

Added mass in the presence of density stratification

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Abstract:

Added mass represents the additional inertia experienced by a body as it moves through a fluid, owing to the flow that this motion generates within the fluid. Being involved in the energy and impulse of the fluid, together with the dipole strength of the body and the hydrodynamic force exerted on it, added mass characterizes the flow both globally and locally. When the fluid can support the propagation of waves, inertial added mass, a real number at any given frequency, is complemented by an imaginary part associated with wave drag.

The concept of added mass is extended to stratified fluids and internal gravity waves. The stratification is assumed uniform, the fluid inviscid, the waves linear and the Boussinesq approximation valid. The added mass coefficients of two oscillating bodies, an elliptic cylinder in two dimensions and a spheroid of vertical axis in three dimensions, are determined based on results obtained with the boundary integral method [1]. Fourier inversion allows the frequency variations of the coefficients to be interpreted as a new memory integral for the force exerted on the body, with kernel represented in Fig. 1.

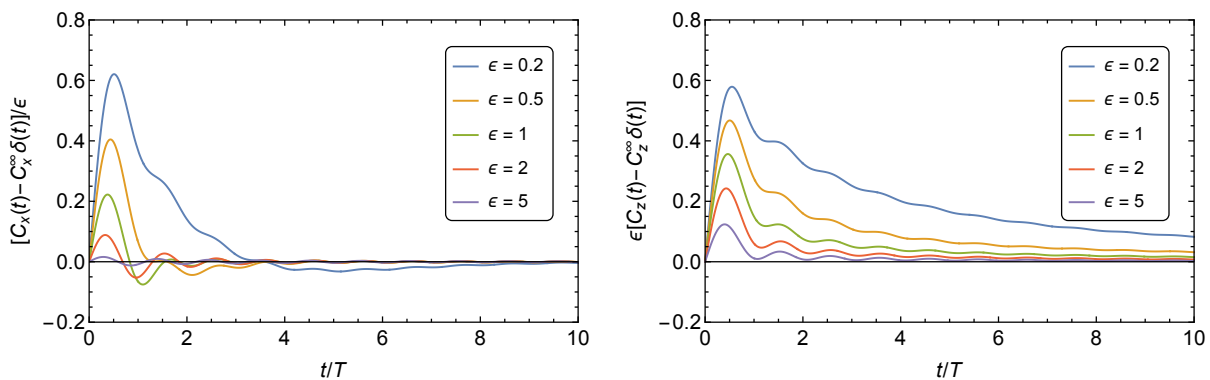


Figure 1: Memory kernel for the oscillations either horizontal (left) or vertical (right) of a spheroid of vertical axis, axial length $2b$, transverse diameter $2a$ and aspect ratio $\varepsilon = b/a$, as a function of time t . The buoyancy period is denoted as T , and the added mass coefficients of the spheroid in the absence of stratification as $C_{x,z}^{\infty}$.

Two applications are discussed: the wave energy, of particular importance for the internal tide, in whose context it is known as conversion rate [2]; and buoyancy oscillations, namely the free oscillations by which a displaced body returns back to equilibrium, of particular importance for midwater floats [3, 4, 5, 6]. These theoretical results are compared to experiment for two types of buoyancy oscillations: those undergone by a sphere displaced from its neutral buoyancy level then released [7, 8, 9, 10]; and the oscillations of a ludion or Cartesian diver [11]. The influence of viscosity at high Stokes number is discussed, together with the relation to measurements of added mass by the impulse response method (see [12, 13] and the references therein).

References

- [1] Voisin B. *Boundary integrals for oscillating bodies in stratified fluids.*, J. Fluid Mech. **927**, A3 (2021).
- [2] Garrett C. & Kunze, E. *Internal tide generation in the deep ocean.*, Annu. Rev. Fluid Mech. **39**, 57–87 (2007).
- [3] Voorhis A.D. *Response characteristics of the neutrally buoyant float.*, Tech. Rep. 71-73, Woods Hole Oceanographic Institution (1971). Available at <https://doi.org/10.1575/1912/24369>.
- [4] Cairns J., Munk W. & Winant C. *On the dynamics of neutrally buoyant capsules; an experimental drop in Lake Tahoe.*, Deep-Sea Res. A **26**, 369–381 (1979).

- [5] Goodman L. & Levine E.R. *Vertical motion of neutrally buoyant floats.*, J. Atmos. Ocean. Technol. **7**, 38–49 (1990).
- [6] D’Asaro E. *Oceanographic floats: principles of operation.*, Observing the Oceans in Real Time (ed. R. Venkatesan, A. Tandon, E. D’Asaro & M.A. Atmanand), pp. 77–98, Springer (2018).
- [7] Larsen L.H. *Oscillations of a neutrally buoyant sphere in a stratified fluid.*, Deep-Sea Res. **16**, 587–603 (1969).
- [8] Levitskii V.V. & Chashechkin Y.D. *Natural oscillations of a neutrally buoyant body in a continuously stratified fluid.*, Fluid Dyn. **34**, 641–651 (1999).
- [9] Biró I., Szabó K.G., Gyüre B., Jánosi I.M. & Tél T. *Power-law decaying oscillations of neutrally buoyant spheres in continuously stratified fluid.*, Phys. Fluids **20**, 051705 (2008).
- [10] Vasil’ev A.Y. & Chashechkin Y.D. *Damping of the free oscillations of a neutral buoyancy sphere in a viscous stratified fluid.*, J. Appl. Maths Mech. **73**, 558–565 (2009).
- [11] Le Gal P., Castillo Morales B., Hernandez-Zapata S. & Ruiz Chavarria G. *Swimming of a ludion in a stratified sea.*, J. Fluid Mech. **931**, A14 (2021).
- [12] Ermanyuk E.V. *The use of impulse response functions for evaluation of added mass and damping coefficient of a circular cylinder oscillating in linearly stratified fluid.*, Exp. Fluids **28**, 152–159 (2000).
- [13] Brouzet C., Ermanyuk E.V., Moulin M., Pillet G. & Dauxois T. *Added mass: a complex facet of tidal conversion at finite depth.*, J. Fluid Mech. **831**, 101–127 (2017).