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# Pre-service teachers' pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools

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*Dealing with simulation and modelling tasks with digital tools in mathematics lessons puts high demands on teachers when it comes to the preparation and implementation of lessons. These must be met in teacher education. To measure the pre-service mathematics teachers' professional knowledge for teaching simulations and mathematical modelling with digital tools, we propose a theory-based model and subsequently present items of an associated test instrument. Using a one-parameter Rasch model, we show that the underlying model can be confirmed empirically and discuss the potentials and limitations of the results.*

*Keywords: Mathematical modelling, simulations, digital tools, pedagogical content knowledge, measurement.*

## Introduction

Large-scale studies – such as Kunter et al. (2013) and Blömeke et al. (2014) – have shown how the pedagogical content knowledge of mathematics teachers can be described theoretically and empirically. Wess et al. (2021b) were able to use these conceptualizations and other research results from recent years to examine the pedagogical content knowledge of pre-service mathematics teachers specifically for mathematical modelling. Due to the increasing importance of digitalization in teaching and learning, it now seems sensible to integrate digital tools into this existing concept and to reinterpret the construct of Wess et al. (2021b). Based on this, the first results of the development of a test instrument that focuses on the professional knowledge for teaching simulations and mathematical modelling with digital tools will be presented in the following. The main question is: *To what extent can the pre-service teachers' pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools be empirically captured as a construct?*

## Theoretical Background

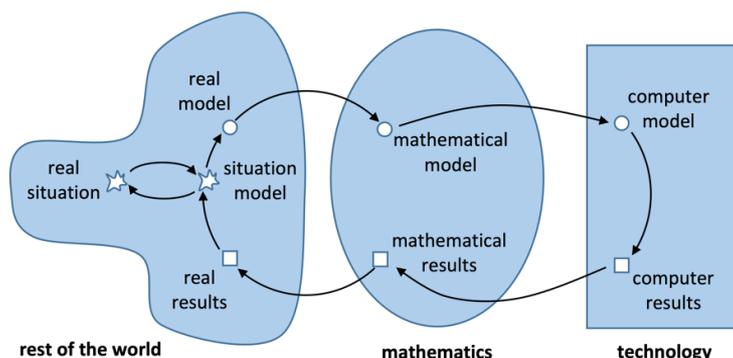
The term **mathematical modelling** describes the investigation of extra-mathematical processes and relations with mathematical tools. This includes the structuring of the extra-mathematical situation, the well-justified construction of a model to describe the reality, translation processes between the extra-mathematical and intra-mathematical world (in both directions), mathematical considerations, and the interpretation and validation of the results obtained (Niss et al., 2007). These modelling processes can sometimes be carried out several times as well.

If a mathematical model, which can be used for experimentation, of a reality-related situation is already available, **simulations** can also contribute to the exploration of reality. Simulations then

enable dynamic, experiment-like processes that provide insights into the real system represented in the model (Greefrath & Siller, 2018). Simulations can also help to validate and optimise the mathematical model (Greefrath & Vorhölter, 2016).

**Digital tools** that are used in mathematics education represent a subcategory of digital media. While digital media are used, among other things, to communicate and document information, digital mathematics tools specifically support mathematical learning processes and the investigation of mathematical relations (Drijvers et al., 2016; Hillmayr et al., 2020). For example, they can generate and process larger amounts of data, visualize interrelationships dynamically, take over calculation processes, reduce complex function terms and offer new possibilities for information research (Greefrath et al., 2018). Thus, on the one hand their use therefore enables – especially in simulation and modelling processes – the treatment of previously inaccessible content. On the other hand, other focal points in mathematical considerations are now made possible (Greefrath & Siller, 2018). Examples of digital mathematics tools (we will speak of “digital tools” in the following) are Computer Algebra Systems, Dynamic Geometry Software, spreadsheets, and function plotters.

Several authors, such as Molina-Toro et al. (2019), investigated the integration of **digital tools in modelling processes**. They showed that digital tools can be used to support various processing phases and sub-competencies of mathematical modelling. Communication with the digital tool is essential here: on the one hand, mathematical descriptions must be translated into the language that the digital tool can understand and process and on the other hand, the results of the digital tool have to be translated back into the mathematical terms and operations. Greefrath et al. (2018) therefore extend the modelling cycle of Blum and Leiss (2007) by a technological world that takes into account the translation processes with the digital tool (Figure 1).



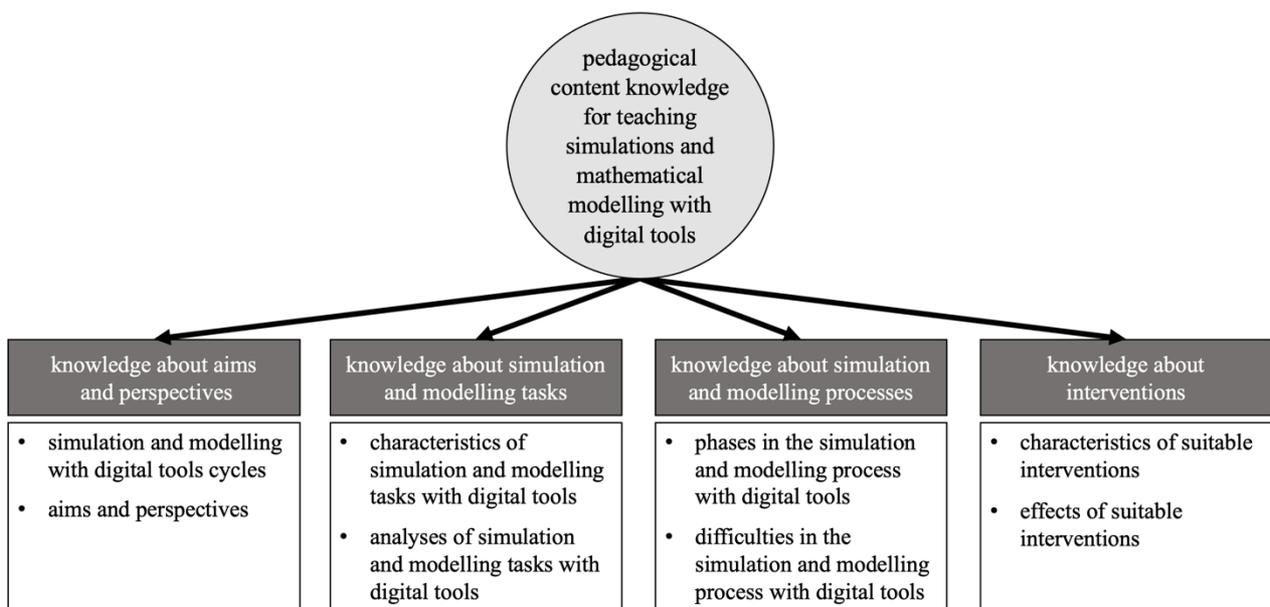
**Figure 1: Extended modelling cycle (cf. Greefrath et al., 2018, p. 235)**

Concrete functions of the digital tool can be implemented in different modelling phases: investigate, experimentalize, visualize, simulate, calculate, control (Greefrath et al., 2018). As mentioned above, mathematical simulation fits into reality-related contexts as an experiment-like process with the already existing mathematical model.

To investigate **professional knowledge for teaching mathematical modelling**, Wess et al. (2021b) developed a structural model of professional competence for teaching mathematical modelling. It serves as the initial basis of our test instrument and uses the model of Kunter et al. (2013) and research

by Borromeo Ferri (2018). Therefore, the pedagogical content knowledge for teaching mathematical modelling includes a *theoretical dimension* (e.g. knowledge about modelling cycles as well as aims, perspectives and criteria for the use of modelling tasks), a *task dimension* (e.g. knowledge about solution processes, analyses and development of modelling tasks), a *diagnostic dimension* (e.g. recognition of modelling phases and difficulties in the modelling process) and an *instruction dimension* (e.g. knowledge about interventions during students' modelling processes) (cf. Borromeo Ferri, 2018; Wess et al., 2021b).

For the current test development to **measure the pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools**, mathematical simulation is included in this structural model at the above-mentioned intersection with mathematical modelling. Additionally, the four teaching competencies are focused on the use of digital tools (cf. Figure 2).



**Figure 2: Pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools (following Wess et al., 2021a)**

## Test construction

Based on the model shown in Figure 2, we developed 84 items in a deductive test construction to empirically (quantitatively) capture the construct described above in four dimensions. The preliminary test design was first qualitatively pre-piloted with experts on simulation, modelling, and digital tools ( $N = 11$ ). Based on their feedback and edits, the content of the test draft was revised. We then presented the test draft to ten pre-service mathematics teachers at the University of Muenster and the University of Wuerzburg for further qualitative pre-piloting. Using think-aloud and verbal probing methods, items that were difficult to understand were identified and then revised or eliminated. Finally, a test draft with 79 closed items in the four theoretically derived dimensions *aims and perspectives* (13 items), *tasks* (10 items), *processes* (28 items) and *interventions* (28 items) was developed. As an example, we would first like to present one item each from the dimensions *aims and perspectives* (Figure 3) and *tasks* (Figure 4):

5.3	The use of digital tools ...	
	... requires a standardized approach to mathematical modelling.	<input type="checkbox"/>
	... in mathematical modelling is only possible in calculation.	<input type="checkbox"/>
	... makes it possible to work on mathematical models with complex function terms.	<input type="checkbox"/>
	... is not helpful in understanding the factual context.	<input type="checkbox"/>

**Figure 3: Example item of the dimension *aims and perspectives***

6.3	Modelling tasks with digital tools in pre-built configurations (e.g., dynamic worksheets) ...	true	false
	... prevent different ways of solving the problem.	<input type="checkbox"/>	<input type="checkbox"/>
	... enable a targeted reduction of task complexity.	<input type="checkbox"/>	<input type="checkbox"/>
	... enable students to use the digital tool independently right from the start.	<input type="checkbox"/>	<input type="checkbox"/>

**Figure 4: Example item of the dimension *tasks***

Following Wess et al. (2021b), the scales consist of multiple-choice and combined-true-false formats, which are to be evaluated dichotomously. Both items (as well as the following) were initially constructed in German and then translated into English for this paper.

The dimensions *processes* and *interventions* are captured with case-based text vignettes. The text vignettes each contain a simulation and/or modelling task and associated conversations between students in a concrete processing phase of the task with digital tools. The text vignette “Traffic Jam” serves as an example here (cf. Figure 5, task, and Figure 6, conversation).

### 7.4 Traffic Jam (9th grade)

At the beginning of the summer vacations, traffic jams often occur. Christina is stuck in a 20 km traffic jam for six hours. She thinks about how many people are in the traffic jam with her.

*Estimate the number of people in the traffic jam.*

*Also use the GeoGebra applet on the right.*

[The simulation shows a traffic jam with ten vehicles per lane. The students can make different assumptions for the traffic jam using the sliders. The simulation then calculates the length of the traffic jam, the number of people in the traffic jam and the number of people per kilometer in this traffic jam.]

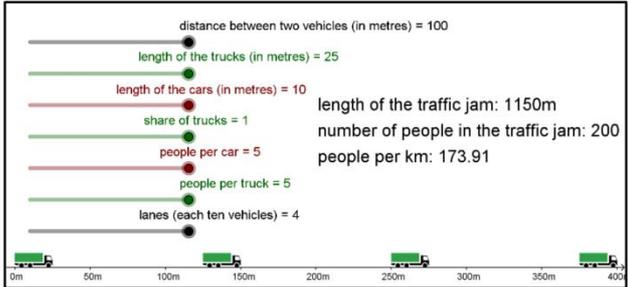
**Figure 5: Task “Traffic Jam” (following Maaß & Gurlitt, 2011; Wess et al., 2021b)**

STUDENT 1: First, we need to know how many cars and how many people are even in the traffic jam. Or rather in an average traffic jam. Otherwise we can't estimate.

STUDENT 2: Exactly. In the simulation, by the way, you can also adjust the trucks. There are fewer people in them, or at least they are longer than a normal car.

STUDENT 3: I'll set everything so that the traffic jam is as big as possible.

STUDENT 3: [Sets the maximum distance and the maximum vehicle length, among other things, on the sliders, see right].



STUDENT 3: Huh? I don't even get to 20 km. The traffic jam is always shorter in the simulation. We can't even solve the problem with that.

STUDENT 2: I have no idea how to do that.

**Figure 6: Conversation of the students while solving the task “Traffic Jam”**

Based on the task and conversation, the participants should subsequently diagnose the students’ problem in the solving process (cf. Figure 7) and derive suitable interventions (cf. Figure 8) in these situations by answering the corresponding items.

7.4.2	Which function of the digital tools do the students <b>mainly</b> use in this situation? Please place <b>one</b> mark.	
	investigate	<input type="checkbox"/>
	simulate	<input type="checkbox"/>
	visualize	<input type="checkbox"/>
	control	<input type="checkbox"/>

**Figure 7: Example item of the dimension *processes***

	Please mark whether each of the following interventions is suitable for the autonomy-oriented promotion of modelling or simulation competencies in this situation. Please place <b>one</b> mark for <b>each</b> intervention.	suitable	unsuitable	do not know
7.4.5	“Check that the simulation allows you to make statements about at least part of the problem.”	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

**Figure 8: Example item of the dimension *interventions* (following Wess et al., 2021b)**

## Evaluation methods and results

To make the quantitative evaluation of the test design in the context of ongoing item revision and selection possible, the test was presented to a suitable sample under standardized conditions. The sample consisted of  $N = 128$  pre-service mathematics teachers from the University of Muenster, the University of Wuerzburg and the University of Erlangen-Nuremberg.

The four dimensions were each scaled with a one-parameter Rasch model (cf. e.g., Rost, 2004). For the calculations, the software R with the packages TAM (Robitzsch et al., 2021) and eRm (Mair & Hatzinger, 2007) was used. Following PISA (OECD, 2012), items with a discrimination index under 0.2 were removed from the test. In line with Bond and Fox (2007), we only left items with adequate mean square fit (MNSQ) statistics in the test. Thus, items whose infit and outfit values were not between 0.8 and 1.2 were gradually eliminated. We made an exception for two items from the dimension *interventions*. The two items show an overfit that, however, is not significant at a level of five per cent. Since the two items are of great importance from a didactic point of view, they remain in the test nonetheless and only their phrasing is revised. In future evaluations, they are to be critically examined again.

After selection and revision, the test contains 54 items in the four dimensions. The one-dimensionality of the scales was tested globally with the help of Andersen tests (cf. e.g., Rost, 2004). According to Lienert and Raatz (1998), the EAP reliabilities of the individual dimensions are sufficient for group comparisons. The results of the tests are summarized in Table 1.

**Table 1: Results of the analyses**

Scale	Number of items	EAP reliability	Andersen test	MNSQ	Pt.-bis. corr.
Aims and persp.	9	.57	1	0.82* to 1.15	> 0.22
Tasks	9	.56	.95	* Exception: Two items are closely below 0.8 (overfit).	
Processes	18	.62	.26		
Interventions	18	.78	.16		

## Summary and Outlook

This article focused the extent to which the pre-service teachers' pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools can be empirically captured as a construct. Based on the structural model of professional competence for teaching mathematical modelling (Wess et al., 2021b), items were constructed using a deductive test theory. It was found that – in the studied group – pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools can be adequately captured as a construct using the developed test instrument. The data collected confirms the four scales *tasks*, *aims and perspectives*, *processes* and *interventions*. Nevertheless, the scales *tasks* and *aims and perspectives* need to be

focused on separately in the further course of the investigations due to the comparatively poorer EAP reliabilities. It needs to be checked whether a multidimensional approach, which takes into account correlations between the latent traits, increases the EAP reliabilities.

Combined with the results obtained from the Andersen test and in the Mean Square Fit (MNSQ) statistics for the two scales, the developed test in its current form seems to enable the measuring of pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools. At the same time, the results are to be confirmed again in cross-validation.

The promising results must be viewed – analogously to Wess et al. (2021a) – against the background that the dichotomous item construction has to allow for definitive true or false answers. Particularly in the field of reality-based tasks, this leads to an additional narrowing of an already very narrow construct, so that many items and text vignettes had to be excluded at the outset. In addition, the scalability and meaningfulness of the current test instrument have so far only been demonstrated for the participating universities. Although we have taken into account the representativeness of the sample according to objective parameters (e.g., study progress, subject combination, previous achievements if applicable) in our evaluation, differences in teacher education in the area of reality-based tasks cannot be ruled out. The question of generalizing the present results therefore remains open for the time being.

In addition to the previous results, pedagogical content knowledge, beliefs and self-efficacy will now also be evaluated and presented in a structural equation model. The complete test instrument will then be used in the coming semesters in courses at the University of Muenster and the University of Wuerzburg in a pretest-posttest control group design.

## References

- Blömeke, S., Hsieh, F.-J., Kaiser, G., & Schmidt, W. H. (Eds.). (2014). *International perspectives on teacher knowledge, beliefs and opportunities to learn: TEDS-M results*. Springer. <https://doi.org/10.1007/978-94-007-6437-8>
- Blum, W., & Leiss, D. (2007). How do students and teachers deal with modelling problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modelling (ICTMA 12): Education, engineering and economics* (pp. 222–231). Horwood. <https://doi.org/10.1533/9780857099419.5.221>
- Bond, T. G., & Fox, C. M. (2007). *Applying the Rasch model: Fundamental measurement in the human sciences* (2nd Edition). Lawrence Erlbaum Associates.
- Borromeo Ferri, R. (2018). *Learning how to teach mathematical modeling in school and teacher education*. Springer International Publishing. <https://doi.org/10.1007/978-3-319-68072-9>
- Drijvers, P., Ball, L., Barzel, B., Heid, M. K., Cao, Y., & Maschietto, M. (2016). *Uses of technology in lower secondary mathematics education: A concise topical survey*. Springer International Publishing. <https://doi.org/10.1007/978-3-319-33666-4>
- Greefrath, G., Hertleif, C., & Siller, H.-S. (2018). Mathematical modelling with digital tools—A quantitative study on mathematising with dynamic geometry software. *ZDM Mathematics Education*, 50(1–2), 233–244. <https://doi.org/10.1007/s11858-018-0924-6>

- Greefrath, G., & Siller, H.-S. (2018). Digitale Werkzeuge, Simulationen und mathematisches Modellieren. In G. Greefrath & H.-S. Siller (Eds.), *Digitale Werkzeuge, Simulationen und mathematisches Modellieren. Didaktische Hintergründe und Erfahrungen aus der Praxis* (pp. 3–22). Springer Fachmedien. <https://doi.org/10.1007/978-3-658-21940-6>
- Greefrath, G., & Vorhölter, K. (2016). *Teaching and learning mathematical modelling*. Springer International Publishing. <https://doi.org/10.1007/978-3-319-45004-9>
- Hillmayr, D., Ziernwald, L., Reinhold, F., Hofer, S. I., & Reiss, K. M. (2020). The potential of digital tools to enhance mathematics and science learning in secondary schools: A context-specific meta-analysis. *Computers & Education*, *153*, 103897. <https://doi.org/10.1016/j.compedu.2020.103897>
- Kunter, M., Baumert, J., Blum, W., Klusmann, U., Krauss, S., & Neubrand, M. (Eds.). (2013). *Cognitive activation in the mathematics classroom and professional competence of teachers: Results from the COACTIV Project*. Springer. <https://doi.org/10.1007/978-1-4614-5149-5>
- Lienert, G. A., & Raatz, U. (1998). *Testaufbau und Testanalyse*. Beltz Psychologie Verlags Union.
- Maaß, K., & Gurlitt, J. (2011). LEMA – Professional development of teachers in relation to mathematical modelling. In G. Kaiser, W. Blum, R. Borromeo Ferri, & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling: ICTMA14* (pp. 629–639). Springer. [https://doi.org/10.1007/978-94-007-0910-2\\_60](https://doi.org/10.1007/978-94-007-0910-2_60)
- Mair, P., & Hatzinger, R. (2007). Extended Rasch modeling: The eRm package for the application of IRT models in R. *Journal of Statistical Software*, *20*(9), 1–20. <https://doi.org/10.18637/jss.v020.i09>
- Molina-Toro, J. F., Rendón-Mesa, P. A., & Villa-Ochoa, J. A. (2019). Research trends in digital technologies and modeling in Mathematics Education. *EURASIA Journal of Mathematics, Science and Technology Education*, *15*(8), Article em1736. <https://doi.org/10.29333/ejmste/108438>
- Niss, M., Blum, W., & Galbraith, P. (2007). Introduction. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education. The 14th ICMI study* (pp. 3–32). Springer. [https://doi.org/10.1007/978-0-387-29822-1\\_1](https://doi.org/10.1007/978-0-387-29822-1_1)
- OECD. (2012). *PISA 2009 technical report*. OECD Publishing. <http://dx.doi.org/10.1787/9789264167872-en>
- Robitzsch, A., Kiefer, T., & Wu, M. (2021). *TAM: Test Analysis Modules* (R package version 3.7-16). <https://CRAN.R-project.org/package=TAM>
- Rost, J. (2004). *Lehrbuch Testtheorie – Testkonstruktion*. Hans Huber Verlag.
- Wess, R., Klock, H., Siller, H.-S., & Greefrath, G. (2021a). Measuring professional competence for the teaching of mathematical modelling. In F. K. S. Leung, G. A. Stillman, G. Kaiser, & K. L. Wong (Eds.), *Mathematical modelling education in east and west. International perspectives on the teaching and learning of mathematical modelling* (pp. 249–260). Springer International Publishing. [https://doi.org/10.1007/978-3-030-66996-6\\_21](https://doi.org/10.1007/978-3-030-66996-6_21)
- Wess, R., Klock, H., Siller, H.-S., & Greefrath, G. (2021b). *Measuring professional competence for the teaching of mathematical modelling: A test instrument*. Springer International Publishing. <https://doi.org/10.1007/978-3-030-78071-5>