# Geometrical modelling of a Cable-Driven Parallel Robot 

 WinchMarceau Métillon, Camilo Charron, Kévin Subrin, Stéphane Caro

## - To cite this version:

Marceau Métillon, Camilo Charron, Kévin Subrin, Stéphane Caro. Geometrical modelling of a CableDriven Parallel Robot Winch. 25ème Congrès Français de Mécanique Nantes, Aug 2022, Nantes, France. hal-03758222v1

## HAL Id: hal-03758222 <br> https://hal.science/hal-03758222v1

Submitted on 23 Aug 2022 (v1), last revised 24 Aug 2022 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Geometrical modelling of a Cable-Driven Parallel Robot winch 

Marceau Métillon ${ }^{\text {a }}$, Camilo Charron ${ }^{\text {a }}$, Kévin Subrin ${ }^{\text {a }}$, Stéphane Caro ${ }^{\text {a, }}{ }^{\text {, }}$<br>a. Nantes Université, École Centrale Nantes, CNRS, LS2N, UMR 6004, F-44000 Nantes, France<br>* Corresponding author stephane.caro@ls2n.fr

## Résumé :

Les robots parallèles à câbles utilisent des câbles pour déplacer et orienter une plateforme mobile dans l'espace de travail du robot. Les câbles sont guidés depuis les actionneurs jusqu'à la plateforme mobile au travers de poulies. Les actionneurs les plus répandus consistent en des enrouleurs motorisés qui permettent de contrôler l'enroulement ou le déroulement des câbles et ainsi contrôler la situation de la plateforme mobile. La géométrie et la position relative des enrouleurs par rapport aux poulies ont une influence sur la modélisation géométrique du robot. Ce papier présente la modélisation avancée d'un enrouleur et son influence sur l'erreur commise en terme de longueur de câble déroulée.


#### Abstract

:

Cable-Driven Parallel Robots (CDPRs) use cables instead of rigid legs to connect the Moving-Platform (MP) to the base frame. Cables are routed from the actuators to the MP trough pulleys. The actuators commonly used to control the cable length, and therefore the MP pose, are winch actuated by motors that coil and uncoil the cable. The winch geometry and relative position to the pulleys have an influence on the geometric modelling of the robot. This paper presents the geometric modelling of the winch and its influence on the error of uncoiled cable.


## Mots clefs : robots parallèles à câbles, enrouleurs, modélisation géométrique

## 1 Introduction

A Cable-Driven Parallel Robot (CDPR) is a parallel robot where the moving-platform is connected to a base frame using cables coiled on winches. Cables are routed through pulleys attached on a rigid base frame. Motors actuate the winches to control the cable lengths. The moving-platform pose (position and orientation) is a function of the cable lengths, cable elasticity, cable mass and the external wrench exerted onto the moving-platform.

CDPRs are particularly well fitted for tasks requiring high dynamics motion [1], a large translational workspace, reconfigurabilities [2] and manipulation of large and/or heavy parts [3].

The ANR CRAFT project ${ }^{1}$ aims to design, model and control CDPRs for agile operations in manufacturing facilities. The CDPRs developed in the framework of this project should be able to work in

[^0]a cluttered environment [4] and assist operators in carrying and manipulating large and heavy parts. The robots will either be autonomous, realizing some tasks that are set up offline or the user will safely co-manipulate objects with the cable-driven parallel cobot (CDPC) as shown in Fig. 1. The robots developed in the framework of the CRAFT project will provide the user with a good agility in large workspace while ensuring the safety of the human operator [5].


Figure 1 - A collaborative Cable-Driven Parallel Robot for manipulation task

Due to high costs, complexity and significant processing time of exteroceptive measurement systems, a measurement of the moving-platform pose is not always available for the control of CDPRs [6]. CDPRs can be controlled using a feed-forward computed-torque model with a joint space feedback and tracking controller. Such control strategy consists in an open-loop in operational space and requires fine modelling of the geometrical behaviour of the CDPR including its actuation to ensure a suitable performance level. Actuation of the cable can be done using different actuations. It is possible to distinguish two main categories : the rotary and linear actuators. In term of rotary actuation the cable can be wound around a drum as a part of a winch. This cable actuation method is the more common and includes different winding and spooling techniques. It is possible to denotes the single layer winding and the multi-layer winding. The single layer winding allows for higher cable length accuracy assuming that knowing the winch revolution, the unwound cable length can be determined. Single layer also reduces the mechanical wear as the wound cable is not overlapping and sliding on the cable already wound on the drum. In the other hand, the single layer winding is more complex as it requires a system to guide the spooling. Another rotary actuation to consider is the twisting wire actuation. A rotary actuator is directly connected to the cable and the cable is twisted which reduce its length [7]. In term of linear actuation, linear motor are used with a block and tackle and act as a hoist to increase the total cable length range and the transmission ratio between the linear speed and the cable velocity [8]. The winch geometry has an influence on the exact cable length uncoiled to control the robot. Its shape and position relative to the pulley have an influence on the uncoiled cable length. The cable cross-section tends to ovalization when subjected to important cable tension, modifying the mean radius of the winding and thus the cable length coiled around the drum [9]. In [10] the geometric model of a suspended CDPR is written including a geometric model of the coiling of the cable on the drum of the winches. However, the described model does not account for any position and orientation of the winch relative to the pulley.

As a consequence, this paper deals with an advanced geometric modelling of the winch of CRAFT. The obtained geometrical model accounts for the cable displacement on the winch and the cable coiling on the winch drum. The winch geometric model is then used in the robot inverse geometric model to control the joint position and accounts for the winch.

## 2 Winch geometric modelling

On the CRAFT prototype, single-layer rotary winches are used as depicted in Figure 2.


Figure 2 - Schematics of CRAFT reel and winch

Figure 3 shows the cable lengths in the CRAFT actuation chain. Cable length $l_{c}$ is the coiled cable length on the winch drum. Cable length $l_{d}$ is the uncoiled cable length from drum exit point $E$ to pulley entry point $A$. Cable length $l_{p}$ is the cable length that is wrapped on the pulley between points $A$ and $A^{\prime}$. Cable length $l_{m}$ is the cable length uncoiled from the pulley exit point $A^{\prime}$ to the moving-platform anchor point $B$.

A drum with a spiralling groove allows to coil a single layer of cable at a given and constant radius. The drum is attached to a shaft describing a rotation relative to the holder which is located on the floor of the cell. A key placed between the shaft and the drum transmits the torque from the shaft to the drum to convert the shaft torque into cable tension. The uncoiled cable length is proportional to the drum rotation. The winches are not equipped with a spoolguide mechanism and the correct cable coiling is guaranteed by the drum groove. The cable is exiting the winch vertically between the drum cable exit point and the panning pulley entry point. While the pulley cable entry point $A_{i}$ is considered fixed in the base frame, the drum cable exit point is moving relative to the base frame. Depending on the winch characteristics and the position of the winch relative to the pulley entry point, this motion affects the cable total length between the drum and the moving-platform.

Figure 4 shows the area of displacement of the cable and the geometric entities associated to its determination.

Plane $\Pi_{1}$ is the plane containing the attachment point $F_{1}$ of the cable on the drum. It can be seen from fig. 5, that the higher $q$, the more cable is coiled on the drum. When $q=0$ no cable is coiled on the drum therefore points $E$ is coincident with point $F_{1}$ at the point $E_{1}$. Plane $\Pi_{2}$ is the plane containing the


Figure 3 - Cable length in the actuation chain
point $F_{2}$ which is the maximum coiling point. Points $A_{1}$ and $A_{2}$ are the projection of point $A$ on planes $\Pi_{1}$ and $\Pi_{2}$ respectively. Points $D_{1}$ and $D_{2}$ are the intersection of the drum rotation axis with planes $\Pi_{1}$ and $\Pi_{2}$ respectively. Point $E$ is the exit point of the cable on the drum. Points $E_{1}$ and $E_{2}$ belong to planes $\Pi_{1}$ and $\Pi_{2}$ respectively so that $A E_{1}$ and $A E_{2}$ are tangent to the drum. Segments $D_{1} E_{1}$ and $D_{2} E_{2}$ are perpendicular to $A E_{1}$ and $A E_{2}$ respectively. During the drum rotation the point $E$ is moving on the segment $E_{1} E_{2}$. Therefore the distance between point $A$ and $E$ is dependant on the drum rotation. Figure 5 shows the geometric modelling of the winches used in the CRAFT prototype.

A frame $\mathcal{F}_{w}=\left(D_{1}, \mathbf{x}_{w}, \mathbf{y}_{w}, \mathbf{z}_{w}\right)$ is attached to the drum holder. The transformation matrix between the winch frame and the base frame is denoted ${ }^{w} \mathbf{T}_{b}$ and is expressed :

$$
{ }^{w} \mathbf{T}_{b}=\left[\begin{array}{c:c}
{ }^{w} \mathbf{R}_{b} & { }^{b} O D_{1}  \tag{1}\\
\hdashline 0 & 0
\end{array}: 1\right]
$$

A frame $\mathcal{F}_{d}=\left(D_{1}, \mathbf{x}_{d}, \mathbf{y}_{d}, \mathbf{z}_{d}\right)$ is attached to the drum which is revolving about the axis $\mathbf{y}_{d}$ with the angle $q$ from vector $\mathbf{r}_{1}$ to vector $\mathbf{x}_{d}$ where $\mathbf{r}_{1}$ is a unit vector collinear on the $D_{1} E_{1}$ segment and is expressed as :

$$
\mathbf{r}_{1}=\left(\begin{array}{c}
\cos (\kappa)  \tag{2}\\
0 \\
\sin (\kappa)
\end{array}\right)_{w}
$$

with $\kappa$ being the angle from vector $\mathbf{x}_{w}$ to $\mathbf{r}_{1}$. The angle $\kappa$ depends on the position of the pulley entry point $A$ relative to the point $D_{1}$ and is expressed as :

$$
\begin{equation*}
\kappa=\chi-\xi \tag{3}
\end{equation*}
$$

with

$$
\begin{equation*}
\chi=\operatorname{atan} 2\left(\left({ }^{w} \overrightarrow{D_{1} A}\right) \mathbf{z}_{w},\left({ }^{w} \overrightarrow{D_{1} A}\right) \mathbf{x}_{w}\right) \tag{4}
\end{equation*}
$$



Figure 4 - CRAFT winch geometric problem formulation
and

$$
\begin{equation*}
\xi=\arccos \left(\frac{r_{w}}{\left\|w \overrightarrow{D_{1} A}\right\|_{2}}\right) \tag{5}
\end{equation*}
$$

where $r_{w}$ is the coiling radius of the cable on the drum and ${ }^{w} \overrightarrow{D_{1} A}$ is the vector pointing from point $D_{1}$ to $A$ expressed in the winch frame.
Using the transformation from frame $\mathcal{F}_{b}$ to $\mathcal{F}_{w}$ vector ${ }^{w} \overrightarrow{D_{1} A}$ is expressed as :

$$
\begin{equation*}
{ }^{w} \overrightarrow{D_{1} A}={ }^{w} \overrightarrow{D_{1} O}+{ }^{w} \overrightarrow{O A}={ }^{w} \mathbf{R}_{b}\left({ }^{b} \mathbf{a}-{ }^{b} \mathbf{d}_{1}\right) \tag{6}
\end{equation*}
$$

The coiling radius of the cable $r_{w}$ is obtained as :

$$
\begin{equation*}
r_{w}=r_{d}-r_{g}+\frac{r_{c}}{2} \tag{7}
\end{equation*}
$$

where $r_{d}$ is the drum radius, $r_{g}$ is the groove radius and $r_{c}$ is the cable radius.
Vector $\mathbf{e}_{1}$ is the coordinates vector of point $E_{1}$ pointing from point $O$ to point $E_{1}$ and is expressed as :

$$
\begin{equation*}
{ }^{b} \mathbf{e}_{1}={ }^{b} \overrightarrow{O D_{1}}+{ }^{b} \overrightarrow{D_{1} E_{1}}={ }^{b} \mathbf{d}_{1}+{ }^{b} \mathbf{R}_{w}{ }^{w} \overrightarrow{D_{1} E_{1}} \tag{8}
\end{equation*}
$$



Figure 5 - CRAFT winch parametrization
where ${ }^{w} \overrightarrow{D_{1} E_{1}}$ is the vector pointing from point $D_{1}$ to point $E_{1}$ expressed in the winch frame and is obtained as :

$$
{ }_{w} \overrightarrow{D_{1} E_{1}}=\left(\begin{array}{cc}
r_{w} & \cos (\kappa)  \tag{9}\\
0 \\
r_{w} & \sin (\kappa)
\end{array}\right)_{w}
$$

Vector $\mathbf{e}_{2}$ is the coordinates vector of point $E_{2}$ pointing from point $O$ to point $E_{2}$ and is expressed as :

$$
\begin{equation*}
{ }^{b} \mathbf{e}_{2}={ }^{b} \overrightarrow{O D_{1}}+{ }^{b} \overrightarrow{D_{1} E_{2}}={ }^{b} \mathbf{d}_{1}+{ }^{b} \mathbf{R}_{w}{ }^{w} \overrightarrow{D_{1} E_{2}} \tag{10}
\end{equation*}
$$

where ${ }^{w} \overrightarrow{D_{1} E_{2}}$ is the vector pointing from point $D_{1}$ to point $E_{2}$ expressed in the winch frame and is obtained as :

$$
w \overrightarrow{D_{1} E_{2}}=\left(\begin{array}{c}
r_{w} \cos (\kappa)  \tag{11}\\
l_{E_{1} E_{2}} \\
r_{w} \sin (\kappa)
\end{array}\right)_{w}
$$

with $l_{E_{1} E_{2}}$ being the winch coiling length.
It is noteworthy that the coordinates of points $E_{1}$ and $E_{2}$ expressed in the base frame are only dependant on the knowledge of the relative position of the pulley entry point $A$ to the winch origin $D$ and the winch characteristics. These coordinates are constant as the pulley entry point $A$ is fixed in the global frame. Vector $\mathbf{e}$ is the coordinates vector of the point $E$ pointing from the base frame origin $O$ to the cable exit point on the winch $E$. As the point $E$ moves on the segment $E_{1} E_{2}$ as a function of the drum rotation, $\mathbf{e}$
can be expressed as a function of $\mathbf{e}_{1}, \mathbf{e}_{2}$ and $q$ so that :

$$
\begin{equation*}
{ }^{b} \mathbf{e}={ }^{b} \mathbf{e}_{1}+\left({ }^{b} \mathbf{e}_{2}-{ }^{b} \mathbf{e}_{1}\right) \frac{q}{q_{\max }} \tag{12}
\end{equation*}
$$

with $q_{\text {max }}$ being the maximum drum rotation angle where the maximum cable length is coiled on the drum, that is to say point $E$ and point $E_{2}$ coincides. As the helix describing the groove centreline on the drum has an integer number of complete revolution, $q_{\max }$ is obtained as :

$$
\begin{equation*}
q_{\max }=\frac{2 \pi l_{E_{1} E_{2}}}{\mu} \tag{13}
\end{equation*}
$$

When $q=0$, there is no cable length coiled on the drum and the cable connection point $F_{1}$ coincides with the exit point $E$ and therefore $E_{1}$. The drum rotation angle $q$ is bounded within the range $\left[0, q_{\max }\right]$. The total uncoiled cable length $l_{u}$ is obtained as :

$$
\begin{equation*}
l_{u}=l_{d}+l_{p m} \tag{14}
\end{equation*}
$$

where $l_{p m}=l_{p}+l_{m}$ is the desired cable length from point $A$ and $B$ and is expressed as a function of the moving-platform pose using the pulley modelling.

The cable length $l_{d}$ between points $E$ and $A$ is obtained as :

$$
\begin{equation*}
l_{d}=\left\|^{b} \overrightarrow{A E}\right\|_{2}=\left\|^{b} \mathbf{e}-{ }^{b} \mathbf{a}\right\|_{2} \tag{15}
\end{equation*}
$$

Considering known the initial state of the robot, namely the initial joint position $q_{0}$ and moving-platform pose $\mathbf{x}_{0}$ the initial total cable length $l_{u_{0}}$ is known :

$$
\begin{equation*}
l_{u_{0}}=l_{d_{0}}+l_{p m_{0}} \tag{16}
\end{equation*}
$$

Knowing the initial total cable length, the relative cable length changes are expressed as :

$$
\begin{equation*}
l_{u}=l_{u_{0}}-k_{w} \delta q \tag{17}
\end{equation*}
$$

where $\delta q$ is the relative joint motion so that $\delta q=q-q_{0}$ and $k_{w}$ is the coiling coefficient linking the drum rotation to the coiled cable length so that :

$$
\begin{equation*}
k_{w}=\sqrt{r_{w}^{2}+\left(\frac{\mu}{2 \pi}\right)^{2}} \tag{18}
\end{equation*}
$$

where $\mu$ is the pitch of the winch.
Injecting eq. (17) and eq. (15) in eq. (14), the following equation is obtained :

$$
\begin{equation*}
\left\|^{b} \mathbf{e}-{ }^{b} \mathbf{a}\right\|_{2}=l_{u_{0}}+k_{w}\left(q-q_{0}\right)-l_{p m} \tag{19}
\end{equation*}
$$

Squaring eq. (19), the equation becomes :

$$
\begin{equation*}
\left({ }^{b} \mathbf{e}-{ }^{b} \mathbf{a}\right)^{\top}\left({ }^{b} \mathbf{e}-{ }^{b} \mathbf{a}\right)=\left(l_{u_{0}}+k_{w}\left(q-q_{0}\right)-l_{p m}\right)^{2} \tag{20}
\end{equation*}
$$

Using eq. (12), eq. (20) becomes :

$$
\begin{equation*}
\left({ }^{b} \mathbf{e}_{1}+\left({ }^{b} \mathbf{e}_{2}-{ }^{b} \mathbf{e}_{1}\right) \frac{q}{q_{\max }}-{ }^{b} \mathbf{a}\right)^{\top}\left({ }^{b} \mathbf{e}_{1}+\left({ }^{b} \mathbf{e}_{2}-{ }^{b} \mathbf{e}_{1}\right) \frac{q}{q_{\max }}-{ }^{b} \mathbf{a}\right)=\left(l_{u_{0}}+k_{w}\left(q-q_{0}\right)-l_{p m}\right)^{2} \tag{21}
\end{equation*}
$$

Developing right hand side of eq. (21) gives :

$$
\begin{equation*}
\frac{q^{2}}{q_{\max }^{2}} l_{E_{1} E_{2}}^{2}+\frac{2 q}{q_{\max }} l_{A E_{1}} l_{E_{1} E_{2}} \cos \left(\alpha_{E_{1}}\right)+l_{A E_{1}}^{2}=\left(l_{u_{0}}+k_{w}\left(q-q_{0}\right)-l_{p m}\right)^{2} \tag{22}
\end{equation*}
$$

where $l_{E_{1} E_{2}}^{2}=\left({ }^{b} \mathbf{e}_{2}-{ }^{b} \mathbf{e}_{1}\right)^{\top}\left({ }^{b} \mathbf{e}_{2}-{ }^{b} \mathbf{e}_{1}\right)$ is the square of the cable length between points $E_{1}$ and $E_{2}$, $l_{A E_{1}}^{2}=\left({ }^{b} \mathbf{e}_{1}-{ }^{b} \mathbf{a}\right)^{\top}\left({ }^{b} \mathbf{e}_{1}-{ }^{b} \mathbf{a}\right)$ is the square of the cable length between the points $E_{1}$ and $A$, and $\cos \left(\alpha_{E_{1}}\right)$ the angle between vector $\overrightarrow{A E_{1}}$ and $\overrightarrow{E_{1} E_{2}}$ as depicted in Figure 6. It can be noted that points $A, E_{1}$ and $E_{2}$ are known and fixed in the base frame so $\cos \left(\alpha_{E_{1}}\right)$ can be computed knowing the winch situation and the pulley coordinates.


Figure 6 - Winch parameters in polyomial coefficients
Eq. (22) is a second order polynomial function of $q$ expressed as :

$$
\begin{equation*}
a q^{2}+b q+c=0 \tag{23}
\end{equation*}
$$

where $a, b$ and $c$ are the second, first and zero degree coefficients respectively. The second degree coefficient $a$ is expressed as:

$$
\begin{equation*}
a=\frac{l_{E_{1} E_{2}}^{2}}{q_{\max }^{2}}-k_{w}^{2} \tag{24}
\end{equation*}
$$

The first degree coefficient $b$ is expressed as :

$$
\begin{equation*}
b=\frac{2}{q_{\max }} l_{A E_{1}} l_{E_{1} E_{2}} \cos \left(\alpha_{E_{1}}\right)+2 k_{w}\left(l_{u 0}+k_{w} q_{0}-l_{p m}\right) \tag{25}
\end{equation*}
$$

The zero degree coefficient $c$ is expressed as :

$$
\begin{equation*}
c=l_{A E_{1}}^{2}-\left(l_{u 0}+k_{w} q_{0}-l_{p m}\right)^{2} \tag{26}
\end{equation*}
$$

Analysing the polynomial coefficients sign gives the sign of the discriminant and therefore the possible solutions for eq. (23) of the polynomial. Using eq. (13) and eq. (18) in eq. (24) the coefficient $a$ is written as :

$$
\begin{equation*}
a=\frac{\mu}{2 \pi}-\sqrt{r_{w}^{2}+\left(\frac{\mu}{2 \pi}\right)^{2}} \tag{27}
\end{equation*}
$$

It can be seen from eq. (27) that the coefficient $a$ is always negative as $r_{w}>0$.
Eq. (25) and eq. (26) shows that sign of coefficients $b$ and $c$ depends on the term $\left(l_{u_{0}}+k_{w} q_{0}-l_{p m}\right)$. In fact, the cable length $l_{p m}$ is always strictly less than $l_{u_{0}}+k_{w} q_{0}$ because it is the maximum cable length stored in the system at any time, $l_{u_{0}}$ being the initial uncoiled cable length and $k_{w} q_{0}$ being the initial cable length coiled on the winch. If $l_{p m}$ is equal to $l_{u_{0}}+k_{w} q_{0}$, it would mean that the whole cable length of the winch is located between point $A$ and $B$ and no cable remain on the winch and between point $E$ and $A$ which is physically impossible. Under the hypothesis that $A$ is located on the segment $A_{1} A_{2}, \cos \alpha_{E_{1}}>0$. The term $\left(l_{u_{0}}+k_{w} q_{0}-l_{p m}\right)$ is always strictly positive. Therefore in eq. (25), as $\frac{2}{q_{\text {max }}} l_{A E_{1}} l_{E_{1} E_{2}} \cos \left(\alpha_{E_{1}}\right)$ is strictly positive, $b$ is strictly positive.
The term $l_{A E_{1}}$ is always strictly inferior to $\left(l_{u_{0}}+k_{w} q_{0}-l_{p m}\right)$ so coefficient $c$ is always strictly negative.
As the sign of $a, b$ and $c$ are known, the sign of the discriminant is determined, defining the solutions of the polynomial. The discriminant of the polynomial is expressed as :

$$
\begin{equation*}
\Delta=b^{2}-4 a c \tag{28}
\end{equation*}
$$

Given the sign of the coefficients, the polynomial discriminant is always positive. Therefore the polynomial eq. (23) admits two solutions $q_{1}$ and $q_{2}$ expressed as :

$$
\begin{align*}
& q_{1}=\frac{-b+\sqrt{\Delta}}{2 c}  \tag{29}\\
& q_{2}=\frac{-b-\sqrt{\Delta}}{2 c} \tag{30}
\end{align*}
$$

| parameter | unit | value |
| :---: | :---: | :---: |
| $k_{w}$ | $[\mathrm{~m}]$ | 0.07460068 |
| $l_{u_{0}}$ | $[\mathrm{~m}]$ | 5.9261 |
| $q_{0}$ | $[\mathrm{rad}]$ | 94.2478 |
| $q_{\max }$ | $[\mathrm{rad}]$ | 188.4956 |
| $l_{A E_{1}}$ | $[\mathrm{~m}]$ | 2.6848 |
| $l_{E_{1} E_{2}}$ | $[\mathrm{~m}]$ | 0.06 |
| $\alpha_{E_{1}}$ | $[\mathrm{rad}]$ | 1.6906 |

Table 1 - Polynomial simulation parameters

Both solution $q_{1}$ and $q_{2}$ were computed for a cable length $l_{p m}$ ranging from 0 to 5 m . Parameters defining the polynomial coefficients are presented in Table 1.

Figure 7 plots numerical results of the model of one winch using the polynomial equation (23) as a function of the desired cable length $l_{p m}$. Figure 7(a) plots both the solutions $q_{1}$ and $q_{2}$. It can be seen that only the $q_{1}$ curve go trough the point representing the initial state of the robot $\left(l_{p m_{0}}, q_{0}\right)$. Indeed, the first solution $q_{1}$ ensures the desired cable length $l_{p m}$ while respecting the initial robot state equation $l_{u_{0}}=l_{d_{0}}+l_{p m_{0}}$. As the sign of coefficient $c$ is negative, $q_{1}$ is smaller than $q_{2}$ leading to a longer cable coiled around the drum. As the cable has a finite length when set up on the winch, the more cable is coiled the less cable there is between the winch and the moving-platform e.g. between points $E$ and $B$. If $q_{1}$ satisfies the equation and $q_{2}$ is higher then more cable is coiled, leading to an incorrect cable length $l_{p m}$. Furthermore solution $q_{2}$ curve does not pass through the initial joint position $q_{0}$.


Figure 7 - Numerical results for geometric model polynomial

Figure 8 illustrates the physical meaning of the two solutions of the polynomial $q_{1}$ and $q_{2}$. Points $E_{q_{1}}$ and point $E_{q_{2}}$ are the winch exit point corresponding to the solution $q_{1}$ and $q_{2}$ respectively. The points $B_{q_{1}}$ and $B_{q_{2}}$ correspond to the moving-platform anchor point position for the solution $q_{1}$ and $q_{2}$ respectively It can be seen that the cable coiled using solution $q_{2}$ leads to insufficient uncoiled cable length between points $E$ and $B$. This can be interpreted as if the anchor points $B_{q_{2}}$ went through the pulley due to the excessive cable length coiled on the winch. Therefore the solution $q_{2}$ is not feasible and do not comply with the desired $l_{p m}$ cable length.


Figure 8 - Illustration of both solution of polynomial

Figure 7(b) plots the error in term of cable length when using the winch geometric modelling compared to not considering the winch modelling but only the coiled and uncoiled cable length. The uncoiled cable length error $\delta l$ is obtained as :

$$
\begin{equation*}
\delta l=k_{w}\left(q_{1}-q_{n w}\right) \tag{31}
\end{equation*}
$$

with $q_{n w}$ being the joint position obtained using a model neglecting the winch geometric. Such model considers a constant cable length $l_{d}$ between the winch and the pulley. The joint position $q_{n w}$ is obtained as a function of the initial state of the robot $\left(l_{p m_{0}}, q_{0}\right)$ and the winch coiling ratio $k_{w}$ as :

$$
\begin{equation*}
q_{n w}=q_{0}+\frac{l_{p m_{0}}-l_{p m}}{k_{w}} \tag{32}
\end{equation*}
$$

In Figure 7(b) it can be seen that there is no error on the initial cable length $l_{p m_{0}}$ and the further from this initial point the more the error. The cable length error $\delta l$ spans from 1.46 mm for $l_{p m}=0 \mathrm{~m}$ to -0.82 mm for $l_{p m}=5 \mathrm{~m}$.

## 3 Conclusion

This paper deals with the advanced geometric modelling of the single-layer constant radius winch drum of a Cable-Driven Parallel Robot. The presented model accounts for the winch position and orientation relative to the pulley entry point as well as the winch characteristics such as the winch length, the drum pitch and the winding radius. The proposed model is a 2 nd order model polynomial on the joint position which solutions are identified and analysed. The proposed model was simulated and results shows that an error in term cable length is accounted for.

## Références

[1] R. Pannequin, M. Jouaiti, M. Boutayeb, P. Lucas, and D. Martinez, "Automatic tracking of freeflying insects using a cable-driven robot," Science Robotics, vol. 5, no. 43, p. eabb2890, 2020.
[2] L. Gagliardini, S. Caro, M. Gouttefarde, P. Wenger, and A. Girin, "A reconfigurable cable-driven parallel robot for sandblasting and painting of large structures," in Cable-Driven Parallel Robots
(A. Pott and T. Bruckmann, eds.), vol. 32 of Mechanisms and Machine Science, pp. 275-291, Cham : Springer International Publishing, 2015.
[3] E. Picard, F. Plestan, E. Tahoumi, F. Claveau, and S. Caro, "Control strategies for a cable-driven parallel robot with varying payload information," Mechatronics, vol. 79, p. 102648, 2021.
[4] U. A. Mishra, M. Metillon, and S. Caro, "Kinematic stability based afg-rrt* path planning for cabledriven parallel robots $\dagger$," in 2021 IEEE International Conference on Robotics and Automation (ICRA), pp. 6963-6969, IEEE, 5/30/2021-6/5/2021.
[5] S. Caro and J.-P. Merlet, "Failure analysis of a collaborative 4-1 cable-driven parallel robot," in New Trends in Mechanism and Machine Science (D. Pisla, B. Corves, and C. Vaida, eds.), vol. 89 of Mechanisms and Machine Science, pp. 440-447, [S.1.] : Springer, 2020.
[6] A. Pott, Cable-Driven Parallel Robots, vol. 120. Cham : Springer International Publishing, 2018.
[7] M. Shoham, "Twisting wire actuator," Journal of Mechanical Design, vol. 127, no. 3, pp. 441-445, 2005.
[8] J.-P. Merlet, "Kinematics of the wire-driven parallel robot marionet using linear actuators," in Robotics and Automation, 2008, ICRA 2008, IEEE International Conference on, ([Pitscataway, NJ]), pp. 3857-3862, IEEE Xplore, 2008.
[9] V. Schmidt and A. Pott, "Investigating the effect of cable force on winch winding accuracy for cable-driven parallel robots," Proceedings of the Institution of Mechanical Engineers, Part K : Journal of Multi-body Dynamics, vol. 230, no. 3, pp. 237-241, 2016.
[10] D. Q. Nguyen, "Kinematics of a 2-dof planar suspended cable-driven parallel robot," in CableDriven Parallel Robots (M. Gouttefarde, T. Bruckmann, and A. Pott, eds.), vol. 104 of Mechanisms and Machine Science, pp. 13-23, Cham : Springer International Publishing, 2021.


[^0]:    1. ANR CRAFT project, grant ANR-18-CE10-0004, https://anr.fr/Project-ANR-18-CE10-0004
