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Modelling while problem posing – A case study of preservice teachers

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Because problem posing might enhance activities that are necessary for solving real-world problems, it has the potential to foster modelling. However, systematic research on the connection between problem posing and modelling is largely missing. Therefore, in the present study, we investigated (1) the modelling activities that take place when posing problems that are based on given real-world situations and (2) the extent to which modelling activities occur with different problem-posing activities. To address these questions, we asked seven preservice teachers to first pose a problem based on a given real-world situation and then to solve their self-generated problem. A qualitative content analysis revealed that modelling activities that are close to the real-world situation (e.g., understanding, simplifying, and structuring the given pieces of information) are involved in problem posing. This result indicates that problem posing has the potential to foster mathematical modelling.

Keywords: Problem posing, modelling, teaching approach, preservice teachers.

Introduction

Modelling is one of the key competencies of mathematical learning as it enables students to understand their environment with the help of mathematics (Niss & Blum, 2020). However, modelling is a demanding process for both students and teachers alike (Schukajlow et al., 2018). Therefore, beneficial approaches for fostering mathematical modelling are needed (Schukajlow et al., 2018). Problem solving research has indicated that problem posing has a positive influence on problem solving because problem posing has been claimed to trigger important problem solving processes, for example, analyzing the given situation in an in-depth manner (Cai & Leikin, 2020). As modelling can be characterized as real-world problem solving and begins with a given real-world situation that has to be understood, simplified, and structured, it is possible that problem posing based on given real-world situations (i.e., modelling-related problem posing) is beneficial for fostering mathematical modelling. Surprisingly, there is only a little research on modelling through problem posing. To investigate the potential of problem posing for modelling, we aimed to analyze the connection between problem posing and modelling from a cognitive perspective in this study (Schukajlow et al., accepted).

Theoretical Background

Mathematical Modelling

Mathematical modelling can be characterized by a demanding process of translating information between the real world and the mathematical world with the goal of solving a real-world problem with the help of mathematics (Niss & Blum, 2020). The modelling process can be described as idealized in a circular theoretical model consisting of various activities (Blum & Leiß, 2007). The process begins with activities that are located in the real world. The given real-world situation, which is often presented as a textual description, first has to be *understood* by reading the text and supplementing the information with experience, thus ending up in the construction of an individual

situation model. In the next step, the situation model has to be transformed into the real model by *simplifying and structuring* the given situation. Through *mathematization*, the translation from the real world into the mathematical world begins. The real model is translated into a mathematical model or problem. By *working mathematically*, a mathematical result can be calculated, and then it has to be *interpreted* with respect how it applies to the real world so that a real result is achieved. Finally, the real result must be *validated* with respect to whether the existing models and results are appropriate for describing the given situation.

Each of the activities described above can be demanding for students and may represent a potential barrier in the solution process (Blum & Leiß, 2007; Schukajlow et al., 2018). Especially the activities located in the real world (understanding, simplifying, and structuring) are challenging as real-world aspects are often neglected (Krawitz et al., 2018). Therefore, there is a need for approaches that can help overcome these barriers.

Problem Posing

In recent years, problem posing has become an important topic in mathematics education as it has great potential for both the teaching and learning of mathematics (Cai & Leikin, 2020). Problem posing includes a variety of processes. In addition to the generation of new problems, it also comprises the reformulation of given problems that can take place before, during, or after problem solving (Silver, 1994). Further, different stimuli can initiate problem posing. In addition to the categorization of these stimuli on the basis of the structure of the given situation (Stoyanova, 1997), the stimuli can also be differentiated on the basis of their connection to reality. With regard to the differentiation of real-world problems (Blum & Niss, 1991), problem posing can be initiated by situations with and without a connection to reality. In our study, we define problem posing as the generation of new problems on the basis of given real-world situations before problem solving and refer to this type of problem in the following as modelling-related problem posing. An example of a given real-world situation is depicted in Figure 1. An exemplary question that can be posed based on the “Cable Car” situation is a question about the length of cable needed for the new cable car.

<p>Cable Car For more than 90 years, the <i>Nebelhorn</i> cable car has taken numerous guests up into the heights. Now it can go into well-earned retirement. Beginning in the summer of 2021, a new cable car will transport enthusiastic outdoor fans to the Nebelhorn mountain. The aim of the project is to avoid long waiting times, to provide seated transportation with an optimal view from every seat, and to increase carrying capacity.</p>	
<p>Technical data of the old cable car:</p>	
Model:	Großkabinen-Pendelbahn
Weight empty cabin:	1,600 kg
Weight full cabin:	3,900 kg
Height valley station:	1,933 m
Height top station:	2,214.20 m
Horizontal difference:	905.77 m
Speed:	8 m/s
Carrying capacity:	500 person/h
Power unit:	120 PS

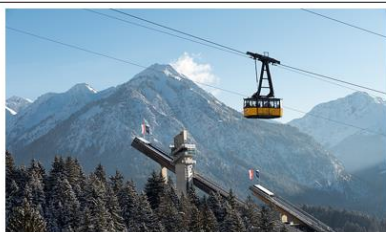


Figure 1: Real-world “Cable Car” situation

Mathematical Modelling in Problem Posing

The connection between mathematical modelling and problem posing can be regarded from two perspectives. On the one hand, questions can be raised during modelling (e.g., Barquero et al., 2019). On the other hand, modelling activities can be already involved in posing a problem. In the following we want to focus the latter connection. In out-of-school modelling scenarios, even though discovering or generating a problem typically takes place before the problem is solved, not much research has investigated the relationship between problem posing and modelling. On the basis of theoretical considerations, generating one's own problems can have a positive influence on the subsequent modelling process. To pose problems that are based on given real-world situations, the given situation has to be *understood* and *explored* with regard to possibilities for posing a problem by distinguishing relevant from irrelevant information and establishing relationships between the relevant information (Bonotto & Santo, 2015). Based on these relationships, possible mathematical problems can be *generated* and *evaluated* with respect to whether or not the given information is coherent and sufficient for solving the problem (Bonotto & Santo, 2015). The associated in-depth analysis of the situation might therefore already involve the modelling activities that are needed to construct an adequate real model and have benefits for the solution process (Hartmann et al., 2021). Empirical research on modelling-related problem posing has supported this assumption. Bonotto and Santo (2015) showed that students included real-world aspects in their solutions when solving self-generated problems using real-world artefacts as problem-posing stimuli (e.g., supermarket bills, restaurant menus). Further, empirical results indicate that the sequence of posed problems is guided by the problem-solving strategies that are typically employed (Cai & Hwang, 2002). Therefore, it is possible that problem posers are involved in *problem solving* while posing a problem that is based on given real-world situations by planning possible solution strategies. As part of planning a possible solution strategy, students might therefore already be engaged in the activities (e.g., structuring and mathematizing) that are needed to solve the self-generated problem. However, an open question is which modelling activities are already taking place during problem posing.

Research Questions and Method

The goal of the study was to analyze the connection between problem posing and modelling by investigating the occurrence of modelling activities in problem posing process. Therefore, we focused on the following research questions:

1. Which modelling activities take place when posing problems based on given real-world situations?
2. With which problem posing activities do the modelling activities co-occur?

Sample

To find an answer to these research questions, we conducted a study with seven preservice mathematics teachers from a large university in Germany (3 men, 4 women) between the ages of 20 and 26 years old ($M = 22.86$, $SD = 1.95$). Five of them participated in a program for a higher track secondary school teachers' degree and two of them for a middle track secondary school teachers' degree. All of them already had experience in solving modelling problems and six of them in problem posing. We used heterogeneity sampling to select preservice teachers with different mathematics

performance levels, with different levels of experience in problem posing and solving, and who were participating in different university programs.

Procedure and Instruments

To collect the data, we used a qualitative design that included thinking aloud and stimulated recall in order to get deep insights in cognitive processes. The preservice teachers were given three real-world situations and were asked to first pose a problem that was based on the given situation and after posing it to solve their self-generated problem while thinking aloud. We recorded both processes. To supplement the thinking aloud data, we conducted a stimulated recall for every posing and solving process. We used the recorded videos of their posing and solving processes, including their writing, speaking, gestures, and facial expressions, to stimulate the processes. As stimuli for problem posing, we used real-world situations as they are described in modelling problems and extended them by adding further authentic information to allow them to pose a variety of problems. An example of a real-world situation is displayed in Figure 1.

Data Analysis

For data analysis, we first transcribed the videos that had been recorded of the problem posing and stimulated recall and paraphrased the transcripts into sequences, each describing an activity in the process. Then, we analyzed the transcripts by using Mayring's (2015) content analysis. The coding scheme is based on the problem posing and modelling activities described in the literature, and the problem-posing activities were extended on the basis of the given material. The coding schema for the problem-posing activities and the modelling activities are presented in Figure 2. The activity of understanding is included in both coding schema and conceptualized in the same way.

Problem-Posing Activities		Modelling Activities	
Understanding	Comprehending and understanding the given situation and information using the description that is given about the situation.	Understanding	Comprehending and understanding the given situation and information using the description that is given about the situation.
Exploring	Discovering/ gathering relevant information to develop possible questions and organizing the information.	Simplifying/ Structuring	Simplifying and structuring the given real-world situation by differentiating between relevant and irrelevant information, identifying missing information, making assumptions on the basis of this information, and identifying possible solution steps.
Generating	Raising and formulating possible questions and defining a question.	Mathematizing	Translating the selected information into a mathematical model (e.g., table, term, diagram).
Problem Solving	Planning a more or less concrete way to solve the generated problem.	Working Mathematically	Performing the mathematical operations to generate a mathematical result.
Evaluating	Evaluating possible questions on the basis of individual criteria (solvable, meaningful, complete, appropriate formulation, difficulty, suitable for a particular target group).	Interpreting	Interpreting back the mathematical result with respect to the real-world situation and question.
		Validating	Checking models/results for plausibility and appropriateness with respect to the situation.

Figure 2: Coding Schemes for Problem Posing and Modelling Activities

All data were coded by the first author, and over 50% of the data were coded by a second well-trained rater. Interrater reliability was at least moderate for problem posing (Cohen's Kappa between $\kappa = .81$ and $\kappa = .95$) and for modelling activities (Cohen's Kappa between $\kappa = .76$ and $\kappa = .92$). To gain an overall picture to which extent the modelling activities are already involved in the problem posing process, we analyzed the transcripts regarding the duration (time of sequences) and the frequency (number of sequences) of the sequences assigned to individual modelling activities.

Results

To assess the occurrence of modelling activities in problem posing, we analyzed the frequency and duration of the sequences in which modelling activities took place while participants posed a problem (see Figure 3). The figure shows that all modelling activities except validation took place during the problem-posing process. However, there were strong differences in the frequencies of the individual activities. Simplifying and structuring could be identified most frequently in problem posing. Second most, understanding could be identified during problem posing, whereas mathematizing, working mathematically, and interpreting could be identified only rarely in the problem-posing process.

Regarding the duration of the individual activities, the overall picture was similar, but it was noticeable that the duration of understanding was significantly higher when compared with the number of sequences. Accordingly, the sequences to which understanding was assigned included a long duration. Overall, participants spent most of their time understanding, simplifying, and structuring, whereas they addressed the activities of mathematization, working mathematically, and interpreting for only very short periods of time.

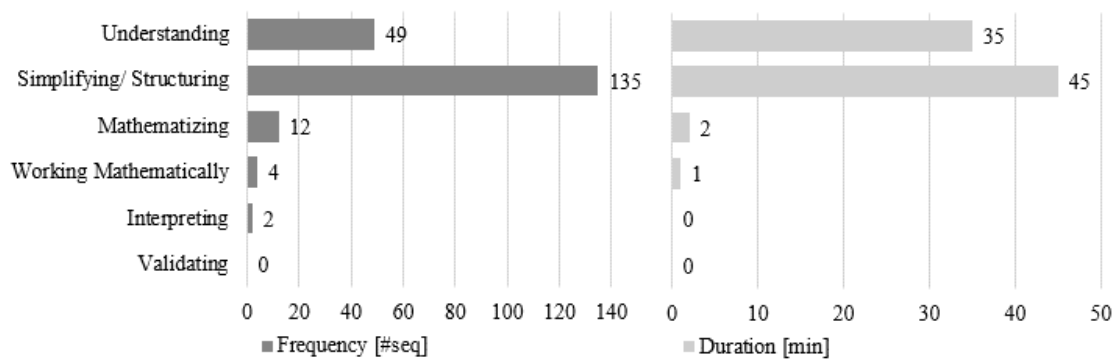


Figure 3: Frequency (left) and duration (right) of modelling activities during problem posing

To understand the connection between mathematical modelling and problem posing, it is important to analyze the co-occurrence of modelling and problem-posing activities. Table 1 presents an overview of the co-occurrence of these activities.

Table 1: Co-occurrence of problem posing and modelling activities

Problem Posing Modelling	Understanding	Exploring	Generating	Problem Solving	Evaluating	Σ
Understanding	49	0	0	0	0	49
Simplifying/ Structuring	0	94	6	14	21	135
Mathematizing	0	0	0	12	0	12

Working math.	0	2	0	2	0	4
Interpreting	0	1	0	1	0	2
Validating	0	0	0	0	0	0

Overall, the results revealed that the modelling activity of understanding exclusively occurred with the problem posing activity of understanding. Simplifying and structuring took place most often when participants were exploring the given situation. In the following excerpt, Max is exploring the given real-world situation by filtering relevant from irrelevant data.

Max: Let's take another look at the data for the old Nebelhornbahn. Um, I find the large-cabin aerial tramway rather irrelevant now. Weight empty cabin 1,600 kg and full cabin 3,900 kg.

However, simplifying and structuring also occurred during generation, problem solving, and evaluation. In the context of generation, simplifying and structuring occurred while participants were making assumptions or mentioning relevant information for solving the problem in the formulation of the self-generated problem. For example, in the following excerpt, Nina supplemented her self-generated problem (*What is the best way to shorten the waiting time for the new cable car?*) with the information that the number of people and the speed should be taken into account when solving the problem.

Nina: (*Supplements problem*) Consider the number of people and speed. Something like this.

During problem solving, simplifying and structuring occurred when participants were planning the solution steps they would follow to solve the self-generated problem. In the following excerpt, Max is planning a possible solution for his self-generated problem.

Max: But then we also have the travel speed of 8 m/s. This means that one could theoretically also determine the travel time if we have the length of the route. How long the cable car needs from one station to the next. That would be the next solution step so to speak.

In the context of evaluation, the possibility of solving the self-generated problem is evaluated by checking whether all the information is given. For example, in the following excerpt, Lea evaluated her posed problem by identifying the information that was relevant for checking whether all the information needed to solve the problem was given in the situation.

Lea: Because we know how fast it is, we know where it starts, we know how it's going, and we can say that it's just going straight, so it's kind of going up as a linear function; Then you could/This is a nice question.

Mathematizing came up exclusively for problem solving and working mathematically, and interpreting occurred during exploration and problem solving.

Discussion and Conclusion

In the present study, we investigated the connection between problem posing and mathematical modelling and in particular the occurrence of modelling activities during modelling-related problem posing. The analysis of the problem-posing processes of seven preservice teachers revealed the

involvement of nearly all modelling activities. We found that especially the modelling activities of understanding, simplifying, and structuring were already involved in posing a problem that was based on given real-world situations. Understanding was involved in problem posing because it is an essential part of both conceptualizations (problem posing and modelling). Simplifying and structuring are similar to the problem-posing activity of exploring as they are aimed at analyzing the given situation in an in-depth manner, and they typically co-occurred with this activity. Hence, while posing a problem, a situation and real-world model might already be developed. Mathematizing is less involved in the problem-posing process, but it typically co-occurred with the problem-posing activity of problem solving. Problem-solving activities while posing a problem may help problem posers to plan possible solution steps by creating a partial mathematical model. The other modelling activities (i.e., working mathematically, interpreting, and validating) occurred only rarely or not at all, and therefore, these activities might not be triggered by posing a problem.

Our results indicate that especially the modelling activities located in the real world occur while posing a problem. Consequently, problem posing might stimulate an in-depth analysis of the context, something that is important to do for modelling. This result contributes to a theoretical model of the relationship between problem posing and modelling, and it needs to be examined in future studies. As the modelling activities in the real world represent a major cognitive barrier (Krawitz et al., 2018), posing a problem with respect to a real-world situation might help problem solvers overcome these cognitive barriers. Some indication about the importance of problem posing for modelling activities in the real world comes from results in a study by Bonotto and Santo (2015) who found that after problem posing, students often considered real-world aspects of the problem. Further research should investigate how self-generated problems are solved and whether problem posing can affect modelling performance. As a practical implication, our results suggest that modelling-related problem posing could be an innovative and fruitful approach for teaching modelling activities that are located in the real world.

Our study has some limitations that we want to acknowledge. We used a qualitative research approach with a small sample to describe the connection between problem posing and modelling in an in-depth and detailed manner. Due to the design, we could make only hypothetical generalizations, which must be verified in future studies. Further limitations result from the real-world situations we used. These limitations should be kept in mind when interpreting the results of the study. Despite the listed limitations, the study allowed us to contribute to research on modelling by qualitatively exploring the connection between problem posing and modelling. We conclude that there is a close relationship between problem posing and modelling and that, therefore, modelling-related problem posing has great potential for fostering modelling.

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