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► **To cite this version:**

Sevinç Göksen-Zayim, Derk Pik, Carla van Boxtel. An observation instrument to analyze the collaboration between students while solving mathematical modelling problems. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03759012

**HAL Id: hal-03759012**

**<https://hal.science/hal-03759012v1>**

Submitted on 23 Aug 2022

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# **An observation instrument to analyze the collaboration between students while solving mathematical modelling problems**

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*Collaborative learning is a well-established approach to elicit reasoning. The ability to solve mathematical modelling problems depends much on the ability to reason, because mathematical modelling problems are usually presented to students through texts. Decisions have to be made at all stages of the mathematical modelling cycle, for instance on assumptions, simplifications and feasibility of the model. In this paper we present an observation instrument to study collaborative modelling. There are three core components: collaborative learning, mathematical modelling and the language that students use while working together.*

*Keywords: Mathematical modelling, collaborative learning, observation, language proficiency.*

## **Introduction**

In the Netherlands, mathematical modelling has become a small but obligatory part of the upper secondary curriculum. However, it is missing in the greater part of lower secondary education. To improve the vertical coherence in the curriculum, our research focuses on modelling assignments in lower secondary education.

The purpose of our larger research is to determine whether the open-ended group assignments we developed promote collaborative mathematical modelling. In this study, we aim to construct an observation instrument. The research question we address is: How do we examine the quality of students' collaborative learning in tasks that aim at mathematical modelling?

## **Theoretical framework**

Many studies have shown that collaborative learning has a positive effect on students' mathematics learning (e.g. Dekker & Elshout-Mohr, 1998, 2004; Pijls, 2007; Webb, 2009; Webb et al., 2020; Yackel et al., 1991). Discussions in small groups can also promote the development of modelling competencies (Maaß, 2006; Galbraith & Clatworthy, 1990). We discuss three features of the interaction between students, that - based upon the literature - can be considered relevant when aiming at mathematical modelling with group tasks: (1) collaborative learning, (2) mathematical modelling and (3) the language that students use while working collaboratively on the modelling task.

*Collaborative learning:* Various researchers have shown that collaborative learning in heterogeneous groups raises the level of abstract reasoning as suggested by van Hiele and Freudenthal when the task enhances students to verbalize their understanding, to explain their thinking to one another and to criticize one another's way of thinking (Calor et al., 2019; Dekker & Elshout-Mohr, 1998; Palha, 2013; Pijls, 2007; Yackel et al., 1991).

Dekker and Elshout-Mohr (1998), for example, defined four key activities in their Process Model for structuring small group discussions of students when working at a mathematical task. Initially the

students are working individually on the same mathematical task. Their works will be different. After some time, they compare the findings: they show their own work, they explain their own work, they justify their own work and eventually reconstruct their own work.

Collaborative learning activities that have been identified as key activities for mathematical level raising, such as explaining and justifying, have also been mentioned by scholars who focus on features of productive collaborative learning (Mercer, 2006; van Boxtel et al., 2000; van der Linden et al., 2000). According to Mercer (2006) three types of talk can be discerned: disputational talk, cumulative talk and exploratory talk. Disputational talk is characterized by disagreement and individualized decision making, cumulative talk takes place when students share knowledge but contributions are uncritically accepted by the group. In exploratory talk students engage critically and constructively with each other's ideas.

*Mathematical modelling:* In the field of mathematical modelling various definitions of the modelling cycle are found. The modelling cycle starts with a situation or problem in the real world. Thus it is important to understand the situation. Blum and Leiß (2005) define this understanding of the original situation as the situation model. Then, the given situation has to be simplified, structured and made more precise, which results in a real model (Blum & Leiß, 2005). In the first step of the modelling cycle it is essential to become aware of the meaning of the problem. Furthermore, modelling encompasses various activities that are carried out consecutively, including mathematising, working mathematically, interpreting, and validating (Blum & Leiß, 2005). Maaß (2006) identifies specific sub-competencies related to the mathematical modelling process and argues that modelling competencies include more than the steps of a modelling process. Assumption making is one of these sub-competencies to understand the real problem and to set up a model based on reality.

*The use of language:* The language used at school often forms an obstacle to learning mathematics (Van Eerde & Hajer, 2009). Plath and Leiß (2018) point out that a low language proficiency may result in comprehension problems and a low performance on modelling tasks.

Mathematical modelling problems are usually presented to students through texts in context-rich assignments. In the context of modelling tasks, it is important to examine what language students use while modelling. Cummins (2000) distinguishes between Basic Interpersonal Communicative Skills (BICS) for communication every day and Cognitive Academic Language Proficiency (CALP) for communication in education. During collaboration, students may explain difficult words to each other, and use mathematical terminology during their collaborative work. Webb (1991) classifies this as the explainer: the learner who translates unknown vocabulary into language that is known to other students. Especially for modelling tasks it is important to understand the situation of the given problem (Blum & Leiß, 2005).

## **Method**

In this study, a series of five different modelling tasks for the domains algebra and geometry have been developed for grade eight students (age 13-14). In the Netherlands, grade eight is part of lower secondary education. The modelling tasks were developed according to design principles that we derived from Galbraith (2006) and Geiger et al. (2021) and literature on collaborative learning.

For collaborative learning, a task should be complex and rely on multiple skills (van Boxtel et al., 2000; Cohen, 1994). Furthermore, the designed modelling tasks are open-ended problems that require making the necessary assumptions, are linked to the real world and motivate students (Galbraith, 2006; Geiger et al., 2021). The tasks were improved using feedback from educational experts, mathematics teachers and a pilot with grade eight students.

In total, five secondary schools with ten classes and seven mathematics teachers, located in an urban environment participated in this research. The collaborative learning groups consisted of three students. Each group worked on one task during one lesson. Three or four randomly selected groups in each class were video-taped and audio-recorded, and the written group products of all groups were collected. In each class, the tasks were introduced with the same introduction and brief information about mathematical modelling.

To analyze students' verbal interaction while collaborating on mathematical modelling tasks, we need a valid and reliable observation instrument. This observation instrument was designed in three steps: (I) development of the instrument based on theory about collaborative learning and mathematical modelling and recordings of three randomly selected groups; (II) a pilot study in which we used six randomly selected video-recordings of group conversations to further operationalize the categories in the instrument and to investigate the reliability of the instrument; (III) adjustment of the instrument.

## **Results**

*First design of the observation instrument:* Recordings from three randomly chosen groups were selected to design a first version of the observation instrument. A variety of two tasks was included. We defined three main categories of student interaction while working collaboratively on mathematical modelling tasks, that can be considered relevant: (1) collaborative learning, (2) mathematical modelling and (3) the language that students use while working collaboratively on the modelling task. In addition, to improve the designed open-ended group assignments, it is important to examine questions students ask the teacher about the modelling task. This additional category is named questioning. Furthermore, for each subcategory, we added space to note comments.

*Collaborative learning:* It is important to discern whether students work primarily individually or together at our tasks. Therefore, the first subcategory in the main category collaborative learning is collaboration. The second subcategory is critical considerations focused on the solution strategies (critical considerations and strategies). The next four subcategories are the four key activities of Dekker and Elshout-Mohr's (1998) Process Model. The last sub-category is the type of discussion. According to Mercer (2006) three types of talk can be discerned: disputation talk, cumulative talk and exploratory talk.

*Mathematical modelling:* Mathematical modelling encompasses various activities that are carried out consecutively. For these activities we make use of the modelling cycle of Blum and Leiß (2005). The steps in the modelling cycle are sequentially added as subcategories to the main category mathematical modelling of the observation instrument. In the first step of the modelling cycle it is essential to get aware of the meaning of the problem. Therefore, we first focus on whether students have understood the problem by observing whether students clarify the purpose of the modelling task (clarifying). In the second subcategory we observe if students discuss how to tackle the problem

(addressing). In the third subcategory, we observe whether students understand the problem and if they transform the problem for reasoning (transforming). The fourth subcategory, we observe whether students simplify or structure the problem (simplifying). In the next subcategories we observe the assumption making, mathematizing, working mathematically, the way students are solving the problem (solving strategies), interpretation of the solution (interpreting) and whether students validate (in between or at the end) throughout the modelling process (validating).

*The use of language:* In the first subcategory we observe the use of everyday language (BICS). To find out whether the designed tasks contain difficult words for the students, we observe whether students explain certain words to each other while collaborating on the modelling task. In the second subcategory we observe the extent to which students use mathematical terms (CALP). To support coding, a list has been added with the most common mathematical concepts for the designed modelling tasks. In the third subcategory we observe whether students explain the mathematical concepts to each other (CALP to BICS). The last subcategory focuses on students' use of modelling language. We investigate whether students formulate their solution in the context of the modelling task (formulating) and whether they use specific modelling language in the solution process.

Questions about the task: The last category is added to examine if the students are asking the teacher a question and to write down the question(s) asked (questioning).

*Results of the pilot study:* Six randomly chosen video-recordings were coded by two researchers (the first two authors). The percentage of agreement between the researchers are shown in Diagram 1. For the main category collaborative learning the subcategories with the least agreements are: explaining their own work, justifying their own work and reconstructing their own work. For the main category mathematical modelling the subcategories with the least agreements are: clarification of the purpose, assumption making, mathematization and validation. For the main category use of language the agreement is acceptable. This is also for the additional category: questioning.

*Revision:* As a result of these findings, the categories with a low percentage of agreement, were revised. Clarifications have been added to the key activities. For example, what is meant with to show, to explain, to justify and to reconstruct. The fourth category, with subcategory to explain, of the observation instrument is shown in Table 1.

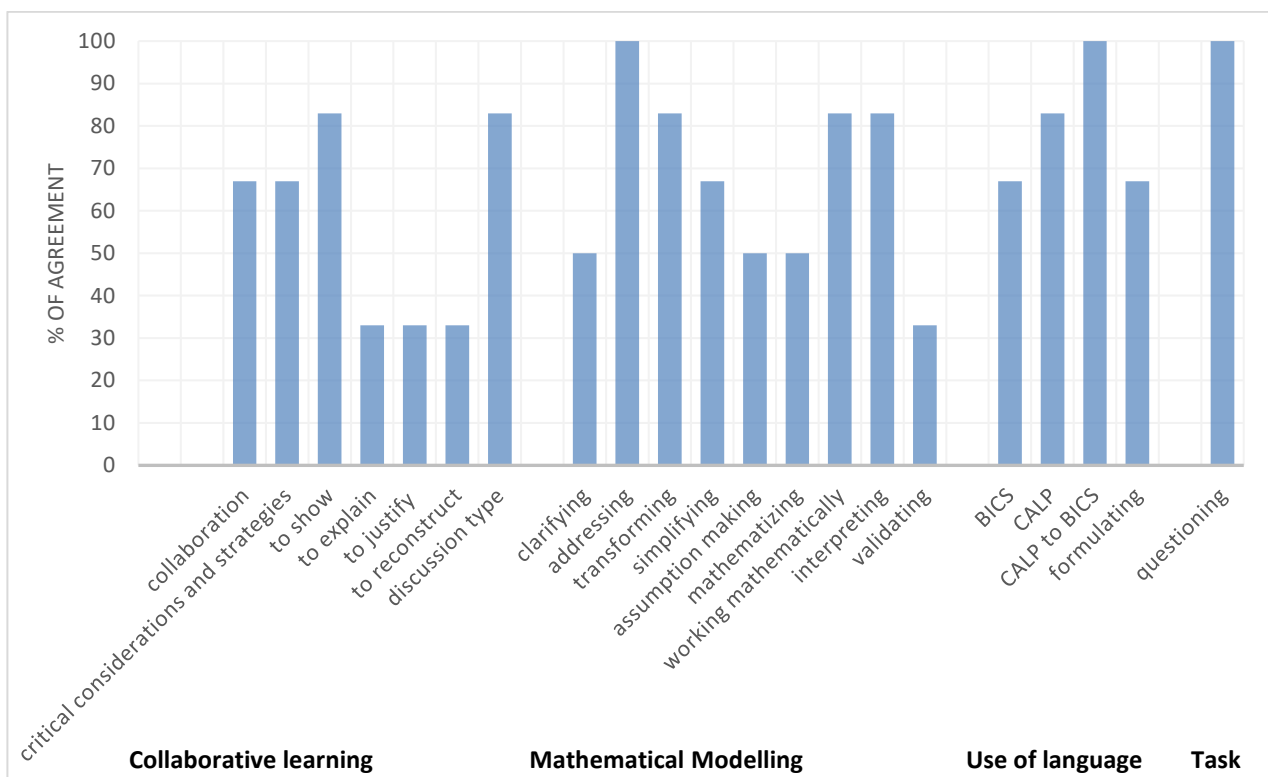
**Table 1: Fourth subcategory of the observation instrument for main category collaborative learning**

Collaborative learning	X	Notes
<b>4. To Explain</b>		
Explaining or clarifying the way of working or thinking		
A. No student explains his way of working or thinking to the rest of the group		
B. One student explains his way of working or thinking to the rest of the group		
C. Several students explain their way of working or thinking to the rest of the group		

An explanation has also been added to the main category mathematical modelling, subcategories making assumptions and validating. In the main category collaborative learning, critical considerations focused on solution strategies have been adapted into two subcategories. Critical considerations students make with each other while collaborating is still a subcategory of collaborative learning. Solution strategies have been moved to the category mathematical modelling, because it fits more in the processes for solving modelling problems. With this category we examine whether students apply one or more strategies to solve the modelling problem.

We also adjusted the sequence of the subcategories in the main category mathematical modelling. In the revised instrument, we started with the subcategory clarification of the purpose of the task, followed by addressing the problem, understanding and simplifying the problem, and assumptions making. After that, the findings of a mathematical model or mathematization were noted, followed by mathematical solution, interpretation and validation. As discussed above, another subcategory had to be added, namely the solution strategies. As a result, the revised observation instrument included ten subcategories for mathematical modelling. By choosing an order that best suits how students work through the task, it becomes easier to observe.

No adjustments were made for the components: language proficiency and question to teacher.



**Diagram 1: Percentages of agreement in each subcategory for the pilot with the observation instrument (six collaborative learning dialogues)**

## **Conclusion and discussion**

In order to evaluate the quality of students' conversation, we have developed an observation instrument with which we can analyze the students collaborative learning while solving mathematical modelling problems. We observe the data on basis of three main categories. The first category is collaborative learning of students. We are using the Process Model (Dekker & Elshout-Mohr, 1998) to observe how students work together to increase their mathematical understanding. The second category is mathematical modelling, using the phases of the modelling cycle (Blum & Leiß, 2005). The last category is the language that students use while working collaboratively on the modelling task. We observe the way students explain difficult words to each other, the use of language related to mathematics and modelling.

For the main category collaborative learning, the least agreement has been reached for the key activities of Dekker and Elshout-Mohr's (1998) Process Model. One reason for this could be that these key activities are applied by Dekker and Elshout-Mohr to students who initially work individually on the same task and then compare it with each other. While the students in our study work together on the tasks throughout the modelling process. For the main category mathematical modelling, subcategories have also been identified in which a low agreement has been reached. These subcategories, have been supplemented with an explanation that has been added or adjusted. Therefore, the revised observation instrument should be retested.

Successful solving of a mathematical modelling problems requires modelling competencies (Maaß, 2006), but also metacognitive strategies in solving complex modelling problems in groups (Vorhölter, 2019). In the instrument we focus on the students' multiple solution strategies, but we do not observe whether students are (collaboratively) engaged in task orientation or planning. It is difficult to include that, because we want to use the observation instrument to know whether the developed modelling tasks are suitable. If there is a lot of regulation during collaboration, it can mean that the task is too open or complex, but it can also mean that students discuss this well. In addition, the developed assignments were performed in groups without teacher guidance and this can lead to confusion of students' thinking (Goos et al., 2002). A more qualitative analysis would then be necessary to gain more insight into this. Follow-up research could focus on how these metacognitive activities and teacher guidance could be included in the observation instrument.

In our next study, we will apply this observation instrument to investigate the quality of student interaction in order to evaluate the suitability of the group tasks we developed in order to develop students' mathematical modelling competencies.

## **Acknowledgment**

This research is funded by the Dutch Research Council (NWO), number 023.009.047

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