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Comparing pre-service teachers' errors in individual and group resolutions of modelling tasks involving estimations

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Real-context estimation problems are useful for introducing modelling in primary school classrooms, although teachers have difficulties associated with the modelling process and also with measurement and estimation senses that may hinder their implementation. Based on a specific categorization, this work analyses the errors of N= 224 pre-service teachers when they draw up individual resolution plans for a sequence of modelling tasks involving estimations, and when they subsequently solve the same sequence in groups and on-site. Individual and group resolutions are compared with two objectives: the first is to determine whether working at the problem site and in groups helps to reduce the number of errors; the second is to find out whether the types of errors that are made are different. This would help to efficiently use learning from errors to improve initial teacher training in solving modelling problems.

Keywords: Fermi problems, errors, measurement, estimation, pre-service primary teachers

Introduction

Real-context estimation problems, also known as Fermi problems, are modelling tasks accessible to primary school students (Ärlebäck, 2009). Hagen (2015) argues that measurement sense and estimation are necessary to successfully solve many modelling tasks. Different studies have found that prospective teachers have difficulties and make many errors when solving modelling problems (Wess et al., 2021; Moreno et al., 2021). Furthermore, deficiencies have been found in the estimation skills of prospective teachers (Castillo-Mateo et al., 2012), especially when they have to reason about estimation and measurement in complex situations (Batur & Nason, 1996). A previous work (Segura & Ferrando, 2021) developed a specific error categorization for one type of Fermi problem: those that require estimating a large number of elements on a delimited surface. This paper analyses errors in the productions of N = 224 pre-service primary school teachers when they are confronted with a sequence of this type of Fermi problems, whose statement is contextualized around the Faculty of Education building. The experience consists in two phases: firstly, individually and in-classroom, they have to make a resolution plan for each problem, secondly, they have to solve the sequence again in groups (N = 62) and perform measurements and estimations at the problems' site.

A comparative analysis allows us to answer the following research questions: Q1. Does group and on-site work help to reduce the number of errors made by prospective teachers when solving Fermi problems? Q2. Do error types evolve from individual resolution plan to group resolution at the problem site?

Theoretical framework

Lesh and Harel (2003) define a *mathematical model* as a system consisting of mathematical concepts, symbolic representations of reality, relationships, regularities or patterns, as well as the procedures, mathematical or otherwise, associated with their use. The development of a mathematical model is

made up of phases, and these phases form a cycle (Borromeo Ferri, 2018) that starts from a real and open situation that has to be *simplified* and *structured*. This process leads to the *real model*, a representation that prefigures the mathematical model. In order to build the *mathematical model*, the real model must be *mathematized*. The next phase requires *working mathematically* within the model to find a solution to the problem in mathematical terms. Finally, the mathematical result must be *interpreted/validated* in the real situation.

Estimation tasks can be used as a means of initiation into mathematical modelling (Borromeo-Ferri, 2018). A specific case of estimation tasks are Fermi problems: these problems present a situation where little concrete information is known, forcing students to make assumptions and estimates to obtain a solution to the initial question (Ärlebäck, 2009, p. 331). In this study we will use a subset of Fermi problems, those that consist of estimating a large number of elements in a delimited surface. Ferrando et al. (2020) categorised the productions of prospective teachers when solving these problems into four types of resolution (counting, linearization, base unit and density). Productions that did not reflect a clear process for arriving at the estimate were classified as *incomplete*.

Studying errors made by pre-service teachers can be useful to understand with what competence and how they solve modelling tasks. Wess et al. (2021) provide a comprehensive categorization that provides an overview of the difficulties at each stage of the modelling cycle. Moreno et al. (2021) established a categorisation of errors (simplification error, mathematization error, resolution error, interpretation error) that has been one of the bases of the categorisation we have developed for Fermi problems. The other basis for establishing a specific error categorisation for Fermi problems consisting of estimating a large number of elements on a surface is the work of Castillo-Moreno et al. (2012). Their research analyses errors of measurement and estimation of length and area, which helps them to establish a categorisation of these errors, both conceptual (errors in perception or meaning of the magnitude; errors in the internalisation of the referents) and procedural (errors in the conversion of units of measurement or incorrect calculations). Basic understanding of quantity management poses difficulties that hinder reasoning about estimation and measurement in complex situations (Baturu and Nason, 1996).

Experimental design

The starting point of this work is a sequence of four problems. The four problems are contextualised in rectangular enclosures located in areas close to the Faculty of Education:

P1-People. How many students can stand on the faculty porch when it rains?

P2-Tiles. How many tiles are there between the education faculty building and the gym?

P3-Grass. How many blades of grass are there in this space?

P4-Cars. How many cars can fit in the faculty parking?

The sample is formed by $N = 224$ prospective teachers. The experience includes two parts: in-class individual work, on-site group work.

In-class individual work

We provided each prospective teacher the written statements of the four problems of the sequence and a little image for each problem. They worked individually for 90 minutes on solving the sequence. The following aspects were emphasized: in each problem they should raise a possible solution plan indicating the measures they would need to obtain the estimation; the work should be done individually; they should explain their procedures in written form and may use drawings or diagrams; and, finally, they were not expected to obtain a numerical estimate but rather to explain how to get the requested estimate. We have called this schematic solution a *resolution plan*.

On-site group work

Pre-service teachers were again confronted with the same sequence of four estimation tasks one week later, but they were randomly grouped into $N = 62$ groups of between 3 and 5 people, and they were taken to the real spaces in the Faculty of Education's environment where the problems are located. They were also provided with measuring instruments to take data and carry out the necessary calculations to obtain a numerical estimate. The session lasted 90 minutes.

Errors categorization

The following categorisation of errors (Table 1), specific to the Fermi problems which require estimating a large number of elements in a delimited surface, is based on a previous analysis of the resolution plans of prospective teachers (Segura & Ferrando, 2021) and on the background review. It is organized following the phases of the modelling cycle and also considers essential processes of estimation and measurement sense.

Table 1. Error categories and associated error types for modelling problems involving area and length estimates

<p>➤ <i>Simplification error:</i></p> <p>E1. Incomplete real model associated with the lack of consideration of elements of the real situation.</p> <p>E2. Incorrect real model due to error of perception of the magnitude.</p> <p>E3. Incorrect real model due to inadequate internalisation of referents of the magnitude to be estimated.</p> <p>E4. Does not build a real model.</p> <p>➤ <i>Mathematization error:</i></p> <p>E5. Mathematical model incoherent with the real model due to an error in the meaning of the terms of the magnitude.</p> <p>E6. Mathematical model incoherent with the real model due to inadequate internalisation of units of measurement of the S.I. of the magnitude to be estimated.</p> <p>E7. Mathematical model incoherent with the real model due to the use of unsuitable units of measurement.</p> <p>E8. Mathematical model is not constructed or is incomplete because elements of the real model are not quantified.</p> <p>➤ <i>Mathematical working error:</i></p> <p>E9. Use of incorrect calculation procedures or calculation errors.</p> <p>E10. Error in conversion of measurement units.</p>

E11. Incomplete resolution procedure.

➤ *Interpretation error:*

E12. Absence of measurement units in the results.

E13. The estimate is clearly incompatible with the real situation.

Results and discussion

The analysis of the N = 224 resolution plans and the N = 62 group resolutions combines descriptive and qualitative analysis of categories and types of errors. Using the categorisation presented in Table 1, Table 2 show the overall results of the analysis of the N = 224 individual resolution plans and the N = 62 group resolutions.

Table 2. Frequency comparison of each type of error in individual resolution plans and group resolutions

Error type/ Category	Frequency in the N = 224 individual resolution plans	Frequency in the N = 62 group resolutions
E1	27 (5,86%)	5 (4,17%)
E2	100 (21,69%)	7 (5,83%)
E3	4 (0,87%)	5 (4,17%)
E4	41 (8,89%)	0
Simplification error	172 (37,31%)	17 (14,17%)
E5	66 (14,32%)	7 (5,83%)
E6	7 (1,52%)	25 (20,83%)
E7	24 (5,21%)	5 (4,17%)
E8	87 (18,87%)	3 (2,50%)
Mathematization error	184 (39,91%)	40 (33,33%)
E9	44 (9,54%)	7 (5,83%)
E10	1 (0,22%)	13 (10,83%)
E11	49 (10,63%)	1 (0,83%)
Mathematical working error	94 (20,39%)	21 (17,49%)
E12	2 (0,43%)	3 (2,50%)
E13	9 (1,95%)	39 (32,50%)
Interpretation error	11 (2,39%)	41 (35%)
<i>Total</i>	<i>461</i>	<i>120</i>

Among the 224 prospective teachers who participated in the in-class individual experience, 166 made at least one error, representing 74.11% of the total. The results show that pre-service teachers made a large number of errors (461) in their individual resolution plans, giving an average of 2.05 errors per solver. In particular, 166 pre-service teachers who made errors had an average of 2.78 errors per

solver. On the other hand, when these prospective teachers were confronted with the same problems in groups and on-site, they made a total of 120 errors, an average of 1.9 errors per group. 51 of the 62 groups made errors, or 82.26% of the total. The 51 groups that made an error had, therefore, an average of 2.35 errors per group. Thus, a higher proportion of groups made errors when they had to find a numerical estimate for each problem, but they made fewer errors than when, individually, each group member had to come up with a resolution scheme.

It is observed that the proportion of errors for each category and for each associated error type is different for individual resolution plans and for group resolutions. In the individual in-class experience 37.31% of the errors belong to the category of *simplification error*, when the real model has to be constructed from the simplification and structuring of the real situation. Specifically, error *E2. Incorrect real model due to error of perception of the magnitude* is the most frequent (21.69%). Qualitative analysis of the individual resolution plans shows that most of these errors are due to confusing length and area. It appears especially in problem P2-Tiles, where the surface area between the gymnasium and the Faculty of Education is confused with the distance between the two. It also appears in P1-People or P3-Grass; in which the width of a person or width of a blade of grass is confused with the surface area they occupy. However, simplification errors are drastically reduced in the on-site group resolutions, accounting for only 14.17%, the category with the lowest error frequency. It is relevant that there is no group resolution that has not developed a real model (E4, leading to incomplete productions) while in the individual resolution plans the number was high (41).

Regarding the category *mathematization error*, number of errors is very high both in the individual in-class experience (39.91% of the total number of errors) and in the group and on-site experience (33.33%). However, the nature and severity of these errors are very different. The most frequent errors in the individual resolution plans (E5 and E8) are the least frequent in the group resolutions. Indeed, *E5. Mathematical model incoherent with the real model due to an error in the meaning of the terms of the magnitude* is a type of error related to the inappropriate use of different quantities in a procedure, mixing them without respecting dimensional homogeneity. A qualitative analysis shows that many errors of this type appear in individual resolution plans which, in order to obtain the number of elements, divide the magnitude measurements of different dimensions. For example, in P1- People, dividing the total area of the porch by the width of a person. We also find other examples of E5, such as the confusion of the concept of area and the concept of perimeter. On the other hand, *E8. Mathematical model is not constructed or is incomplete because elements of the real model are not quantified* relates to resolutions that do not develop the mathematical model sufficiently to know which strategy would give the estimate. There are numerous examples of this error in the individual resolution plans: for example, in P4- Cars it is mentioned that "the car is measured" and that "the parking is measured" but it is not stated precisely what magnitude is plan to be measured are, nor is it written with what mathematical procedures the number of cars would be estimated from these "measurements". There is a lack of definition of relevant variables for the resolution of the problem and the dependencies between these variables, which indicates difficulties in understanding the mathematical concepts involved in the real model of the situation. Errors E8 and E5, numerous in individual resolution plans (18.87% and 14.32%, respectively) are the most serious in this category. They denote important conceptual shortcomings, especially E8 is often linked to resolutions

categorised as incomplete. In contrast, in on-site and group resolutions, the most frequent type of *mathematization error* (20.83% of the total number of errors) is *E6. Mathematical model incoherent with the real model due to inadequate internalisation of units of measurement of the S.I. of the magnitude to be estimated*. This type of error is caused by assigning a value to an estimation reference in conventional units of measurement that does not correspond to reality. We find many examples in group resolutions: for example, in P2-Tiles, those who consider that one foot is equivalent to one metre. Another example: in P3-Grass, a solver estimates that the dimensions of a blade of grass are 1 metre wide and 3 metres long. It is clear from these examples that the conventional measures for the length of a foot or the area of a blade of grass are not well internalised by the solver. These are errors that denote shortcomings in the measurement sense, but do not hinder the development of a mathematical model to obtain an estimate. In group resolutions only 2.50% presented incomplete mathematical models (E8).

Regarding the category *mathematical working error*, number of errors is similar for individual resolution plans (20.39%) and group resolutions (17.49%). However, the associated error types also change drastically. In the individual resolution plans the most numerous are E9 and E11. For *E9. Use of incorrect calculation procedures or calculation errors*, a qualitative analysis shows that, in individual resolution plans, this error is due to the use of the inverse algorithm (Ivars and Fernandez, 2016), that is, make a multiplication in a situation with a measure division structure. For example, the procedure $surface\ area \div element\ area = number\ of\ elements$ is inverted to $surface\ area \cdot element\ area = number\ of\ elements$. Errors of type *E11. Incomplete resolution procedure* are due to the fact that, in many individual resolution plans, the solver has not explicitly written that, in order to find the number of elements of a surface using the area of the surface and the area occupied by the element, he/she must divide these data. Solvers do not write down the process because, perhaps, it is taken for granted. In contrast, in the on-site group experience the most numerous error in this category is E10, which only appears once in the individual plans. E9 and E11 hardly appear at all. Thus, in group resolutions, numerous errors of type *E10. Error in conversion of measurement units* are made, especially when converting units of area (for example, from cm^2 to m^2 by multiplying by 100).

Finally, the most important change is observed in the *interpretation error* category: from representing only 2.35% of the total errors in individual resolution plans, it becomes the most frequent in group resolutions (35% of the total). This is not because working in a group and on-site makes it difficult to interpret and validate the results, but because the schematic nature of the resolution plan, which did not require a numerical estimate, meant that there was no solution to interpret, as most plans only point out how they would get there. In group resolutions, the most frequent error is *E13. The estimate is clearly incompatible with the real situation*, representing 32.50% of the total errors. In fact, it is the most frequent error made in all phases of group resolutions. A qualitative analysis shows that most of these errors occur because the numerical nature of the result is incompatible with reality. For example, a decimal number is given as an estimate of the number of cars that fit in a car park, or people that fit on the porch. This error is because the solvers have not checked the mathematical result and have presented it as a solution without first interpreting it in the real situation.

Conclusions

The discussion of the results above allows us to answer the two research questions:

Q1: Does group and on-site work help to reduce the number of errors made by prospective teachers when solving Fermi problems?

Working in groups and on-site does not reduce the proportion of participants who make errors, but increases slightly. This may be because the group resolution is more comprehensive than the individual resolution plan, as it requires measurements and numerical estimations. A larger number of processes may allow more errors to be made. However, working in groups and on-site does slightly reduce the number of errors made and, above all, their seriousness. Errors that prevent the development of a real model (simplification errors, especially E4) and those that prevent the development of a mathematical model (especially E8) are drastically reduced. This explains, from the point of view of errors, a previous work (Segura et al., 2021) which shows that, for this same sample, the percentage of incomplete productions drops from 15.6% in the individual experience to 2% in the group productions. Working in groups and on-site, therefore, is a scaffolding tool that, although it does not reduce the number of errors committed, it does drastically reduce their relevance. Errors made in groups do not impede the development of a model and the achievement of a reasoned estimate, even if it may be wrong.

Q2: Do error types evolve from individual resolution plan to group resolution at the problem site?

The nature of the errors indeed differs from individual resolution plans to group and on-site resolutions. The evolution at each stage of the modelling process is as follows: in group resolutions there is a sharp reduction of error rates of the simplification phase compared to individual resolution plans. Working in the place of the problem improves the perception of space and, therefore, of the magnitudes involved in the real model and which will be quantified in the mathematical model. This allows for a low number of incomplete or incoherent real models, when in the resolution plans this number was high.

In the mathematization phase overall proportion of errors does not change much, but their nature does. The types of errors associated with this phase in individual resolution plans were mostly serious conceptual errors that often impede the development of a mathematical model and result in incomplete productions. However, in group resolutions most errors in this phase denote a lack of skills in the measurement sense, but do not prevent the development of a mathematical model to reach an estimate. Again, working in groups and on-site helps in the modelling process, although the detailed measurement work brings out errors in the sense of measurement and estimation.

The proportion of errors in the mathematical working phase is similar in the individual in-class experience and in the on-site group experience. But the types of error are different in each experience. In the individual resolution plans there is a majority presence of errors of a structural type in the estimation calculation procedures (use of the wrong algorithm, or lack of calculations). However, in the group resolutions, most of the errors in this phase are due to failures in the conversion between units of measurement, which implies a poor understanding of the concept. This difference is due to the schematic character of the individual resolution plan: it is not required to execute the numerical operations, only to explain them, which may lead to errors in the choice of algorithms that would not be made if actually executed, but avoids making numerical errors (such as in conversion).

The change in the proportion of errors of the interpretation phase is drastic: these errors increase considerably in group resolutions. The explanation is obvious: the schematic nature of the individual resolution plan makes it unnecessary to interpret the result, as it only explains how to get an estimate, and very few solvers provide a numerical estimate. However, in the group resolution they have to reach a numerical estimate, and many groups do not reflect on the nature of the result obtained: they use decimal numbers to express the number of cars or people, or give clearly implausible estimates (e.g. estimating that there are 30 blades of grass in a garden).

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