To give an answer, this study aims to analyse written productions in order to access the assumptions made during the transition simplifying/structuring. The methods used are explained below.

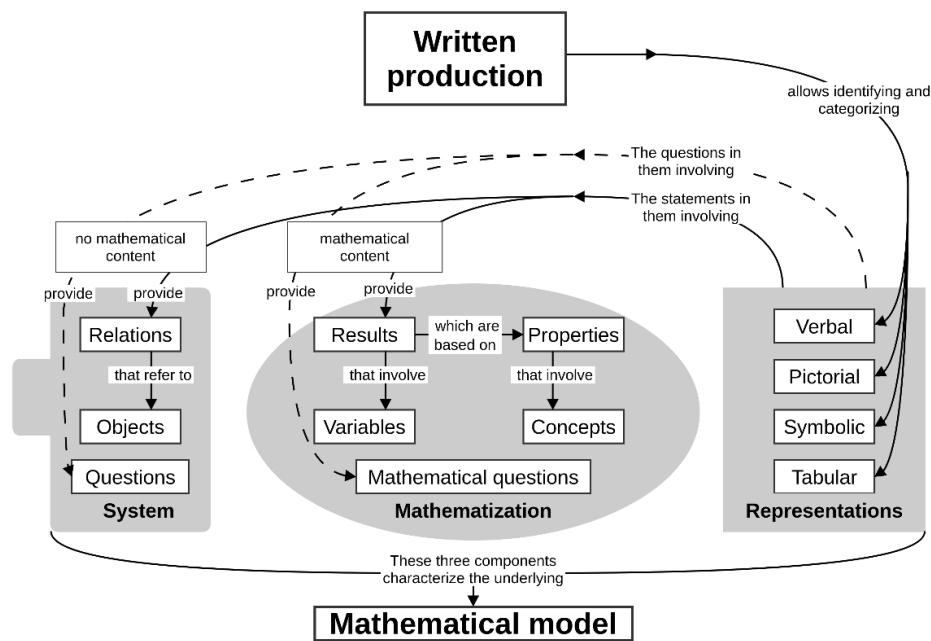
## **Methods**

A qualitative methodology was used for the development of the research. The sample used was made up of  $N = 74$  students of the degree in Primary Education at the University of Granada (45 women and 29 men). The participants were organised according to the usual working groups in class for them: 1 group of two people, 12 groups of three people and 9 groups of four. The 22 groups worked for an hour and a half, without interactions between groups, to solve the following task, adapted from Kaiser (2014): *The Cabo Mayor lighthouse is situated at the North of Santander. The focus of this lighthouse is situated 91 m above sea level; thus, it is useful to warn ships that they were approaching the coast. How far, approximately, is a ship from the coast when it sees the lighthouse for the first time?* The work of the groups generated 22 written productions that reflected the models proposed by the students.

## **Analysis procedure**

The research question was approached on the basis of a three-stage analysis. (1) The first one was to characterise and categorise the models developed. For this, the instrument and procedure presented by Montejo-Gámez et al. (2021) were used, which are schematised in Figure 1. The procedure starts by deciding what is to be considered mathematical content. Next, per each written production, the representations used in such production were identified, categorised and then analysed: On one hand, the questions of the system and the mathematical questions of the model were identified (depending on whether or not they contained mathematical content). On the other hand, the statements involved in the representations drove to identifying the relations of the system (those without mathematical

content) and mathematical results (those with mathematical content) of the model. Then, the entities referred to in the relations were identified as the objects of the system, whereas the relevant quantities involved in the results were identified as the variables of the mathematisation. Finally, the underlying mathematical properties from the results were abstracted, and hence, the concepts involved. In this way, the system and the mathematisation that made up the mathematical model in the written production were obtained. The synthesis of the differentiating elements provided a summary characterisation which gives an overall idea of the model developed and highlights its main attributes (see Table 1 below). Once this was done with all the written productions, models were organised into emergent categories, according to such attributes and the validity of the models (to provide a reasonable answer to the task).



**Figure 1: Flux diagram of the procedure used to characterise the models from the written productions in the first stage (Montejo-Gómez et al., 2021, p. 6)**

(2) The second stage of the analysis consisted of identifying the information that students assume to be valid in their models, which was based on Fernández-Ahumada and Montejo-Gómez's (2019) notion of premise. In this way, the relations and the results and properties of the models analysed in the first stage were retrieved, and those statements that were used without prior justification were selected. Note that such statements were obtained from the representations of the models. For example, the first pictorial representations in Figure 2 below (from a to e) indicate that students assume as valid the flatness of the earth's surface. Once the statements assumed as valid were recovered, they were organised into emerging categories with a common meaning, and each category was assigned a synthetic formulation that summarises this meaning. These categories were obtained by two of the researchers independently and then discussed and reviewed by the third researcher. In order to find out the incidence of each category, the following variables were quantified: frequency of appearance in the productions, contextuality (percentage of statements in that category that were represented without any reference to mathematical content), awareness (percentage of statements in

that category that the authors recognise as assumptions in an obvious way) and validity (percentage of statements that have been used in an acceptable way in the context in which they were used). For example, the 82.4% validity shown in row S6 in Table 2 indicates that 82.4% of the times students used the Pythagorean theorem it was to apply it to right-angled triangles (on the remaining occasions they used it in situations where it does not apply). (3) Finally, in the third stage, the distribution of the statements that students considered valid according to each type of model was obtained. These data made it possible to visualise the impact of the information students assumed to be valid on the models produced.

## Results

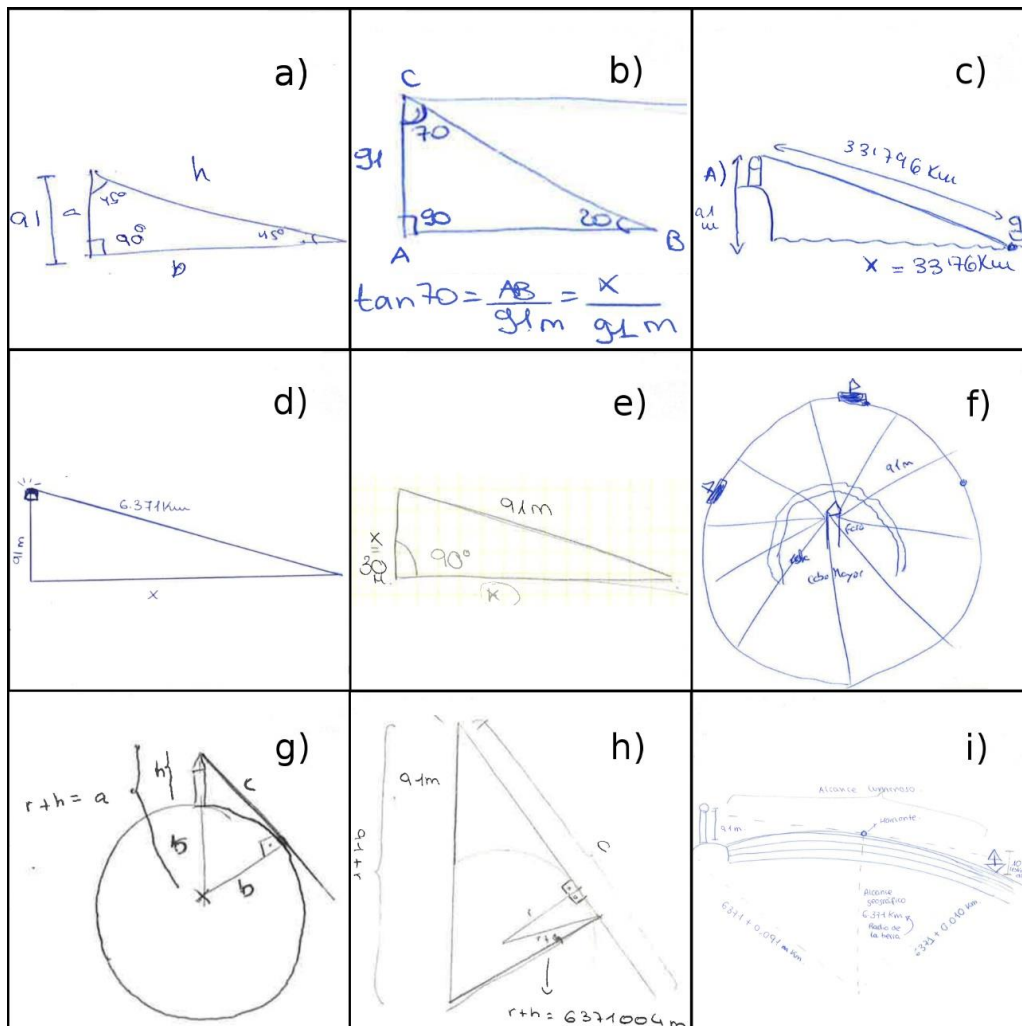
The characterisation and categorisation of the models developed by the students (first stage of the analysis) started from the choice of what was to be considered mathematical content: arithmetic operations, units of measurement, geometric properties of the circle and the Pythagorean theorem, content that belongs to the syllabus of the university course in which these students were enrolled. This first stage resulted in six categories, which are summarised in Table 1 and illustrated in Figure 2. The most repeated models (11 of the 22 models analysed) were those based on a right-angled triangle with the sea surface (flat) and the lighthouse. Nine of these constitute the M1 category and the remaining two make up the M2 category. M1 consists of the models that correctly identified the height of the lighthouse. None of them were valid for solving the task, as four of them assumed ad hoc data on the angles (Figure 2, a and b) and four others assumed values for the sides of the triangle (Figure 2, c and d). A final group gave no answer after considering the right-angled triangle.

**Table 1: Summary characterisation and validity of the models developed by students**

<i>Model</i>	<i>Frequency</i>	<i>Summary characterisation</i>	<i>Valid?</i>
M1	9	A right-angled triangle whose base is the Earth's surface (considered to be flat). The lighthouse height datum is correctly identified (Figure 2a-d).	No
M2	2	A right-angled triangle whose base is the Earth's surface (considered to be flat). The lighthouse height datum is incorrectly identified (Figure 2e).	No
M3	1	Circle of radius 91 m (Figure 2f).	No
M4	5	Right-angled triangle whose legs are the radius of the Earth and the lighthouse-horizon line of sight. The height of the ship is neglected (Figure 2g).	Yes
M5	3	Triangle whose legs are the radius of the Earth plus the height of the ship and the lighthouse-ship line of sight. The triangle is assumed to be right-angled (Figure 2h).	No
M6	2	Two right-angled triangles with common leg the radius of the Earth, and uncommon legs the ship-horizon and lighthouse-horizon lines of sight, respectively (Figure 2i).	Yes

M2 is made up of the models based on a right-angled triangle that misidentified the 91 m height datum (Figure 2e). The remaining categories use the circumference to address the task. In particular, category M3 has been assigned to a single model that interpreted the range of the lighthouse as 91 m and described the situation using a circle, which could not lead to an answer to the question posed (Figure 2f). In turn, category M4 encompasses the 5 models that used the curvature of the Earth's surface to model the situation and disregarded the height of the ship, which allowed them to obtain close-to-reality solutions (Figure 2g). Category M5 includes models that did not disregard the height of the ship but considered right-angled triangles that are not really right-angled, which was the case

for two of the groups (Figure 2h). Finally, M6 comprises models that considered the ship's height and used two right-angled triangles to calculate the ship-to-horizon and horizon-to-lighthouse distances (Figure 2i).



**Figure 2: Pictorial and symbolic representations corresponding to the models found**

Table 2, regarding the second stage of the analysis, summarises the information that students considered valid for the development of the models without the need for justification. There were 116 such statements, which were organised into seven categories. Four of them included statements expressed with hardly any mathematical content (rows S1-S4 in Table 2), and the remaining three were mostly expressed in mathematical language. In the case of S1-S4, we found hypotheses that students used without showing evidence that they were making assumptions. Of particular note were the assumptions of the flatness of the Earth's surface, the assumption that the height of the ship can be neglected, and different approximations to useful numerical parameters (angles in the M1 models, triangle side lengths in the M1, M2 and M3 models, and the radius of the Earth in M4, M5 and M6). In contrast, those models that used the curvature of the Earth or the height of the ship showed a higher degree of awareness. Furthermore, data indicate that statements with a higher degree of awareness have higher validity within the corresponding model. With regard to the last three categories (rows S5-S7 in Table 2), which describe applied mathematical knowledge, it is observed that students

generally use valid mathematical content (the Pythagorean Theorem in most cases) without justifying it, and that they assume its applicability as a matter of course. However, about a quarter of the statements in S5 expose that the students show explicit awareness of having identified a variable whose numerical value provides the answer to the task.

**Table 2: Categories of statements that students considered valid in their models**

<i>Synthetic formulation</i>		<i>Frequency</i>		<i>Contextuality</i>		<i>Awareness</i>		<i>Validity</i>	
S1a. The Earth's surface is flat	S1b. The Earth's surface is curved	12	10	100%	100%	8.3%	80%	0%	100%
S2a. The height of the ship is negligible	S2b. The height of the ship is not negligible	18	4	100%	100%	0%	50%	100%	100%
S3. A variable has a certain numerical value		21		90.4%		28.6%		57.1%	
S4. There are no clouds or haze		2		100%		100%		100%	
S5. Calculating certain information answers the problem (identification of the unknown).		22		4.5%		22.7%		81.8%	
S6a. Pythagorean Th. is applicable to solve the problem	S6b. Trigonometry is applicable to solve the problem	17	2	0%	0%	0%	0%	82.4%	100%
S7. When the ship starts to sight the lighthouse, the ship-lighthouse line is tangent to the Earth.		8		0%		12.5%		100%	

Regarding the impact of the statements that students used without justification, Table 3 shows the distribution of these statements in the different models. In particular, row S1 shows that the use of the curvature of the Earth was decisive in the development of the models, so that M1, M2 and M3 are based on the flatness of the Earth's surface, while the remaining three types of models are based on taking advantage of its curvature. Among the latter are also concentrated the claims related to the tangency of the ship-lighthouse line of sight (line S7 in Table 3), although it seems reasonable that these claims are a consequence of using the curvature of the Earth, rather than a cause of the development of the models. In contrast, the use of ship height does discriminate between M4, which disregards it, and M5 and M6, which introduce it as a model parameter (row S2 of Table 3). As for the rest of the statements, the applicability of certain mathematical contents such as trigonometry could be a differentiating feature of the models (especially with regard to mathematics), but it has not been considered as such due to its residual nature in the context of pre-service education. Finally, it is observed that statements related to the adoption of values for the parameters and the identification of the unknown are evenly distributed across the different models, so that they did not exert any relevant influence on the students when constructing their models either.

**Table 3: Distribution of the statements that students considered valid according to each type of model**

<i>Synthetic formulation</i>		<i>M1 (9)</i>		<i>M2 (2)</i>		<i>M3 (1)</i>		<i>M4 (5)</i>		<i>M5 (3)</i>		<i>M6 (2)</i>	
S1a. The Earth's surface is flat.	S1b. The Earth's surface is curved.	9	0	2	0	1	0	0	5	0	3	0	2
S2a. The height of the ship is negligible	S2b. The height of the ship is not negligible	9	0	2	0	1	0	5	0	1	2	0	2



S3. A variable has a certain numerical value	7	3	1	5	3	2						
S4. There are no clouds or haze	0	0	0	1	1	0						
S5. Calculating certain information answers the problem (identification of the unknown).	9	2	1	5	3	2						
S6a. Pythagorean Th. is applicable to solve the problem	6	2	1	0	0	0	5	0	3	0	2	0
S6b. Trigonometry is applicable to solve the problem												
S7. When the ship starts to sight the lighthouse, the ship-lighthouse line is tangent to the Earth.	0	0	0	4	2	2						

Note: The number in brackets next to  $M_i$  indicates the number of models that fell into the  $i$ -th model category

## Discussion and conclusion

This paper characterises the mathematical models developed by prospective elementary teachers when dealing with a task in which a property of the reference system, which is not explicit in the task but known to all participants, is key to finding a valid answer. The aim of this study was to analyse the information that prospective elementary teachers put into play without justifying it and its impact on the models developed. The main novelty of this paper is the analysis strategy used for this purpose and the first results found about this impact.

Analysis of the statements that prospective elementary teachers assumed to be true without justification revealed a large volume of information that was used in the models without obvious justification, with a balance between mathematical and extra-mathematical information. These results are aligned with those of Wozniak (2012), who found a large amount of mute praxeologies during modelling activity of prospective elementary teachers. In addition, it was observed that statements that students consciously impose are more valid for the model than those that are assumed without obvious awareness. As for the impact of these types of statements on the models developed, it was found that the use of the flat or curved character of the earth's surface was decisive. Indeed, all three types of models that assumed the flatness of the Earth led to non-valid answers, while two of the three types of models based on the curvature of the Earth did provide reasonable answers. The use of ship height was not decisive for the validity of the models, but it did differentiate them. On the other hand, data or information on the applicability of certain mathematical content had less impact on the models developed.

In brief, it has been found that the information that students use without justification when developing a model (especially in the simplification and structuring of the initial contextualised situation) can determine the modelling outcomes. Therefore, knowing this information is useful for understanding their models, finding difficulties (Fernández-Ahumada and Montejo-Gómez, 2019) and accounting for the contextual mathematical knowledge of prospective elementary teachers. The present study highlights the usefulness of written productions when studying this information that is used without justification and provides an analysis strategy for this purpose. The instrument developed by Montejo-Gómez et al. (2021) has been used, although tools for analysing written productions such as that of Ferrando et al. (2017) could also be used for this purpose. The didactic implication that emerges from this study is the relevance of expressing the mathematical knowledge that is activated during the modelling activity, which is especially necessary in prospective elementary teachers. Tools such as



the Study and Research Paths (Barquero et al., 2019) or the implementation of modelling tasks focused on the specific activities of using, formulation or detection of hypotheses could be useful in this sense. Exploration of these possibilities will be developed as future work.

## Acknowledgment

This research was funded by José Castillejo program of the Spanish Ministry of Education, Culture and Sports, grant CAS18/00474 with the support of the Andalusian Research Group FQM-193.

## References

- Ärlebäck, J.B., & Albarracín, L. (2019). An extension of the MAD framework and its possible implication for research. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the CERME11* (pp. 1128-1135). Freudenthal Group & Freudenthal Institute, Utrecht University.
- Barquero, B., Bosch, M., & Wozniak, F. (2019). Modelling praxeologies in teacher education: the cake box. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the CERME11* (pp. 1144-1151). Freudenthal Institute, Utrecht University.
- Czocher, J. A. (2016). Introducing Modeling Transition Diagrams as a Tool to Connect Mathematical Modeling to Mathematical Thinking. *Mathematical Thinking and Learning*, 18(2), 77-106. <https://doi.org/10.1080/10986065.2016.1148530>
- Dede, A. T. (2016). Modelling difficulties and their overcoming strategies in the solution of a modelling problem. *Acta Didactica Napocensia*, 9(3), 21-34.
- Fernández-Ahumada, E., & Montejo-Gámez, J. (2019). Dificultades en el aprendizaje matemático del profesorado en formación: análisis de las premisas utilizadas al modelizar. In J. M. Marbán, M. Arce, A. Maroto, J. M. Muñoz-Escolano, & Á. Alsina (Eds.), *Investigación en Educación Matemática XXIII* (pp. 273-282). SEIEM.
- Ferrando, I., Albarracín, L., Gallart, C., García-Raffi, L. M., & Gorgorió, N. (2017). Análisis de los modelos matemáticos producidos en la resolución de problemas de fermi. *Bolema*, 31, 220-242. <https://doi.org/10.1590/1980-4415v31n57a11>
- Kaiser, G. (2014) Mathematical Modelling and Applications in Education. In S. Lerman (Ed.) *Encyclopedia of Mathematics Education* (pp. 553-561). Springer.
- Kaiser, G., Schwarz, B., & Tiedemann, S. (2010). Future teachers' professional knowledge on modeling. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling Students' Mathematical Modeling Competencies* (pp. 433-444). Springer.
- Maaß, K. (2006). What are modelling competencies? *ZDM*, 38, 113–142. <https://doi.org/10.1007/BF02655885>
- Montejo-Gámez, J., Fernández-Ahumada, E., & Adamuz-Povedano, N. (2021). A Tool for the Analysis and Characterization of School Mathematical Models, *Mathematics*, 9(13), 1569. <https://doi.org/10.3390/math9131569>
- Montejo-Gámez, J., & Fernández-Plaza, J. A. (2021). “Flatten the curve”: Characterisation of the modelling cycle based on the adaptation of teaching to e-learning. Submitted for publication.
- Wozniak, F. (2012). Des professeurs des écoles face à un problème de modélisation: une question d'équipement praxéologique. *Recherches en Didactique des Mathématiques*, 32(1), 7-55.