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Basic Mental Models of Derivative and Integral: First Hypothesis

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Introduction

In recently published contributions we have developed a multiple-choice test (<https://hal.archives-ouvertes.fr/hal-03103685>) to measure the ability of university students to think and argue with basic mental models (BMMs) of derivatives and integrals, see Greefrath et al. (2021).

Derivative – BMM of ...

Tangent Slope (TS): The derivative gives the slope of the tangent to the graph.

Local Rate of Change (RC): The derivative gives the local rate of change.

Amplification Factor (AF): The factor amplifying small change: $\Delta y \approx m \cdot \Delta x$.

Local Linearity (LL): The slope of a locally almost straight part of the graph.

Integral – BMM of ...

Area (AR): The integral is the measure of the oriented area under the function graph.

(Re)construction (RC): The integral determines a quantity from its rate of change.

Average (AV): The integral averages the function values on an interval.

Accumulation (AC): The integral is a generalized sum of small pieces.

Building on this we now intend to understand better the development of these basic mental models. In a first approach we look at differences in students' perceptions and argumentations at different stages of their mathematical studies.

We defined four BMMs for the derivative and also four for the integral. For each concept we developed tasks each with four explanations that argued on the basis of four different BMMs. Students were then asked to rate if the given explanations are alike to their own thinking.

Results and Hypothesis on the development of BMMs

How do BMMs preferences change under the influence of university level mathematics courses? We based our first tentative answers on two data sets. The first is a cross sectional data set of 262 students across several study programs (e.g., teacher students, mathematics and physics bachelor students), approximately one half of the students were in the first week of their studies at the time of testing, the remaining half had at least completed one semester. Results are shown in table 1.

Table 1: Development comparison from cross sectional data - values are average agreement with the explanation according to the given BMM on a Likert scale from 1 to 5 (3 is neutral)

BMM	TS	RC	AF	LL	AR	RC	AV	AC
First term	4.46	3.83	2.75	3.69	2.90	2.27	1.91	2.10
Higher term	4.28	3.76	2.76	3.58	2.71	2.21	1.89	2.04
Wilcox p	0.002*	0.32	0.96	0.29	0.037*	0.66	0.98	0.72

From these results it seems that the intensity of BMMs either is constant or decreases. This result may be surprising but there are two plausible hypothesis that should be examined in further studies: 1st: BMMs are important in the learning process and hence students in higher semesters no longer need them to guide them in their argumentation, they can solve the tasks without consciously activating the BMMs. 2nd: Depending on the subject (e.g. physics) students may develop further BMMs that we have not investigated, and these interfere here (such further BMMs may be e.g. the integral as curve in a direction (vector) field, the derivative as degree one term in a power series).

Even more surprising is that a second data set yields quite different results. The BMMs of derivative were tested with 35 teacher students in the beginning and at the end of a first semester course. Results in table 2 indicate significant boosts for all BMMs but the amplification factor. Although this points in the opposite effect compared to the above results, nevertheless both hypotheses formulated above may explain these results: In the first study the students from higher terms were experienced students, while in the second study post-test students were still very close to the phase where the concepts are learned and BMMs are needed according to the 1st hypothesis. Moreover, in the second study students were from a homogenous group of teacher students and for this group, BMMs are explicit part of their studies so that the 2nd hypothesis may be applied.

Table 2: Development comparison pre-post-test design

BMM	TS	RC	AF	LL
Pre test	4.29	3.83	2.68	3.61
Post test	4.50	4.02	2.85	4.03
Wilcox p (matched)	0.000*	0.035*	0.300	0.000*

Conclusion

The results presented here strongly suggest that further research is needed to better understand the development of BMMs and especially their role in on-going calculus courses and especially in the thinking of experienced students.

References

Greefrath, G., Oldenburg, R., Siller, H.-S., Ulm, V., & Weigand, H.-G. (2021). Basic mental models of integrals: Theoretical conception, development of a test instrument, and first results. *ZDM Mathematics Education* 53, 649–661. <https://doi.org/10.1007/s11858-020-01207-0>