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# Professional content knowledge of mathematical modelling of pre-service teachers: First considerations for a new test instrument

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*Modelling competence is a central concept in research on teaching and learning mathematical modelling. Numerous studies support the theoretical foundation of modelling competence as well as the existence of tested and already modified test instruments for different age levels. In addition to other aspects of professional competence, such as pedagogical content knowledge, pre-service teachers must also possess modelling competence themselves. Even though test instruments for modelling competence already exist, the aspect of professional content knowledge of mathematical modelling as a competence facet of teachers has received less attention thus far. In this theoretically oriented paper, first theoretical considerations for the development of such a test instrument are presented on the one hand, and on the other hand, ideas for further development are discussed based on sample items.*

*Keywords: Professional competences, content knowledge, test instrument, mathematical modelling.*

## Introduction

The importance of pedagogical content knowledge in mathematical modelling teacher education has already been highlighted as significant, but only more systematically in recent research. Therefore, a focus will now also be placed on the content knowledge of modelling. In this paper, we mainly present theoretical considerations regarding the content knowledge of modelling in the overall context of teacher education on modelling. In addition, we present examples of concrete items. Accordingly, we would like to address the challenge of developing a test instrument, which will differ from already existing tests in the still pending development process.

Therefore, we first review the following theoretical foundations, which form a theoretical framework for us as a basis for the systematic construction of items. This concerns professional competence in the models of Kunter et al. (2013) and Shulman (1986). Especially regarding Krauss et al. (2013), we derive our considerations of levels of mathematical expertise, which we concretise with respect to calculus, with which we start as a mathematical subject area with item construction. Relevant aspects of modelling sub-competencies follow, as well as a discussion of existing test instruments. Finally, the item examples illustrate first considerations and show the challenge of construction. The main goal of this paper is to demonstrate we need a new test instrument for the content knowledge of mathematical modelling in teacher education, including which aspects of test construction need to be considered.

## **Professional competence**

The professional competencies of mathematics teachers and pre-service mathematics teachers have been extensively studied (e.g. Blömeke & Delaney, 2012; Kunter et al., 2013). The concepts of professional competence developed in these studies share key commonalities. In particular, professional knowledge is seen as consisting of different knowledge domains: content knowledge, pedagogical content knowledge and pedagogical–psychological knowledge (Shulman, 1986). In addition to these cognitively oriented knowledge dimensions, affective aspects are also considered. Further conceptions of pedagogical content knowledge for mathematics teachers, some of which also include content knowledge and pedagogical–psychological knowledge, are relevant (Depaepe et al., 2013). The importance of teachers’ content knowledge is highlighted in various studies (Blömeke & Delaney, 2012). The high correlation of content knowledge and pedagogical content knowledge is well known (Krauss et al., 2008; Yang et al., 2021), highlighting the importance of the content knowledge of pre-service teachers in particular.

## **Levels of mathematical content knowledge**

Mathematical content knowledge can be described using different models. Based on Shulman (1986), Krauss et al. (2013) proposed four levels to describe mathematical content knowledge. They distinguish everyday mathematical knowledge, the mastery of school-level mathematical knowledge, a deep understanding of the content of the secondary school mathematics curriculum and university-level knowledge (Krauss et al., 2013, p. 155). This classification begins with a first level of mathematical content knowledge that all adults must possess. The second level is school knowledge. At this level, we consider competence to use mathematics in the context of the knowledge usually taught at school. This *school mathematical competence* goes beyond everyday mathematical competence and includes, for example the competences required to complete a task in the Abitur examination (KMK, 2012). A third level describes the mathematical content knowledge required for a deeper understanding of the subject content at the secondary level. We refer to this level as *in-depth school mathematical competence*. This includes elementary mathematics from a higher standpoint (Klein, 2016) as taught at university. The fourth level can be called *university competence*. This includes mathematical knowledge taught at university with virtually no connection to the school curriculum, for example algebraic number theory (Krauss et al., 2013). Depending on the course of study, *in-depth school mathematics competency* or university mathematics competency are achieved at university in a teaching degree with mathematics as a subject. Content knowledge, as conceptualised in COACTIV, lies between the second and the third level, as shown in the published example item ‘Is  $2^{1024} - 1$  a prime number?’ (Krauss et al., 2013, p. 152).

## **Modelling cycle and sub-competencies of modelling**

Mathematical modelling processes can be illustrated by a modelling cycle that depicts the sub-processes or sub-competencies of simplifying, mathematising, interpreting and validating. Former research describes many cycles that vary mainly in the number of individual sub-processes (Borromeo Ferri, 2006). The modelling cycles from applied mathematics (Pollak, 1977) describe the processes deterministically in three or four phases. In didactically oriented discussion, there are four-phase, but also six- and seven-phase cycles (Blum & Leiß, 2007) or extended cycles concerning the

use of technologies (Greefrath et al., 2018). Regardless of the number of sub-processes, modelling competence is required. In this context, modelling competence is understood as the ability to identify a real-world problem in a given situation, translate it into mathematics and interpret and validate the solution to the corresponding mathematical problem about the given situation (Niss et al., 2007). *Global modelling competence* refers to the entire modelling process and to general competences, such as a structured and goal-oriented approach to tasks, reasoned argumentation and independent reflection on the modelling process (Kaiser & Brand, 2015). The *sub-competencies of mathematical modelling*, however, refer to the sub-processes in the modelling cycle mentioned above or identified in the various cycles (Maaß, 2006).

The assessment of students' modelling competencies using tests has already been empirically demonstrated in many studies for different age groups (Haines et al., 2001; Hankeln et al., 2019; Kaiser & Brand, 2015). Two basic principles can be distinguished. On the one hand, there are items that focus on the sub-processes of modelling (atomistic approach) and, on the other hand, items that require the complete pass through of a modelling cycle (i.e. a holistic approach; Blomhøj & Jensen, 2003). In our test construction, we prefer an atomistic approach and limit ourselves to the sub-competencies simplify, mathematise, interpret and validate.

### **Mathematical modelling and analysis**

In his characterisation of applied mathematics and modelling, Pollak (1977) attributed early special importance to analysis. To work on a modelling task, a knowledge of mathematical content is required in addition to process-related competences, which are described within the framework of the German educational standards for the Abitur examination. The central subject here is calculus (KMK, 2012).

Textbooks often contain examples from calculus as modelling tasks, for example for modelling growth processes. These tasks are particularly suitable for students because of their accessibility through a reference to real life and use of a well-known mathematical topic. With the help of differential calculus, many real or application situations can be modelled, such as topics in the context of traffic (Siller, 2013). Simultaneously, calculus is a central subject area in the German Abitur examination and in mathematics teacher education; it is receiving special attention in research (Rach & Heinze, 2017).

### **Development of a test instrument**

Previous test instruments on university students' modelling-specific mathematical content knowledge use either rather clear Level 2 mathematical content knowledge for modelling, that is the school level (Yang et al., 2021), or Level 3 to Level 4, that is university knowledge or knowledge clearly beyond the secondary level (Czocher et al., 2021; Haines et al., 2001). While Level 2 may seem too low for pre-service teachers in higher semesters, Levels 3 to 4 may overwhelm some pre-service teachers. Sample items from the test of content knowledge from COACTIV could be used as a guide for items on modelling specific content knowledge between Levels 2 and 3.

Based on the previous considerations, we design an *atomistic* test instrument for the modelling-specific content knowledge of pre-service teachers with subject-specific content at *Levels 2 to 3* from

the subject area of *calculus* and the focus on one of the four *selected sub-competencies*: simplify, mathematise, interpret and validate. Four example items will illustrate this direction in the following.

**Example item 1 (mathematising)**

Spruces represent an important timber species in Germany. The temporal development of the thickness of spruces is modelled by a function  $d$ . The planted seedling has a diameter of 0.04 m. After 160 years, a spruce has typically reached a diameter of 0.96 m. Which of the following mathematical models best fits the problem described?

$$d(t) = \frac{1}{1 + e^{-0,04(t-80)}}$$



$$d(t) = 0,00575 t + 0.04$$



$$d(t) = 0,04 e^{\frac{1}{50}t}$$

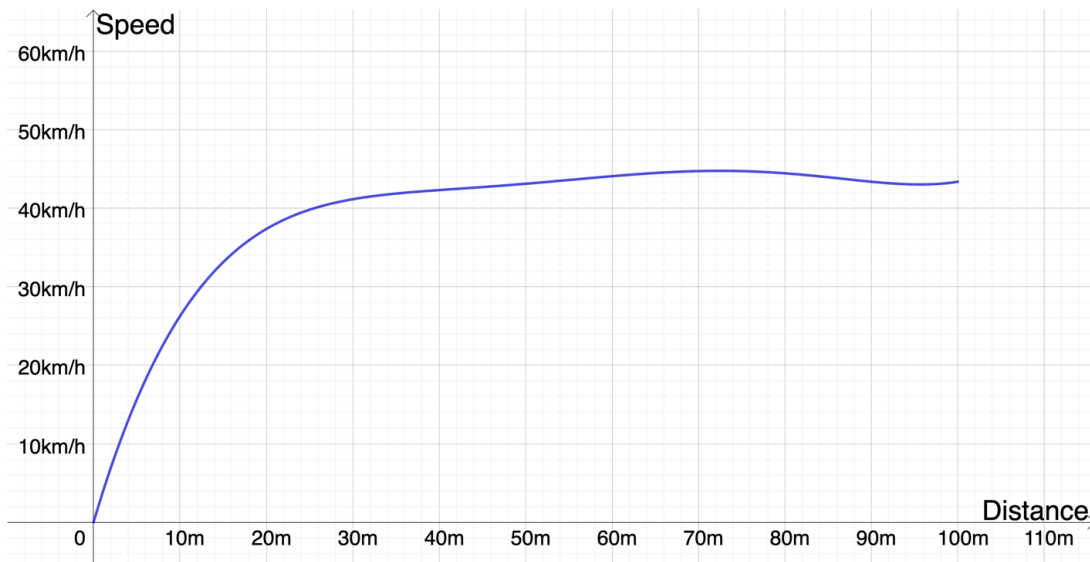


$$d(t) = \frac{1}{e^{-0,04(t-80)} - 1} + \frac{t}{80}$$



**Example item 2 (interpreting)**

In 2009, Usain Bolt set a new world record over 100m running with a time of 9.58s. In the figure, the course of his speed during the world record race is approximated.



Describe the progression of Bolt's speed during the race, considering the real-world context of the record run.

**Example item 3 (interpreting)**

Tim and his friends are standing on a federal highway in the slow-moving traffic that has seemingly formed ‘out of nowhere’. Caro is annoyed: ‘These damn 70 zones! If they would lift the speed limit here, then everyone could drive faster, there wouldn't be so many vehicles piling up, and we'd get there sooner!’

Tim has come up with the following ‘half-speed rule’:  $d = \frac{v}{2}$ :

- $v$  = speed
- $l$  = vehicle length
- $d$  = distance between two vehicles
- $S$  = route of the convoy
- $D$  = vehicle throughput per hour

$$D = \frac{S}{l+d} = \frac{1000 \cdot v}{l+d} = \frac{1000 \cdot v}{6+d} = \frac{1000 \cdot v}{6 + \frac{v}{2}}$$

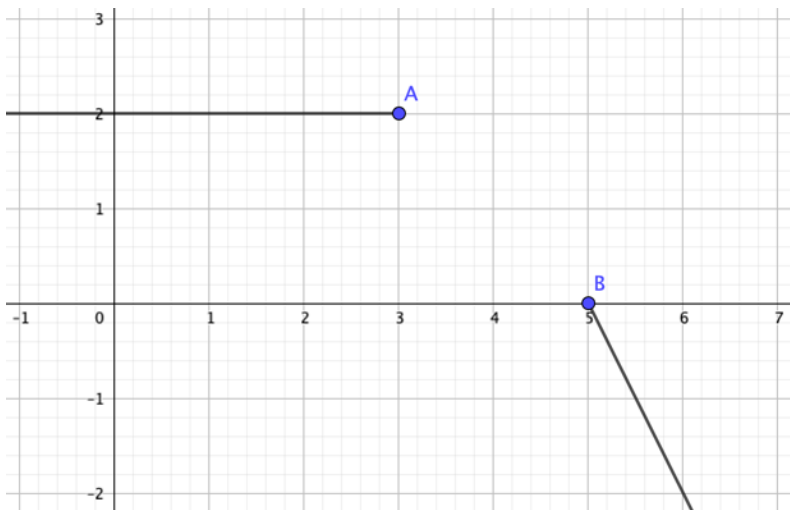
Tim achieves an approximate vehicle throughput of 1,600 vehicles per hour.

What did Tim calculate with his result?

- The throughput grows infinitely with increasing speed.
- The risk of rear-end collisions increases due to a guideline speed of 50km/h.
- The throughput grows limited with increasing speed.
- The maximum vehicle throughput is only achieved at a higher speed.

#### Example item 4 (validating)

The shape of a road between two existing roads should be modelled mathematically. The roads can be presented in the coordinate system as follows.



The missing piece of the road between A and B should be modelled mathematically. The model chosen was the appropriate arc of the function  $f(x) = \sqrt{4 - (x - 3)^2}$ . Explain why this result would not be used by engineers in reality.

#### Discussion and Conclusion

Previous research shows that there is a need for a special test instrument on professional content knowledge in mathematical modelling, which is, on the one hand, more demanding in content than average modelling tasks for school students and, on the other hand, below the level of university students who study mathematics as a major (cf. tests by Czocher et al., 2021; Haines et al., 2001). Furthermore, it is desirable that sub-competencies of modelling are considered atomistical, so a separate consideration of sub-competencies becomes possible. In this case, there is already experience with modelling tests for students (e.g. Hankeln et al., 2019; Kaiser & Brand, 2015), whereas in Haines et al. (2001), there is a focus on the first steps of the modelling cycle and no aim at a distinction between sub-processes. Another aspect concerns the common mathematical content of teacher education. Here, several items from the well-established test instrument of Haines et al. (2001) seem less suitable, as they mainly target linear optimisation. Therefore, we focused on calculus as a well-

known subject area from upper secondary education and developed items for (pre-service) teachers that go beyond the content of lower secondary education, such as in Yang et al. (2021).

Example item 1 refers to finding a suitable mathematical model in the context of exponential functions. Hence, it is an important sub-area of calculus that is also covered in school. Not all the models given (e.g. logistic growth) are typically school subjects, but go beyond this. Therefore, this test item is to be placed between Levels 2 and 3 of mathematical knowledge. Basic experience with (plant) growth is sufficient to identify the appropriate mathematical model. Based on the problem, the participants can consider which type of growth is appropriate for this problem. The possible answers include linear growth, exponential growth, and logistic growth. A backward validation by inserting values is not necessarily goal-oriented because the terms available for selection correctly represent the boundary points in each case. The mathematical properties of the function must therefore be recognised from the properties of the real problem. The item, then, targets mathematising. For an objective evaluation, a multiple-choice task is recommended here. The task format is like well-known tasks by Haines (2001), but the level of difficulty is adapted to the target group.

Example item 2 describes a world record race using a graph of a function. This graph represents speed and needs to be interpreted. This belongs to the subject area of calculus and can be assigned to secondary level II. Like example 3, it belongs to interpreting, because the results of a mathematical model need to relate to the respective situation. In example item 3, the mathematical content goes beyond the school material. Example item 4 addresses validation. Students must critically examine the mathematical model used (Item 4) in the context of the real-world situation. Item 4 addresses a familiar high school context with a functional equation not usually used in that context. Further additional items for simplification can be taken from the test by Haines et al. (2001), if necessary. A broad coverage of sub-competencies seems important to us to obtain a valid measurement instrument that considers all steps of the modelling cycle for pre-service teachers.

All items, therefore, can be assigned to the subject area of calculus and use school material from upper secondary school or surpass it. The mathematical requirements, then, are implicit. It should be discussed whether explicit items on working mathematically should also be included. In various test instruments for students (Hankeln et al., 2019), this has not been done. The reason in this case was that the processes of modelling were the focus of the study and the test length should remain within certain limits. The question of the test length is also to be discussed here. Other test instruments (Kaiser & Brand, 2015), however, consider holistic tasks and implicitly include mathematical work. However, we also consider it attractive for teacher educators to use items precisely for certain sub-competencies, so that a very focused diagnosis is possible.

Some of the items developed thus far can be objectively coded using multiple choice items, while others are evaluated using criterion-guided coding manuals in the partial credit model. In addition to the items presented here, further items for a complete test of professional content knowledge for modelling need to be developed and empirically tested. A prerequisite is a discussion of the competencies to be tested (atomistic or holistic), level of difficulty, item format and mathematic subject area.

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