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Characterising the roles of digital resources in mathematical modelling

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Over the last decades digital resources of various kinds have entered mathematics education including mathematical modelling. In this paper we present the a priori analysis of a modelling activity based on Study and Research Path using the notions of media-milieu dialectic and Herbartian schema from the Anthropological Theory of the Didactic. We use these notions to differ between resources: digital tools, textbooks, data and to differ between the pragmatic and epistemic value of the resources. Finally, we discuss how task design can benefit from addressing the potential roles resources can play in modelling processes.

Keywords: Mathematical modelling, media-milieu dialectics, Herbartian schema, digital tools, study and research paths.

Introduction

Through computers, with numbers of digital tools, and smartphones students have access to numerous resources potentially supporting their mathematical work including modelling. Initially, emphasis has been put on Dynamic Geometry Software (DGS) and Computer Algebra Systems (CAS), but other resources emerge continuously. The presence of the resources affects the objects, techniques and practices, students are engaging with in mathematics. Artigue claims that “mathematical objects are not absolute objects but are entities which arise from the practices of given institutions” (2002, p. 248). Thus, when institutions allow the use of more resources it changes the practices for example within mathematical modelling and during the last CERME, the following questions were raised:

”How can digital and physical tools change ways of approaching modelling problems? How do they change the meaning of “working mathematically” today? Do we need to rethink the definition and conceptualizations of the steps of the modelling processes when using technology and simulations?” (Barquero et al., 2020, p. 1108)

Though still open, the questions are not new. Previously, Siller and Greefrath (2010) have argued how the merger of modelling and technology might serve both pedagogical, psychological, cultural and pragmatic aims for the teaching of modelling where more advanced calculations and realistic problems can be addressed. They have described the modelling process with an addition (an extra cycle) to the modelling cycle representing the role of computational media (Greefrath et al., 2011). Doerr, Ärlebäck and Misfeldt challenge this representation of the merger arguing that this approach does not capture the potentials of the new resources: “computational media both empower the mathematical processes involved in modelling activities by providing new “worlds” to explore and potentially shape the world we try to model.” (Doerr et al., 2017, p. 79). In other words, emphasis must be put on epistemic values just as much as pragmatic values. Though more realistic problems, requiring more advanced calculations, and larger amounts of data might favour the pragmatic value of computational media. Hence, the role of computational media should be central in our studies.

Greefrath et al. examined this in a study where performances by students who employed GeoGebra compared to those using pen and paper, did not differ. Though students' attitudes towards GeoGebra and self-efficacy related to the program, were predictors for their modelling competence. They conclude that technological tools "cannot simply be seen as a facilitator of learning mathematical modelling, at least not if the tasks are not changed." (Greefrath et al., 2018, p. 243). Thus, the need for analysing and understanding how we can design learning environments providing high-quality modelling education, which draw on "experimental materials and technology in modelling" (Carreira et al., 2019, p. 48) is still an open question. Schukajlow et al. (2018, p. 11) recommend authors in the future to "[...] take into account how technologies can be used for modelling or more generally what interaction between humans and media are meaningful [...]". This aligns with the claim of Artigue (2002) and the notion of instrumental genesis, which she argues works in two directions: instrumentalization and instrumentation. Trouche formulates the processes as: "the instrumentation process is the tracer of the artifact on the subject's activity, while the instrumentalization process is the tracer of the subject's activity on the artifact" (Trouche, 2020, p. 410), where the artefact can be DGS, CAS or others. In other words, when tools are used for pragmatic purposes, the students operate the tools to gain certain answers, which can be considered instrumentalization. When we identify instrumentation, it is often linked to more epistemic purposes, where students' thinking is shaped by the tool and the explorations done within the tool. Thus, to capture the needs formulated by Schukajlow et al. (2018) and Carreira et al. (2019) we might need to link instrumental genesis with mathematical modelling, as done by Geiger (2017). He discusses the role of instrumental genesis and teachers' orchestration through a technology rich modelling task capturing traits of authenticity, open-endedness and connectivity as characteristics of high-quality modelling education. He concludes that "by improvising and revising his approach to orchestrating students' learning the teacher promoted changes in students' schemas of instrumented action related to both the digital tool and also the task" (Geiger, 2017, p. 299). He continues by stating the need for further research on requirements for designing modelling problems, where technology acts as enabler for other design principles, implementation, and the learning of mathematical modelling.

To address the challenge of designing and implementing modelling activities, where technology acts as enabler, we propose to explore the potentials of the Anthropological Theory of the Didactic (ATD) in terms of media-milieu dialectics and the Herbartian schema when designing Study and Research Paths (SRP).

The media-milieu dialectics and Herbartian schema

Below we present a generating question, Q , initiating a SRP modelling total income for persons with different levels of education. The question invites students to study of the extra-mathematical and mathematical domain further. Thus, we consider the modelling activity a driver for learning mathematics as seen in (Jessen, 2017). This paper presents the a priori analysis of the design. This is one of the four main components of didactical engineering from the perspective of ATD (Barquero & Bosch, 2015). The a priori analysis explore the potential role of digital resources for modelling and afterwards we discuss how this might inform design principles. From an ATD perspective, the modelling activity unfolds as a dialectic between study and research processes initiated by Q . Students pose derived questions, which leads them to study some media. The new knowledge gained from the

media is reworked into partial answers to the derived, and hereby the generating question. This is the research process or reconstruction of knowledge, which takes place in the milieu (Jessen, 2017). The quality of the reconstructed knowledge is validated against the milieu. In ATD we use the Herbartian schema (Chevallard, 2008) to explain the media-milieu dialectic and identify factors affecting the dialectic. The schema describes the didactic system S working together for the development of an answer to the modelling problem Q :

$$S(X; Y; Q) \Rightarrow A^\heartsuit$$

X represents the group of students (which can be a singleton), Y represents the group of teachers supporting X 's study of the question Q . Y can be a group, a single teacher, y , or \emptyset in case of self-study. The \Rightarrow indicates that the system of $X; Y$ and Q bring into being a personal answer A^\heartsuit to the modelling question. A^\heartsuit is different from answers found in any resource, as it is the result of the personal or joined modelling process of a class. The modelling process in ATD is characterised as X and Y 's interaction with the milieu M , which is “a fuzzy and changing set of didactic “tools” of different kinds that X , acting under the supervision of Y , has to bring together (\Rightarrow)” (Chevallard, 2008, p. 2). The entire process can be depicted as:

$$[S(X; Y; Q) \Rightarrow M] \Rightarrow A^\heartsuit$$

$$M = \{A_1^\diamond, A_2^\diamond \dots, A_l^\diamond, W_1, W_2, \dots, W_m, D_1, D_2 \dots, D_n\}$$

The elements of the milieu are media which “designate here any representation system of a part of the natural or social world addressed to a certain audience” (Chevallard, 2007, p. 1, our translation). The A_i^\diamond denote existing answers being students' previously learned knowledge. This includes instrumented techniques and ability to draw on different digital resources. The W_j cover resources to be studied during the modelling process. This can be a page of a textbook, web searches of different nature (Khan Academy, encyclopaedia etc.). The D_k denotes data in various forms. Chevallard (2019) argues that data can be of quantitative or qualitative nature. Students might generate data (from experiments or simulations), they can be found in databases, or they can be part of the formulation of the modelling problem. Thus, the milieu of the Herbartian Schema is the media brought together with the purpose of nurturing the study of the modelling problem Q . Note that the milieu can have an element of a-didactic potential as known from the work of Brousseau (1997), where “a system of objects acting as a fragment of “nature” for Q , able to produce objective feedback about its possible answers” (Kidron et al, 2014, s. 158). Thus, for modelling problems, their real-world contexts become objects to be studied by students when developing and validating an answer A^\heartsuit .

We consider the notions of media-milieu and Herbartian schema tools for designing the rich environments for high-quality modelling education as suggested by Carreira et al. (2019). Below we draw on elements of didactical engineering from the perspective of ATD when we address the research question of this paper: How can media-milieu dialectic and Herbartian schema support the characterisation of the potential roles played by digital resources in the SRP on modelling?

A modelling problem on total income

The generating question for the SRP has been designed to introduce the notion of piecewise linear functions in Danish upper secondary mathematics in grade 10 building on the notion of linear function

from lower secondary mathematics. In Denmark the students are allowed to use CAS-tools for their written exit examination and to visit webpages previously used in class (Danish Ministry of Education, 2017). Therefore, the students are used to search all sorts of information online and familiar with basic uses of CAS-tools. The problem is formulated as:

Three friends completing lower secondary school in Denmark discuss their plans for further education. One is planning to get a job without pursuing further education. The two others plan to become a nurse (bachelor degree) and upper secondary teacher (master degree) respectively. They discuss when each of the different strategies have given the largest total income? Below you find a table of average income in Danish kroner (DKR), as result of years of completed education.

Table 1: The table of average income corresponding to years of education

Years of education	9	12	14	16	18	21
Average Income/year	210.000 DKR	310.000 DKR	365.000 DKR	370.000 DKR	490.000 DKR	520.000 DKR

The entries correspond to completing lower secondary, upper secondary, short further education, bachelor's degree, master's degree, and the PhD degree. The idea behind the problem is to create the need for piecewise defined functions and the specific notation linked to this. The teacher can enlarge the milieu by the end of the modelling process, introducing a work, W_j , presenting the notation. The idea of enlarging the milieu with intentions of the a-didactical situations has been reported on by Grønbæk & Winsløw (2015). Striving to promote self-study element on complex numbers for first year students, they designed media and milieus for students to explore. They argue for their design choice by stating "when students study and encounter an inference they do not follow, they are supposed to consider the text as a milieu that resources and constraints their efforts to fill the gap" (Grønbæk & Winsløw, 2015, p. 2132). The same idea has been explored by Jessen (2017), who explicitly designed a repository offered to the student to consult, when and if they encountered shortcomings of their existing knowledge and techniques. We will return to this idea later.

An a priori analysis of the SRP

As Grønbæk & Winsløw (2015) we here present the potential paths of students working with this SRP, not having a repository. We present the potential strategies, which serve as an argument for generating power of Q . Not all strategies require the use of digital resources. Basic pen and paper methods allow students to add income for each person. This requires knowledge about educational length for nurse and upper secondary teacher. This can be found online through webpages or educational guides. This represents data, D_i , brought into the milieu by the students. The strategy can be eased using spreadsheets as indicated in figure 1, where blue fields mark the years where the nurse's total income is largest, the orange is when the teacher exceeds both. This strategy draws on existing answers, A_i^\diamond , in terms of arithmetic, and instrumented techniques if using spreadsheets.

Some students might consider the spreadsheets tables, they can create scatter plots from. Some might experiment with linear regression using Excel, GeoGebra, etc. Both instrumented techniques and

existing answer, A_i^\diamond ... The tables can be considered self-generated data, D_l . When the digital tool plays the role of media it provides the students with answers. When exploring what function fits data, we consider the digital tool the milieu for exploration. Thus, the scatter plot might be yet another answer or experiments with different types of function. It depends on the context and the students discourse regarding the problem of fitting data. If new knowledge is reconstructed, we consider it research and the digital tool is the milieu supporting this.

Other students might recall the notion of linear functions, A_j^\diamond , identify the average income as the slope of a linear function. The x -axis represents time measured in years. When the linear function is expressed as $f(x) = ax + b$, where f is total income in thousands of DKR, they need to determine the constant b . For the person without further education, we have $b = 0$. In this scenario the students need to determine b for the nurse, where $f(6.5) = 0$, since it takes 6.5 years to complete nursing school. The slope of the function is 370, and students should solve the equation $0 = 370 \cdot 6.5 + b$. Which means $b = -2405$. This reasoning requires several notions and formulas (known answers, $A_1^\diamond, A_2^\diamond, \dots$) put together in new ways. The students' interaction with the milieu provides the students with new answers and further practiced for the teacher.

Some students might choose to use CAS-tools and the 'solve' command to do the same. Depending on their instrumental genesis, this might affect their conception of the models developed. The pragmatic use of the CAS-tool provides them with an answer to be studied, W_i , and incorporated with the models they are building. From here the intersection points can be found solving the equation where two functions are equal or using apps in the CAS-tool. This leads to the construction of different mathematical knowledge. The simple models of total income are presented at the left side of figure 2. When shared with classmates, the models become new answers brought into the milieu by students for their classmates to study. The different approaches will provide them with broader perspective on choices made during the modelling process.

For some students the real world will drive them to elaborate their models. Most students know that Danish government provide financial support to students after turning 18. The amount depends on whether you live at home or away from home, and the parents' income. These data, $D_i \dots$, are to be found online and to be use in the models describing the nurse and the teachers' total income. Other income from student jobs might also be included providing new pieces in each model.

This can be modelled by adding numbers by hand or in spreadsheets as an elaboration of the strategy shown in figure 1. Again, students might choose to experiment with scatter plots and linear regression for certain domains. Here the digital tools have epistemic value, when students' partial answer becomes objects to be explored further using notions, works and existing answers. Alternatively, they might try to use the strategy of being able to find the slope of each part of the function using the potential amounts of income (financial support, student jobs etc.).

Besvarelse 2

År	Arbejder	Sygeplejerske	Gymnasieleærer
9	0	0	0
9,5	105000	0	0
10	210000	0	0
10,5	315000	0	0
11	420000	0	0
11,5	525000	0	0
12	630000	0	0
12,5	735000	0	0
13	840000	155000	0
13,5	945000	310000	0
14	1050000	465000	0
14,5	1155000	620000	182500
15	1260000	775000	365000
15,5	1365000	930000	547500
16	1470000	1085000	730000
16,5	1575000	1240000	912500
17	1680000	1395000	1095000
17,5	1785000	1550000	1277500
18	1890000	1705000	1460000
18,5	1995000	1860000	1642500
19,5	2100000	2015000	1825000
20,5	2205000	2170000	2007500
21,5	2310000	2325000	2190000
22,5	2415000	2480000	2372500
23,5	2520000	2635000	2555000
24,5	2625000	2790000	2737500
25,5	2730000	2945000	2920000
26,5	2835000	3100000	3102500

Figure 1: spreadsheet solution

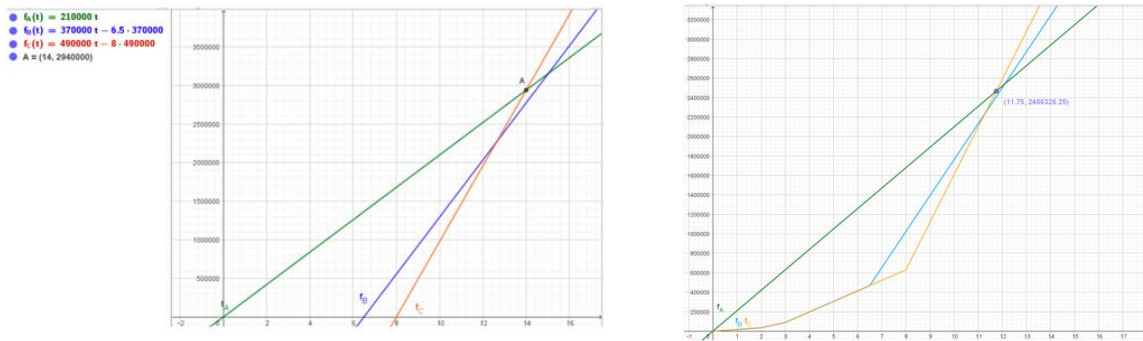


Figure 2: Two different solutions with linear and piecewise linear functions respectively

Then the strategies will be like those used to produce the models of the left side of figure 2 taking each domain into consideration. For this strategy the digital tools will mainly have pragmatic value. Though the tools allow the development of still more elaborate models to be done faster, using CAS-tools. This is valuable, when engaging students in continuously improving their models against the extra-mathematical context. Finally, when the teacher elaborates the milieu by introducing the notation for piecewise linear function and syntax for this using CAS-tools, the teacher provides the students with the a-didactical potential of formalising their previous work. Simple examples of models based on this new knowledge is presented at the right side of figure 2.

Not all students will be able to use this notation, why it is important to orchestrate validation of the different models by asking the students to compare and contrast their answers, helping them to realise which piece of the scatter plot is representing the blue fields from the spreadsheet and how is this represented in the formal notation of piecewise linear functions?

Discussion and concluding remarks

From the a priori analysis we can argue that media-milieu dialectic and Herbartian schema allow us to identify the roles played by digital resources in the SRP on modelling as being the provider of data, and new answers or work to be studied. In some cases, the tools might function as reminders of notions or tools taught previously. Also, the digital resources represent the perceived reality against which hypotheses, notions, newly constructed answers, and still more elaborated models are explored and validated. Moreover, SRP and the media-milieu dialectic seem to provide the students with the autonomy to enlarge the milieu or to choose known methods and answers as starting point for their modelling process. We can argue that SRP based modelling activities where the media-milieu dialectics are explicitly considered in the design process offers a richness in terms of number of strategies, students might pursue depending on previous learning. It allows students to experiment with data from various resources, and different uses of technology. We consider this the outset for high-quality modelling education as requested by Carreira et al., 2019. The notion of Herbartian schema naming resources as answers, works or data, which all can shift between being media to be studied or milieu employed in research processes can highlight changing roles that enrich modelling and enable us to harvest the pedagogical, psychological, cultural, and pragmatic aims of merging modelling and digital tools as Siller and Greefrath (2010) argued for. Jessen (2017) draw on media-milieu dialectics and Herbartian schema for the a posteriori analysis. If we do the same for technology rich modelling activities, we might be able to describe the potential meaningful interaction between

humans and media as asked for by Schukajlow et al. (2018). This resonates with the instrumental genesis approach adopted by Geiger (2017), though his framework does not capture episodes where the digital tools provide new knowledge in terms of search for data or works to be studied. The study process might be the most novel contribution to modelling activities: to explore the potential of small elements of direct instructions, brought into the milieu by the students. Therefore, when Doerr et al. (2017); Geiger (2017) and Greefrath et al. (2018) argue that the presence of technologies in mathematics education calls for a change of tasks designed for modelling activities to capture new potentials. We propose to consider the digital tools and technologies both as media and milieu facilitating students' exploration of modelling as a dialectic study and research processes. Thus, from a task design perspective, we need to experiment with ways to nurture both study processes and research processes. We need to explore how technology and different tools can nurture those processes. What is gained, what is lost in the shaping of mathematical modelling? And if the modelling functions as driver for learning mathematics, how does this approach shape the mathematical object constructed through the modelling process?

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