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# Characterising rationales for using statistical modelling in education research from a mathematical modelling perspective: A review

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*This study developed and discussed a framework for characterising rationales for using statistical modelling from a mathematical modelling perspective based on a systematic literature review. We used this framework to provide an overview of the distributions of these rationales in the analysed studies, focusing on statistical modelling. The results identified three categories of rationales for using statistical modelling, namely, competency, content, and socially oriented types. This work discussed how the conceptualised rationales for using statistical modelling in the teaching and learning of statistics can guide the initial step in task design and curriculum development related to statistical modelling. These rationales may also serve as a common point of departure for discussions and collaboration between the mathematics and statistics education communities on the use of models and modelling.*

*Keywords: Statistical modelling, mathematical modelling, rationale, systematic review.*

## Introduction

Research on the teaching and learning of statistical modelling (SM) has increasingly gained attention in the statistics education community (e.g. Langrall et al., 2017; Pfannkuch et al., 2018) and mathematics education communities (e.g. Ärleback & Frejd, 2021; Frejd & Ärleback, 2021; Kawakami & Mineno, 2021). This trend owes its rise to the advent of data science and the need for models to address uncertainty, and underpinned by the worldwide growth of mathematical modelling (MM) in education research (e.g. Kaiser, 2017; Niss et al., 2007) and the rapid technological development.

The present study is part of an overall research goal to elaborate on the connections, boundaries, and boundary-crossing approaches between the teaching and learning of MM and statistics or SM. In this study, MM refers to the generic modelling process involving back-and-forth transitioning between the extra-mathematical world and mathematics (Niss et al., 2007). SM refers to ‘any one of a number of practices: the development of a distribution (empirical or descriptive model) from data; the process of creating a theoretical (probability) model from an empirical model; and the practice of sampling from a theoretical model (simulation)’ (Langrall et al., 2017, p. 502). MM and SM both include the generation, use, evaluation, and revision of models, and both emphasize the real-world context. However, a characteristic of SM that is not always found in MM is that of making uncertainty and variability central aspects of modelling (Langrall et al., 2017).

Pfannkuch et al. (2018) highlighted interpretations of SM and frameworks for describing students’ reasoning with SM in statistics education literature. However, the design guidelines for SM tasks and curriculum components related to SM have not been identified and elaborated. As with MM in

mathematics teaching and learning (Niss et al., 2007), such guidelines are essential for including SM as a significant component of statistics teaching and learning at all levels of education. This study examined the rationales for using SM, given the particular significance of rationales in designing SM tasks and curricula related to SM. Studies on teaching and learning SM have employed various rationales for using SM (Kawakami, 2019; Pfannkuch et al., 2018), but these rationales have yet to be systematically organised and structured. To facilitate the work of researchers, teachers, and curriculum developers in systematically designing purposeful SM tasks and to position SM practices appropriately within the statistics and mathematics curricula, this study aimed to (i) develop and discuss a framework for characterising the rationales for using SM from an MM perspective based on a systematic literature review; and (ii) provide an overview of the distributions of these rationales in studies focusing on SM.

### **Theoretical framework**

Commonalities between rationales, goals, theories, and practices in research focusing on MM and SM have been noted and broadly pointed out (Frejd & Ärlebäck, 2021; Langrall et al., 2017). To investigate some aspects of these commonalities in more detail, we used the rationales for using MM in mathematics education to clarify the characteristics of SM from an MM perspective.

### **Rationales for using MM in mathematics education**

The reasons for including the teaching and learning of MM in mathematics education have been discussed from at least two perspectives. Niss and colleagues (2007) introduced dual rationales for using MM: *modelling as a means for developing competency in applying mathematics and building mathematical models* and *modelling as a means for learning mathematics*. The former uses MM to develop a general MM competency (analysing and constructing mathematical models of extra-mathematical contexts and situations) by focusing on the use of mathematics in real-world contexts and problem solving. The present work referred to this rationale as *competency oriented*. The latter, modelling as a means for learning mathematics, uses MM to support the learning of mathematical contents through modelling activities ‘by offering motivation for its study as well as interpretation, meaning, proper understanding and sustainable retention of its concepts, results, methods and theories’ (Niss & Blum, 2020, p. 28). Our work referred to this rationale as *content oriented*. Niss and Blum (2020) stressed that these two rationales are not dichotomous but can be pursued simultaneously. However, the priority of either can change the aims and designs of mathematics lessons. In addition to these two rationales, Barbosa (2006) proposed a third rationale from a critical mathematics education perspective, namely, *modelling as a means for reflecting on the nature and role of mathematical models in society*. This perspective uses MM to ‘emphasize critical thinking about the role of mathematics in society, the role and nature of mathematical models, and the function of mathematical modeling in society’ (Kaiser, 2017, p. 274). We referred to this rationale as *socially oriented*.

### **Three potential rationales for using SM in the teaching and learning of statistics**

Pfannkuch et al. (2018) noted and listed four distinct educational purposes for using SM in the teaching and learning of statistics: (P1) enculturating students engaged in the discipline with an

approximation of professional statistical practice and reasoning, thinking, points of view, and beliefs; (P2) instructing students in key statistics knowledge and concepts; (P3) developing students' notions of the power, nature, role, purpose, and utility of SM; and (P4) enabling students to gain insights on particular situations. However, these four objectives have not been elaborated upon, nor are their relations discussed systematically. In this study, we sought to elaborate on the rationales for using SM by relating these four purposes to rationales for MM.

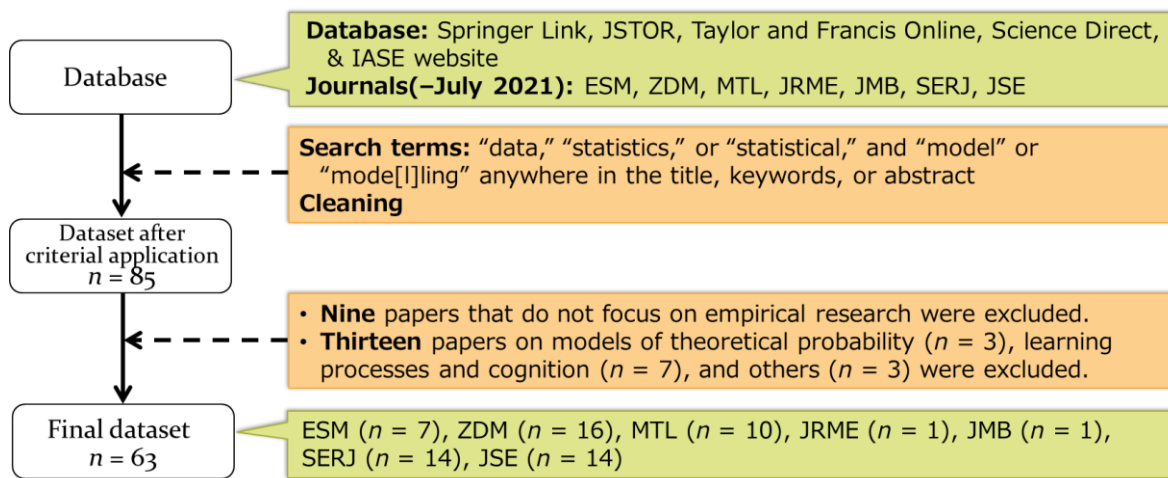
P1 employs SM as a means for developing students' statistical competency. In statistics education, such a statistical competency is often framed and discussed in terms of statistical literacy, statistical reasoning, or statistical thinking (Garfield & Ben-Zvi, 2008). According to Garfield and Ben-Zvi (2008), statistical literacy is the ability to interpret, evaluate, and communicate statistical information and messages; statistical reasoning is the ability to connect statistical concepts and explain statistical processes and results; statistical thinking is the ability to use statistical models, methods, and applications in advancing statistical investigation, such a PPDAC cycle (Problem–Plan–Data–Analysis–Conclusion) (Wild & Pfannkuch, 1999). In terms of the rationales for using MM, P1 can be described as *competency-oriented SM*. P2 and P3 employ SM for promoting the learning of statistical contents, such as statistical knowledge and concepts (e.g. variability, distribution, sample, and sampling) as well as knowledge and concepts related to statistical models and modelling. Employing the rationales for using MM, P2 and P3 can be described as *content-oriented SM*. Lastly, P4 employs SM for decision making in the real-world, social, and societal contexts, in which data are embedded and, in some cases, for developing a critical understanding of the use and role of statistics, statistical models, and modelling in these contexts. In terms of the rationales for using MM, P4 can be described as *socially oriented SM*.

In summary, we have elicited three potential rationales for using SM in the teaching and learning of statistics: (R1) *competency-oriented SM*, (R2) *content-oriented SM*, (R3) *socially oriented SM*. These three rationales for using SM, as well as the rationales for using MM, are not in opposition to one another. Indeed, multiple ones may merge within a single practice. As the priority of any of these rationales can influence the aims and design of statistics lessons, we make a distinction among these three rationales. The present study addressed two questions: (i) How can rationales R1–R3 be further elaborated and understood based on the use of SM as also done in empirical research? (ii) What is the use distribution of rationales R1–R3 in empirical research on SM?

## **Methodology**

We conducted a systematic literature review of peer-reviewed research on SM in mathematics and statistics education. Research papers from the following influential mathematics education journals were identified: *Educational Studies in Mathematics* (ESM) (May 1968–July 2021), *ZDM: Mathematics Education* (ZDM) (1997–July 2021), *Mathematical Thinking and Learning* (MTL) (1999–July 2021), *Journal for Research in Mathematics Education* (JRME) (1970–July 2021), and *Journal of Mathematical Behavior* (JMB) (1995–July 2021). Research papers from the following internationally recognised journals in statistics education were also included: *Statistics Education Research Journal* (SERJ) (2002–July 2021) and *Journal of Statistics Education* (JSE) (1993–July

2021). The paper selection process is illustrated in Figure 1. As part of the systematic review process, we read all identified 63 papers, and then coded the rationales for using SM in the papers by assigning them into the three categories R1, R2, and R3. The coding was determined based on the explicit description(s) of the rationale(s) for using or investigating SM or modelling in the papers by focusing on the (1) purpose and position of the study; (2) intentions and purposes of the used teaching materials, curriculum, and teaching practices; as well as (3) research questions in the papers. Examples of such descriptions and their coding are shown in Table 1. The first author conducted the first analysis, which included classification, and the second author independently checked the assigned papers and the first analysis. Where discrepancies occurred, the authors discussed and resolved these issues.



**Figure 1: Paper selection process**

**Table 1: Examples of the descriptions from which categories R1–R3 were determined**

Category	Examples of the description in the papers
R1	In this article, we have examined one approach to developing primary school students’ statistical literacy, namely, through modelling with data. (English & Watson, 2018, p. 113)
R2	The study relied on classroom video and student artefacts to analyse aspects of the students’ modelling experiences which exposed them to powerful statistical ideas, such as key repeatable structures and dispositions in statistics. (Makar & Allmond, 2018, p. 1139)
R3	Statistical modelling needs to become a tool for critical democracy. (Zapata-Cardona, 2018, p. 1220)

## Results

Table 2 presents the identified rationales for using SM in the analysed literature with the frequency and examples within each category.

**Table 2: SM rationales in the literature in terms of categories R1, R2, and R3 ( $n = 63$ )**

Category	Freq. (%)	Examples (listed by author only given space constraints)
R1	19 (30)	Doerr & English (2003) <sub>JRME</sub> , Biehler et al. (2017) <sub>SERJ</sub> , Doerr et al. (2017) <sub>SERJ</sub> , Noll & Kirin (2017) <sub>SERJ</sub> , Dvir & Ben-Zvi (2018) <sub>ZDM</sub> , Leavy & Hourigan (2018) <sub>ESM</sub>
R2	13 (21)	Prodromou & Pratt (2006) <sub>SERJ</sub> , Lesh et al. (2008) <sub>ESM</sub> , Ainley & Pratt (2017) <sub>SERJ</sub> , Büscher & Schnell (2017) <sub>SERJ</sub>

R1 & R2	21 (33)	English (2012) <sub>ESM</sub> , Manor & Ben-Zvi (2017) <sub>SERJ</sub> , English & Watson (2018) <sub>ZDM</sub> , Kazak et al. (2018) <sub>ZDM</sub> , Makar & Allmond (2018) <sub>ZDM</sub> , Patel & Pfannkuch (2018) <sub>ZDM</sub> , van Dijke-Droogers et al. (2021) <sub>ESM</sub>
R1 & R3	5 (8)	Simonoff (1997) <sub>JSE</sub> , Biehler et al. (2018) <sub>ZDM</sub> , Wilkerson & Laina (2018) <sub>ZDM</sub>
R2 & R3	2 (3)	Zapata-Cardona (2018) <sub>ZDM</sub> , Zapata-Cardona & Martinez-Castro (2021) <sub>MTL</sub>
R1, R2, & R3	3 (5)	Jacobson et al. (2009) <sub>JSE</sub> , Garfield et al. (2012) <sub>ZDM</sub> , Kazak et al. (2021) <sub>MTL</sub>

### Only R1 rationale: Competency-oriented SM

Papers in which R1 was the only rationale used comprised the second largest category of papers, accounting for about 30% ( $n = 19$ ). These studies typically used SM to develop statistical competencies (e.g. statistical literacy, reasoning, and thinking) as well as statistical processes (e.g. *statistical inquiry* in the sense of Wild and Pfannkuch [1999] and *informal statistical inference* in the sense of Makar and Rubin [2009]). These studies emphasised the role of statistical models and modelling in developing statistical competencies; for example, ‘[m]odels are important concepts in statistics and key components of learning to think statistically’ (Noll & Kirin, 2017, p. 213). Exclusively, R1-coded papers often described SM as applicable for real-world problem solving and stressed the applied nature of statistics, emphasising the use of actual and authentic data in the educational setting.

### Only R2 rationale: Content-oriented SM

Papers in which R2 was the only rationale used comprised the third largest category of papers, accounting for about 21% ( $n = 13$ ). These papers used SM to elicit, develop, and deepen the understanding of a wide range of statistical contents, including an aggregate view of data, measures of distribution, signal and noise, variation, population, sample and sampling, theoretical distributions, statistical models, SM process, causality, and statistical inference. These papers tended to reframe statistical concepts and knowledge as models or modelling constructs in support of the notion of *emergent modelling* (Gravemeijer, 1999) and *model-eliciting activities* (Lesh & Doerr, 2003); for example, ‘measures are understood as models, which can either be used to make sense of a given situation or to reason about the statistical measures themselves’ (Büscher & Schnell, 2017, p. 144). Exclusively, R2-coded papers often expressed SM as an epistemic practice of statistics and a pedagogical tool.

### Combined R1 and R2 rationales: Competency- and content-oriented SM

The largest category of papers employed both R1 and R2 (approximately 33%,  $n = 21$ ). These papers used SM to combine and integrate statistical competencies (R1), such as statistical reasoning and informal inference, and statistical contents (R2). In these papers, the emphasis was not only on real-world problem solving, transiting between the real (data) and model (data) world, but also on conceptual developments within the model world. Some papers considered statistical literacy and reasoning as competency to carry out SM, comparable to MM competency. For example, Patel and Pfannkuch (2018) framed *SM reasoning* as the ability to transit between the physical and model worlds involving the following activities: starting with understanding of the real-world problem, applying structure to it toward a transition to the model world, refining the model and analysing

simulated data, and interpreting the results back into understanding regarding the original real-world problem. R1-and-R2-coded papers often used SM as an integrator between statistical methods and statistical content.

### R3 rationale: Socially oriented SM

R3 was the least commonly discerned rationale in literature (approximately 16%,  $n = 10$ ). None of the papers used it as the sole rationale, only using it in tandem with R1 and/or R2. These studies used SM to enhance critical thinking on real-life, social, and societal contexts and to examine the power and limitations of statistical models and modelling. In some cases, the discipline of statistics embedded in these contexts were examined using the notions of authenticity, critical citizenship, ethics, and publicity. These papers focused on the role of statistical models and modelling in thinking critically about life and society. For example, Zapata-Cardona (2018, p. 1220) observed that ‘[s]tatistical modelling needs to become a tool for critical democracy’ and ‘[m]odelling activity should focus on the functions of the applications in society’. R3-coded papers often described SM as a means and object of social criticism and decision-making based on data. They also emphasised the use of social issues and contexts in education.

### Discussion and conclusion

Our analysis showed that all three rationales for using SM in research on the teaching and learning of statistics were used to various extents in empirical studies. In other words, SM is seen as a multifaceted means of achieving applied, epistemic, and social critical-related goals in the teaching and learning of statistics. However, based on the results of the systematic review in terms of R1 to R3, SM can also be understood more holistically: (i) R1 can be seen as stressing *SM as a component of statistical competencies*, i.e. the use of statistical models/modelling as a key element for promoting real-world problem solving and statistical inquiry (e.g. Noll & Kirin, 2017); (ii) R2 can be taken to promote *SM as integrating aspects of statistical contents*, meaning that the structure of statistical content is rooted and reflected in statistical models/modelling (e.g. Büscher & Schnell, 2017); and (iii) R3 portrays *SM as shaping and influencing real-life, social, and societal decision making*, indicating that statistical models/modelling form the basis for data-driven decision making (e.g. Zapata-Cardona, 2018). Figure 2 summarises these three conceptualised rationales for using SM in the teaching and learning of statistics. In each rationale in Figure 2, the goal–means relation is depicted with arrows. Inclusionary relations between the goal and means are shown by the ellipses.

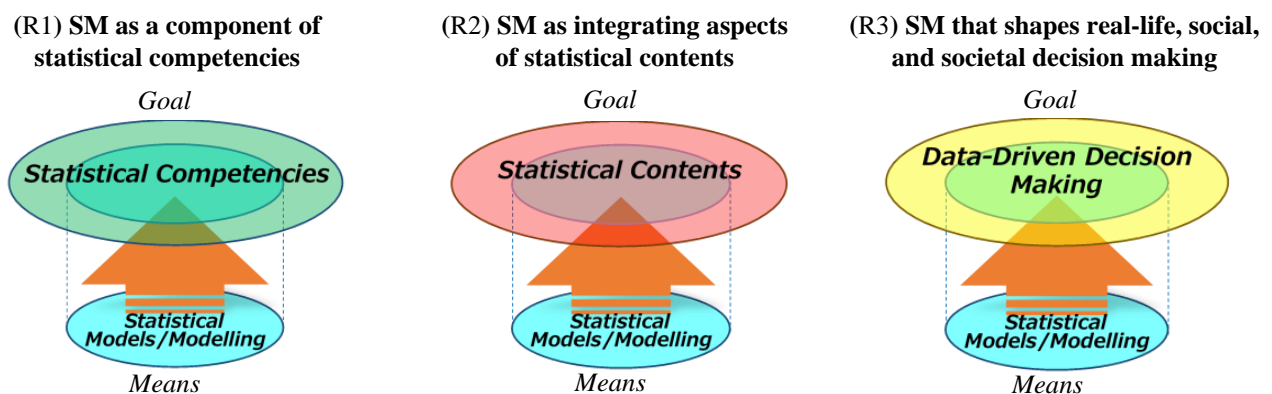


Figure 2: Three conceptualised rationales for using SM in the teaching and learning of statistics

The review results also revealed that these three rationales can be used together in one lesson, unit, curriculum, or project. Indeed, the exclusive use of R3 was not found in any of the 63 analysed papers. This is in contrast to the research on MM, where the lone use of the socially oriented rationale is common (cf. ethnomodelling). The combination of R1 and R2 was the most common category, with three papers using all three rationales (R1–R3). Hence, combining rationales is common in the context of SM, in contrast to much of the research on MM. These results may be related to the nature of statistics, which pertains to statistical methods, statistical content, and data with context (Wild & Pfannkuch, 1999). However, a more detailed review of MM literature is needed to establish further the viability of these differences between SM and MM.

The goal–means relations in Figure 2 support the hypothesis of the learning trajectory of SM. For example, learners can advance competency-oriented SM (R1) using contents constructed through content-oriented SM (R2). They can then perform socially oriented SM (R3) by making full use of competency and contents acquired through the other types. The inclusive relations in Figure 2 also suggest that different rationales have different domains and assumptions in which statistical models and modelling are placed and that, in practice, the functions/roles of statistical models and modelling can be changed dynamically. Therefore, it is also necessary to review the meaning of statistical models, modelling, and the description of SM specified in each study.

The conceptualised rationales for using SM in Figure 2 can guide the initial step in task design and curriculum development related to SM for researchers and teachers. It can also serve as a common lens for shared discussion and collaboration between the mathematics and statistics education communities on issues related to the teaching and learning of models and modelling (e.g. English & Watson, 2018; Langrall et al., 2017). However, to provide more concrete design guidelines, additional research is needed to identify and elaborate on more aspects and characteristics of SMs. The review results specifically demonstrated the applicability of the findings of MM education research to SM education research in terms of goals and rationales. A natural next step is to clarify the similarities and differences in teaching and learning SM and MM. It is also necessary to clarify to what extent we need to distinguish between MM and SM, or whether SM is a sub-form of MM. These, in turn, will shed light on the nature and role of models and modelling, and the underlying assumptions and hypothesis in, and for, MM research.

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