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Marceau Métillon, Camilo Charron, Kévin Subrin, Stéphane Caro. Geometrical modelling of a Cable-Driven Parallel Robot Winch. 25ème Congrès Français de Mécanique Nantes, Aug 2022, Nantes, France. ⟨hal-03758222v2⟩

HAL Id: hal-03758222

<https://hal.science/hal-03758222v2>

Submitted on 24 Aug 2022

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Geometrical modelling of a Cable-Driven Parallel Robot winch

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Résumé :

Les robots parallèles à câbles utilisent des câbles pour déplacer et orienter une plateforme mobile dans l'espace de travail du robot. Les câbles sont guidés depuis les actionneurs jusqu'à la plateforme mobile au travers de poulies. Les actionneurs les plus répandus consistent en des enrouleurs motorisés qui permettent de contrôler l'enroulement ou le déroulement des câbles et ainsi contrôler la situation de la plateforme mobile. La géométrie et la position relative des enrouleurs par rapport aux poulies ont une influence sur la modélisation géométrique du robot. Ce papier présente la modélisation avancée d'un enrouleur et son influence sur l'erreur commise en terme de longueur de câble déroulée.

Abstract :

Cable-Driven Parallel Robots (CDPRs) use cables instead of rigid legs to connect the Moving-Platform (MP) to the base frame. Cables are routed from the actuators to the MP through pulleys. The actuators commonly used to control the cable length, and therefore the MP pose, are winch actuated by motors that coil and uncoil the cable. The winch geometry and relative position to the pulleys have an influence on the geometric modelling of the robot. This paper presents the geometric modelling of the winch and its influence on the error of uncoiled cable.

Mots clefs : robots parallèles à câbles, enrouleurs, modélisation géométrique

1 Introduction

A Cable-Driven Parallel Robot (CDPR) is a parallel robot where the moving-platform is connected to a base frame using cables coiled on winches. Cables are routed through pulleys attached on a rigid base frame. Motors actuate the winches to control the cable lengths. The moving-platform pose (position and orientation) is a function of the cable lengths, cable elasticity, cable mass and the external wrench exerted onto the moving-platform.

CDPRs are particularly well fitted for tasks requiring high dynamics motion [1], a large translational workspace, reconfigurabilities [2] and manipulation of large and/or heavy parts [3].

The ANR CRAFT project¹ aims to design, model and control CDPRs for agile operations in manufacturing facilities. The CDPRs developed in the framework of this project should be able to work in

1. ANR CRAFT project, grant ANR-18-CE10-0004, <https://anr.fr/Project-ANR-18-CE10-0004>

a cluttered environment [4] and assist operators in carrying and manipulating large and heavy parts. The robots will either be autonomous, realizing some tasks that are set up offline or the user will safely co-manipulate objects with the cable-driven parallel robot (CDPR) as shown in Fig. 1. The robots developed in the framework of the CRAFT project will provide the user with a good agility in large workspace while ensuring the safety of the human operator [5].



FIGURE 1 – A collaborative Cable-Driven Parallel Robot for manipulation task

Due to high costs, complexity and significant processing time of exteroceptive measurement systems, a measurement of the moving-platform pose is not always available for the control of CDPRs [6]. CDPRs can be controlled using a feed-forward computed-torque model with a joint space feedback and tracking controller. Such control strategy consists in an open-loop in operational space and requires fine modelling of the geometrical behaviour of the CDPR including its actuation to ensure a suitable performance level. Actuation of the cable can be done using different actuators. It is possible to distinguish two main categories : the rotary and linear actuators. In term of rotary actuation the cable can be wound around a drum as a part of a winch. This cable actuation method is the more common and includes different winding and spooling techniques. It is possible to denote the single layer winding and the multi-layer winding. The single layer winding allows for higher cable length accuracy assuming that knowing the winch revolution, the unwound cable length can be determined. Single layer also reduces the mechanical wear as the wound cable is not overlapping and sliding on the cable already wound on the drum. In the other hand, the single layer winding is more complex as it requires a system to guide the spooling. Another rotary actuation to consider is the twisting wire actuation. A rotary actuator is directly connected to the cable and the cable is twisted which reduce its length [7]. In term of linear actuation, linear motor are used with a block and tackle and act as a hoist to increase the total cable length range and the transmission ratio between the linear speed and the cable velocity [8]. The winch geometry has an influence on the exact cable length uncoiled to control the robot. Its shape and position relative to the pulley have an influence on the uncoiled cable length. The cable cross-section tends to ovalization when subjected to important cable tension, modifying the mean radius of the winding and thus the cable length coiled around the drum [9]. In [10] the geometric model of a suspended CDPR is written including a geometric model of the coiling of the cable on the drum of the winches. However, the described model does not account for any position and orientation of the winch relative to the pulley.

As a consequence, this paper deals with an advanced geometric modelling of the winch of *CRAFT*. The obtained geometrical model accounts for the cable displacement on the winch and the cable coiling on the winch drum. The winch geometric model is then used in the robot inverse geometric model to control the joint position and accounts for the winch.

2 Winch geometric modelling

On the CRAFT prototype, single-layer rotary winches are used as depicted in Figure 2.

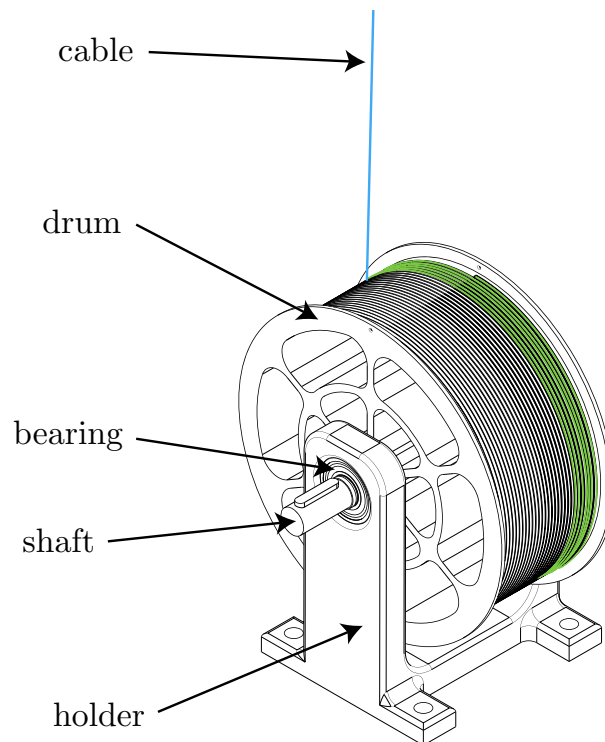


FIGURE 2 – Schematics of CRAFT reel and winch

Figure 3 shows the cable lengths in the CRAFT actuation chain. Cable length l_c is the coiled cable length on the winch drum. Cable length l_d is the uncoiled cable length from drum exit point E to pulley entry point A . Cable length l_p is the cable length that is wrapped on the pulley between points A and A' . Cable length l_m is the cable length uncoiled from the pulley exit point A' to the moving-platform anchor point B .

A drum with a spiralling groove allows to coil a single layer of cable at a given and constant radius. The drum is attached to a shaft describing a rotation relative to the holder which is located on the floor of the cell. A key placed between the shaft and the drum transmits the torque from the shaft to the drum to convert the shaft torque into cable tension. The uncoiled cable length is proportional to the drum rotation. The winches are not equipped with a spoolguide mechanism and the correct cable coiling is guaranteed by the drum groove. The cable is exiting the winch vertically between the drum cable exit point and the panning pulley entry point. While the pulley cable entry point A_i is considered fixed in the base frame, the drum cable exit point is moving relative to the base frame. Depending on the winch characteristics and the position of the winch relative to the pulley entry point, this motion affects the cable total length between the drum and the moving-platform.

Figure 4 shows the area of displacement of the cable and the geometric entities associated to its determination.

Plane Π_1 is the plane containing the attachment point F_1 of the cable on the drum. It can be seen from fig. 5, that the higher q , the more cable is coiled on the drum. When $q = 0$ no cable is coiled on the drum therefore points E is coincident with point F_1 at the point E_1 . Plane Π_2 is the plane containing the

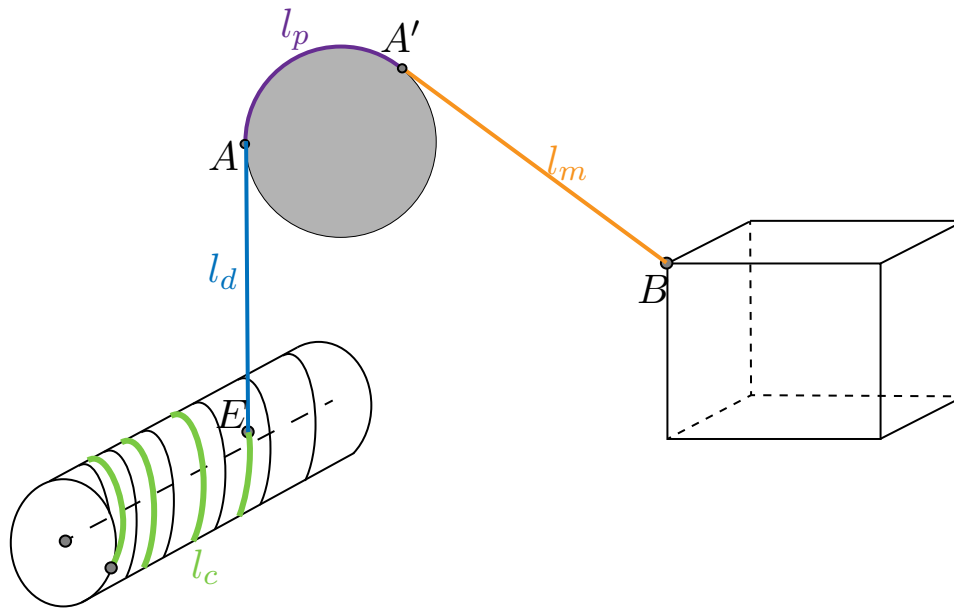


FIGURE 3 – Cable length in the actuation chain

point F_2 which is the maximum coiling point. Points A_1 and A_2 are the projection of point A on planes Π_1 and Π_2 respectively. Points D_1 and D_2 are the intersection of the drum rotation axis with planes Π_1 and Π_2 respectively. Point E is the exit point of the cable on the drum. Points E_1 and E_2 belong to planes Π_1 and Π_2 respectively so that AE_1 and AE_2 are tangent to the drum. Segments D_1E_1 and D_2E_2 are perpendicular to AE_1 and AE_2 respectively. During the drum rotation the point E is moving on the segment E_1E_2 . Therefore the distance between point A and E is dependant on the drum rotation. Figure 5 shows the geometric modelling of the winches used in the CRAFT prototype.

A frame $\mathcal{F}_w = (D_1, \mathbf{x}_w, \mathbf{y}_w, \mathbf{z}_w)$ is attached to the drum holder. The transformation matrix between the winch frame and the base frame is denoted ${}^w\mathbf{T}_b$ and is expressed :

$${}^w\mathbf{T}_b = \begin{bmatrix} {}^w\mathbf{R}_b & {}^bOD_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

A frame $\mathcal{F}_d = (D_1, \mathbf{x}_d, \mathbf{y}_d, \mathbf{z}_d)$ is attached to the drum which is revolving about the axis \mathbf{y}_d with the angle q from vector \mathbf{r}_1 to vector \mathbf{x}_d where \mathbf{r}_1 is a unit vector collinear on the D_1E_1 segment and is expressed as :

$$\mathbf{r}_1 = \begin{pmatrix} \cos(\kappa) \\ 0 \\ \sin(\kappa) \end{pmatrix}_w \quad (2)$$

with κ being the angle from vector \mathbf{x}_w to \mathbf{r}_1 . The angle κ depends on the position of the pulley entry point A relative to the point D_1 and is expressed as :

$$\kappa = \chi - \xi \quad (3)$$

with

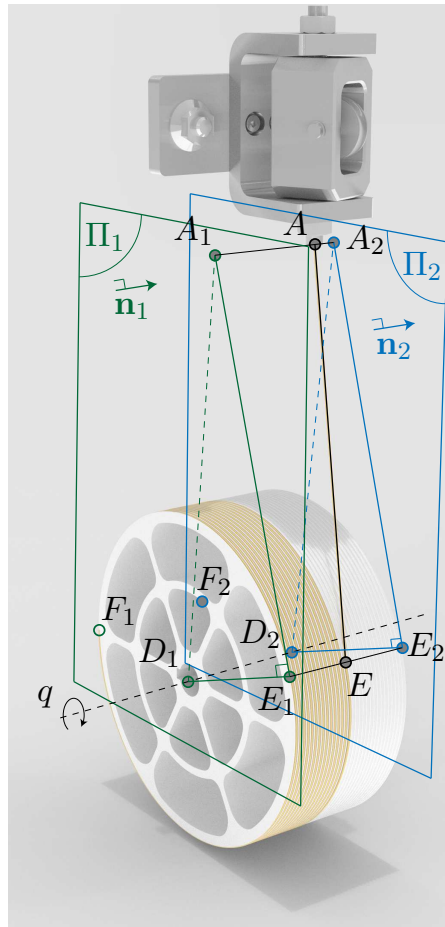


FIGURE 4 – CRAFT winch geometric problem formulation

$$\chi = \text{atan2} \left(\left({}^w \overrightarrow{D_1 A} \right) \mathbf{z}_w, \left({}^w \overrightarrow{D_1 A} \right) \mathbf{x}_w \right) \quad (4)$$

and

$$\xi = \arccos \left(\frac{r_w}{\| {}^w \overrightarrow{D_1 A} \|_2} \right) \quad (5)$$

where r_w is the coiling radius of the cable on the drum and ${}^w \overrightarrow{D_1 A}$ is the vector pointing from point D_1 to A expressed in the winch frame.

Using the transformation from frame \mathcal{F}_b to \mathcal{F}_w vector ${}^w \overrightarrow{D_1 A}$ is expressed as :

$${}^w \overrightarrow{D_1 A} = {}^w \overrightarrow{D_1 O} + {}^w \overrightarrow{OA} = {}^w \mathbf{R}_b \left({}^b \mathbf{a} - {}^b \mathbf{d}_1 \right) \quad (6)$$

The coiling radius of the cable r_w is obtained as :

$$r_w = r_d - r_g + \frac{r_c}{2} \quad (7)$$

where r_d is the drum radius, r_g is the groove radius and r_c is the cable radius.

on the knowledge of the relative position of the pulley entry point A to the winch origin D and the winch characteristics. These coordinates are constant as the pulley entry point A is fixed in the global frame. Vector \mathbf{e} is the coordinates vector of the point E pointing from the base frame origin O to the cable exit point on the winch E . As the point E moves on the segment E_1E_2 as a function of the drum rotation, \mathbf{e} can be expressed as a function of \mathbf{e}_1 , \mathbf{e}_2 and q so that :

$${}^b\mathbf{e} = {}^b\mathbf{e}_1 + ({}^b\mathbf{e}_2 - {}^b\mathbf{e}_1) \frac{q}{q_{max}} \quad (12)$$

with q_{max} being the maximum drum rotation angle where the maximum cable length is coiled on the drum, that is to say point E and point E_2 coincides. As the helix describing the groove centreline on the drum has an integer number of complete revolution, q_{max} is obtained as :

$$q_{max} = \frac{2 \pi l_{E_1E_2}}{\mu} \quad (13)$$

When $q = 0$, there is no cable length coiled on the drum and the cable connection point F_1 coincides with the exit point E and therefore E_1 . The drum rotation angle q is bounded within the range $[0, q_{max}]$.

The total uncoiled cable length l_u is obtained as :

$$l_u = l_d + l_{pm} \quad (14)$$

where $l_{pm} = l_p + l_m$ is the desired cable length from point A and B and is expressed as a function of the moving-platform pose using the pulley modelling.

The cable length l_d between points E and A is obtained as :

$$l_d = \|\vec{{}^bAE}\|_2 = \|\mathbf{e} - \mathbf{a}\|_2 \quad (15)$$

Considering known the initial state of the robot, namely the initial joint position q_0 and moving-platform pose \mathbf{x}_0 the initial total cable length l_{u_0} is known :

$$l_{u_0} = l_{d_0} + l_{pm_0} \quad (16)$$

Knowing the initial total cable length, the relative cable length changes are expressed as :

$$l_u = l_{u_0} - k_w \delta q \quad (17)$$

where δq is the relative joint motion so that $\delta q = q - q_0$ and k_w is the coiling coefficient linking the drum rotation to the coiled cable length so that :

$$k_w = \sqrt{r_w^2 + \left(\frac{\mu}{2\pi}\right)^2} \quad (18)$$

where μ is the pitch of the winch.

Injecting eq. (17) and eq. (15) in eq. (14), the following equation is obtained :

$$\|{}^b\mathbf{e} - {}^b\mathbf{a}\|_2 = l_{u_0} + k_w(q - q_0) - l_{pm} \quad (19)$$

Squaring eq. (19), the equation becomes :

$$\left({}^b\mathbf{e} - {}^b\mathbf{a}\right)^T \left({}^b\mathbf{e} - {}^b\mathbf{a}\right) = (l_{u_0} + k_w(q - q_0) - l_{pm})^2 \quad (20)$$

Using eq. (12), eq. (20) becomes :

$$\left({}^b\mathbf{e}_1 + \left({}^b\mathbf{e}_2 - {}^b\mathbf{e}_1\right) \frac{q}{q_{max}} - {}^b\mathbf{a}\right)^T \left({}^b\mathbf{e}_1 + \left({}^b\mathbf{e}_2 - {}^b\mathbf{e}_1\right) \frac{q}{q_{max}} - {}^b\mathbf{a}\right) = (l_{u_0} + k_w(q - q_0) - l_{pm})^2 \quad (21)$$

Developing right hand side of eq. (21) gives :

$$\frac{q^2}{q_{max}^2} l_{E_1E_2}^2 + \frac{2q}{q_{max}} l_{AE_1} l_{E_1E_2} \cos(\alpha_{E_1}) + l_{AE_1}^2 = (l_{u_0} + k_w(q - q_0) - l_{pm})^2 \quad (22)$$

where $l_{E_1E_2}^2 = \left({}^b\mathbf{e}_2 - {}^b\mathbf{e}_1\right)^T \left({}^b\mathbf{e}_2 - {}^b\mathbf{e}_1\right)$ is the square of the cable length between points E_1 and E_2 , $l_{AE_1}^2 = \left({}^b\mathbf{e}_1 - {}^b\mathbf{a}\right)^T \left({}^b\mathbf{e}_1 - {}^b\mathbf{a}\right)$ is the square of the cable length between the points E_1 and A , and $\cos(\alpha_{E_1})$ the angle between vector $\overrightarrow{AE_1}$ and $\overrightarrow{E_1E_2}$ as depicted in Figure 6. It can be noted that points A , E_1 and E_2 are known and fixed in the base frame so $\cos(\alpha_{E_1})$ can be computed knowing the winch situation and the pulley coordinates.

Eq. (22) is a second order polynomial function of q expressed as :

$$a q^2 + b q + c = 0 \quad (23)$$

where a , b and c are the second, first and zero degree coefficients respectively. The second degree coefficient a is expressed as :

$$a = \frac{l_{E_1E_2}^2}{q_{max}^2} - k_w^2 \quad (24)$$

The first degree coefficient b is expressed as :

$$b = \frac{2}{q_{max}} l_{AE_1} l_{E_1E_2} \cos(\alpha_{E_1}) + 2 k_w (l_{u_0} + k_w q_0 - l_{pm}) \quad (25)$$

The zero degree coefficient c is expressed as :

$$c = l_{AE_1}^2 - (l_{u_0} + k_w q_0 - l_{pm})^2 \quad (26)$$

Analysing the polynomial coefficients sign gives the sign of the discriminant and therefore the possible solutions for eq. (23) of the polynomial. Using eq. (13) and eq. (18) in eq. (24) the coefficient a is written as :

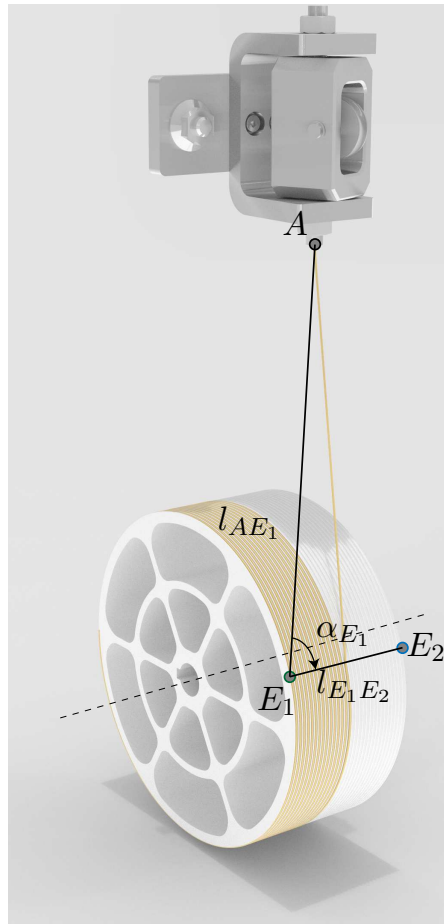


FIGURE 6 – Winch parameters in polynomial coefficients

$$a = \frac{\mu}{2\pi} - \sqrt{r_w^2 + \left(\frac{\mu}{2\pi}\right)^2} \quad (27)$$

It can be seen from eq. (27) that the coefficient a is always negative as $r_w > 0$.

Eq. (25) and eq. (26) shows that sign of coefficients b and c depends on the term $(l_{u_0} + k_w q_0 - l_{pm})$. In fact, the cable length l_{pm} is always strictly less than $l_{u_0} + k_w q_0$ because it is the maximum cable length stored in the system at any time, l_{u_0} being the initial uncoiled cable length and $k_w q_0$ being the initial cable length coiled on the winch. If l_{pm} is equal to $l_{u_0} + k_w q_0$, it would mean that the whole cable length of the winch is located between point A and B and no cable remain on the winch and between point E and A which is physically impossible. Under the hypothesis that A is located on the segment A_1A_2 , $\cos \alpha_{E_1} > 0$. The term $(l_{u_0} + k_w q_0 - l_{pm})$ is always strictly positive. Therefore in eq. (25), as $\frac{2}{q_{max}} l_{AE_1} l_{E_1E_2} \cos(\alpha_{E_1})$ is strictly positive, b is strictly positive.

The term l_{AE_1} is always strictly inferior to $(l_{u_0} + k_w q_0 - l_{pm})$ so coefficient c is always strictly negative.

As the sign of a , b and c are known, the sign of the discriminant is determined, defining the solutions of the polynomial. The discriminant of the polynomial is expressed as :

$$\Delta = b^2 - 4ac \quad (28)$$

parameter	unit	value
k_w	[m]	0.07460068
l_{u_0}	[m]	5.9261
q_0	[rad]	94.2478
q_{max}	[rad]	188.4956
l_{AE_1}	[m]	2.6848
$l_{E_1E_2}$	[m]	0.06
α_{E_1}	[rad]	1.6906

TABLE 1 – Polynomial simulation parameters

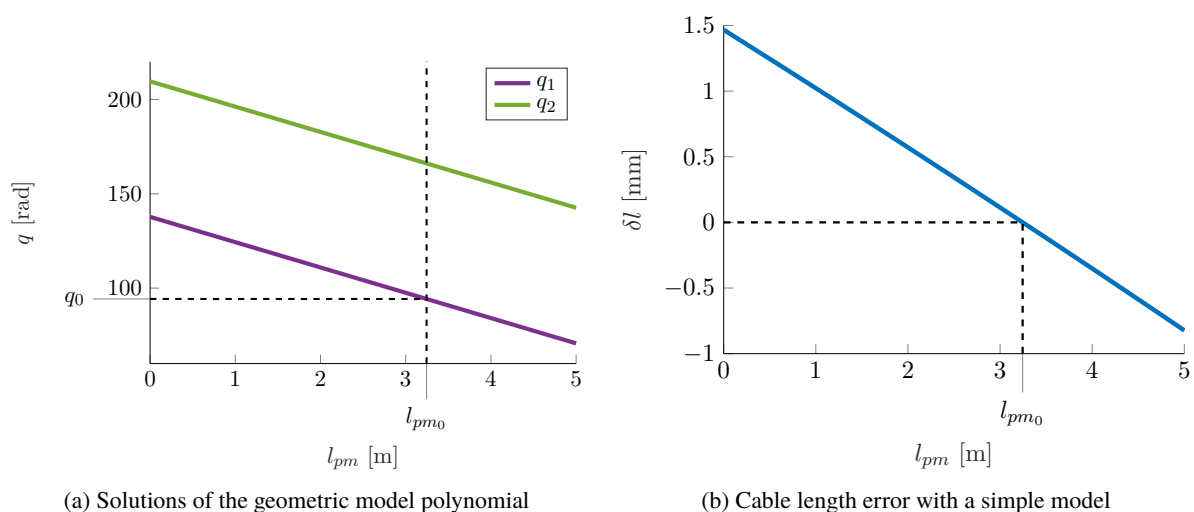
Given the sign of the coefficients, the polynomial discriminant is always positive. Therefore the polynomial eq. (23) admits two solutions q_1 and q_2 expressed as :

$$q_1 = \frac{-b + \sqrt{\Delta}}{2c} \quad (29)$$

$$q_2 = \frac{-b - \sqrt{\Delta}}{2c} \quad (30)$$

Both solution q_1 and q_2 were computed for a cable length l_{pm} ranging from 0 to 5m. Parameters defining the polynomial coefficients are presented in Table 1.

Figure 7 plots numerical results of the model of one winch using the polynomial equation (23) as a function of the desired cable length l_{pm} . Figure 7(a) plots both the solutions q_1 and q_2 . It can be seen that only the q_1 curve go trough the point representing the initial state of the robot (l_{pm_0}, q_0). Indeed, the first solution q_1 ensures the desired cable length l_{pm} while respecting the initial robot state equation $l_{u_0} = l_{d_0} + l_{pm_0}$. As the sign of coefficient c is negative, q_1 is smaller than q_2 leading to a longer cable coiled around the drum. As the cable has a finite length when set up on the winch, the more cable is coiled the less cable there is between the winch and the moving-platform e.g. between points E and B . If q_1 satisfies the equation and q_2 is higher then more cable is coiled, leading to an incorrect cable length l_{pm} . Furthermore solution q_2 curve does not pass through the initial joint position q_0 .



(a) Solutions of the geometric model polynomial

(b) Cable length error with a simple model

FIGURE 7 – Numerical results for geometric model polynomial

Figure 8 illustrates the physical meaning of the two solutions of the polynomial q_1 and q_2 . Points E_{q_1} and point E_{q_2} are the winch exit point corresponding to the solution q_1 and q_2 respectively. The points B_{q_1} and B_{q_2} correspond to the moving-platform anchor point position for the solution q_1 and q_2 respectively. It can be seen that the cable coiled using solution q_2 leads to insufficient uncoiled cable length between points E and B . This can be interpreted as if the anchor points B_{q_2} went through the pulley due to the excessive cable length coiled on the winch. Therefore the solution q_2 is not feasible and do not comply with the desired l_{pm} cable length.

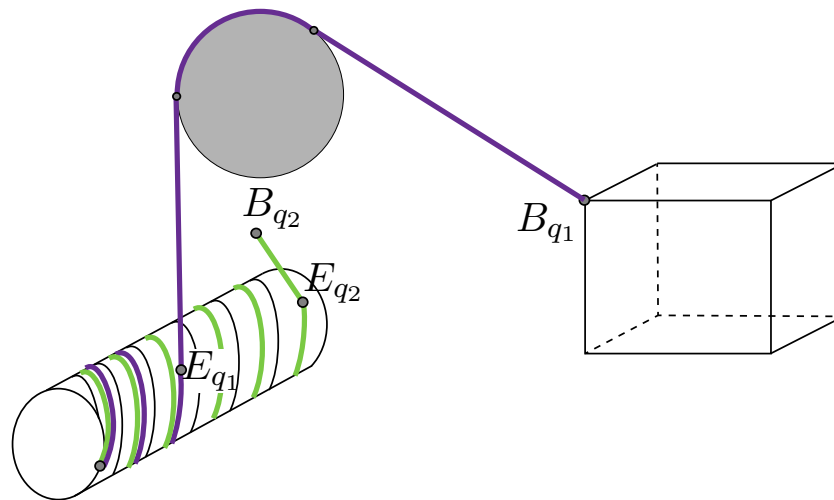


FIGURE 8 – Illustration of both solution of polynomial

Figure 7(b) plots the error in term of cable length when using the winch geometric modelling compared to not considering the winch modelling but only the coiled and uncoiled cable length. The uncoiled cable length error δl is obtained as :

$$\delta l = k_w (q_1 - q_{nw}) \quad (31)$$

with q_{nw} being the joint position obtained using a model neglecting the winch geometric. Such model considers a constant cable length l_d between the winch and the pulley. The joint position q_{nw} is obtained as a function of the initial state of the robot (l_{pm0} , q_0) and the winch coiling ratio k_w as :

$$q_{nw} = q_0 + \frac{l_{pm0} - l_{pm}}{k_w} \quad (32)$$

In Figure 7(b) it can be seen that there is no error on the initial cable length l_{pm0} and the further from this initial point the more the error. The cable length error δl spans from 1.46 mm for $l_{pm} = 0$ m to -0.82 mm for $l_{pm} = 5$ m.

3 Conclusion

This paper deals with the advanced geometric modelling of the single-layer constant radius winch drum of a Cable-Driven Parallel Robot. The presented model accounts for the winch position and orientation relative to the pulley entry point as well as the winch characteristics such as the winch length, the drum pitch and the winding radius. The proposed model is a 2nd order model polynomial on the joint position

which solutions are identified and analysed. The proposed model was simulated and results shows that an error in term cable length is accounted for.

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