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► **To cite this version:**

Johannes Przybilla, Matthias Brandl, Mirela Vinerean, Yvonne Liljekvist. Digital mathematical maps - results from iterative research cycles. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03754749

**HAL Id: hal-03754749**

**<https://hal.science/hal-03754749>**

Submitted on 19 Aug 2022

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# Digital mathematical maps – results from iterative research cycles

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*In this paper, we provide results from the design-based research and development process of a digital didactical tool: mathematical maps. We describe in detail the current research cycle, in which the mathematical maps are integrated into a geometry course designed cooperatively by university mathematics teachers and researchers in mathematics education from different universities. The results of the evaluation affirm the (perceived) usefulness of individual functionalities of the digital tool but also point to suggestions for design improvement. Furthermore, the use of the map within the geometry course seems to promote favorable beliefs regarding the nature of mathematics.*

*Keywords: Teacher education, defragmentation, design-based research, digital tool, mathematical maps.*

## **Introduction: Addressing the double discontinuity via defragmentation**

The second-tertiary transition has received much attention across all kinds of disciplines and countries, partly because of the high drop-out rates of first-year students. In general, as well as in mathematics, a holistic perspective on the transition problem is sought (see e.g. Gueudet et al., 2016). However, a look at the research literature shows that mathematics education research often focuses on cognitive challenges resulting from the inherent characteristics of the subject (e.g. Thomas et al., 2015). Especially in teacher education, this perspective has a long tradition. Almost 100 years ago, the mathematician Felix Klein (1924/2016) defined the term "double discontinuity" to address problems of future teachers when transitioning from school to university and when transitioning back to school. For the first discontinuity, he writes: "[When a student is] confronted with problems, which do not remind him, in any particular, of the things with which he had been concerned at school. Naturally, he forgets all these things quickly and thoroughly" (Klein, 1924/2016, p. 1). Thus, the underlying reason is the different appearance of mathematics at school and at university resulting in students being unable to relate them to each other. Winsløw and Grønbaek (2013) showed that this is still a relevant problem in our current education system. Of course, the goal of courses in teacher education is to make mathematical and didactical insights relevant for future teaching at school. This threatens to fail as a consequence of the double discontinuity. To overcome Klein's double discontinuity, students have to see connections between the "axiomatic-formal" world of university mathematics and the "perceptual-symbolic" or "conceptual-embodied" world of school mathematics (cf. Tall, 2008). Unfortunately, students are not able to find those autonomously or incidentally (Winsløw & Grønbaek, 2013). The lack of perception of intra-mathematical connections leads not only to the fragmentation of school and university content but also to the fragmentation of teacher education as a whole, since the different mathematics courses, as well as mathematics and mathematics education, are likely to be perceived as isolated from each other. As a way to address

this, we develop and evaluate the learning tool *digital mathematical maps* (as in Brandl, 2009), which especially reveal connections between school and university mathematics.<sup>1</sup>

## Methodological Background: design-based research

For the development and evaluation of the learning tool, initially as part of a teaching format (see Schwarz et al., 2017), we use a design-based research approach (Bakker, 2018; see Figure 2). The teaching format and learning tool is applied in courses, followed by an evaluation and a subsequent adaptation of both based on the evaluation results. The objectives of the use of the mathematical maps within the teaching format are manifold. Examples are the demonstration of connections between school and university mathematics, the linking of university mathematics and didactics, and the conscious reflection on treated contents/methods/learning tools so that acquired knowledge and competencies from the course can be made useful for their future teaching practice.

In the process of the research cycles, the created teaching format and the mathematical maps were examined from different points of view (cf. next section). In the present research cycle, the technical implementation and the usefulness of the components of the mathematical maps are in focus. In particular we address the following research question:

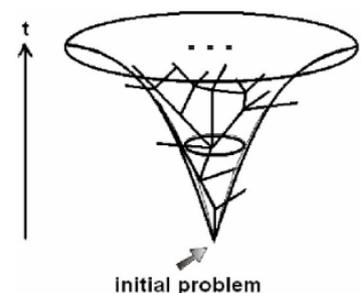
Does the use of mathematical maps within the geometry course help to counteract the double discontinuity by, among other things, promoting favorable beliefs related to the nature of mathematics?

With the help of the TAM framework (Davis, 1985; see Figure 5), conclusions are to be drawn about the students' attitude towards the learning and teaching tool, as well as about their actual intention to use the maps.

## Short description of previous research cycles

The concept of "mathematical maps" was introduced by Brandl (2009) as a supplementing didactical tool that shows interrelations between mathematical discoveries (horizontal dimension) as well as the development of a mathematical content - starting from an initial problem - in time (vertical dimension) (see Figure 1). In addition, (historical) contextual information should be included as a scaffold to attach new mathematical content (see Wood et al. 1976). In the next paragraphs, we briefly describe some research cycles (light cycles in Figure 2), in which the mathematical maps are developed as part of a teaching format. For details, we refer to the references given for each cycle.

As a first attempt to implement the didactical tool, Java script was used "to produce content knots which are connected to related content knots by lines in an automatically evolving 3D-visualization" (Schwarz et al., 2017, p. 2293). Due to programming difficulties in the first research cycle, a timeline of geometry was created and used instead of mathematical maps, so that the final evaluation had its



**Figure 1: Initial Idea for the design of the didactical tool (Brandl, 2009)**

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<sup>1</sup> The tool is freely available at the web address <https://math-map.fim.uni-passau.de/> under the tab Interactive Map.

main focus on the lectures (Datzmann & Brandl, 2019). As a consequence of the evaluation results, the teaching format was supplemented, among other things, by GeoGebra applets for the visualization of geometric facts.

Subsequently, the mathematical maps were redesigned according to the original design idea (Brandl, 2009) by one of the authors and integrated into the improved teaching format. The evaluation in this cycle again had a strong focus on the lectures of the blended learning course (Datzmann et al., 2020), and the mathematical maps were only evaluated by some of the faculty members and slightly modified for the subsequent design research cycle.

### Detailed description of the current research cycle

As a conclusion of the evaluation in the last research cycle, small adaptations to the mathematical maps were made. Subsequently, it was used as a didactical tool in a geometry course within teacher education at Karlstad University Sweden. In the following, the current research cycle up to the evaluation of the mathematical maps will be presented (see Figure 2 dark cycle).

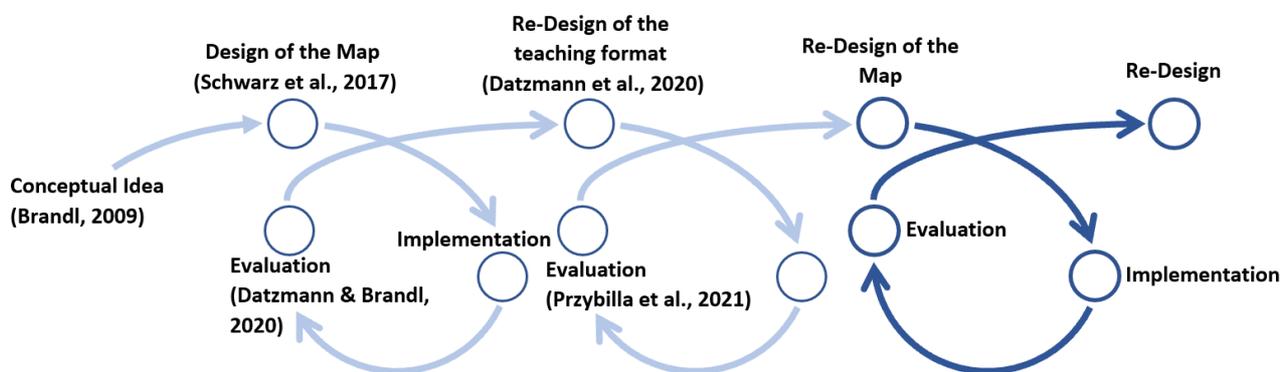
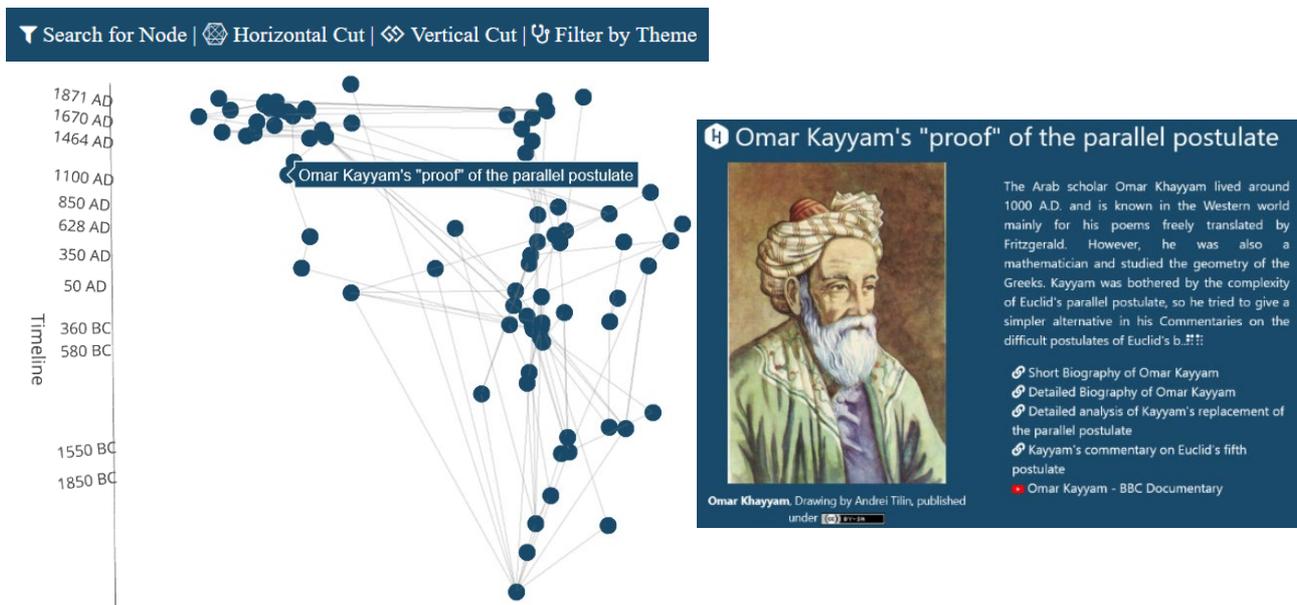


Figure 2: Illustration of the research cycles

### Re-Design of the mathematical maps

Since this article focuses on the development and research process regarding the mathematical maps, only central components that are important for the later evaluation will be outlined here. For detailed explanations of the individual functionalities, as well as technical details, see Przybilla et al. (2021).

In the last re-design of the mathematical maps, a preview of contents was set up so that when clicking on a node, all contents of this node are displayed on the right side of the screen (see Figure 3). Thus, all contents of a node can be directly accessed in the three-dimensional map, so that the user no longer have to take the roundabout route via the timeline if they are interested in a node. The various functionalities of the map can be applied via a bar at the top of the screen (See Figure 3). In the horizontal cut all nodes are projected into a plane so that via the Euclidean distance, statements can be made about the similarity of contents. In this way, inner-mathematical relationships shall become clear. As an additional feature, the nodes can be filtered by topic, so that similarities become even more apparent. In the vertical cut function, any node can be selected, for which then the historical genesis is represented as a directed graph in two dimensions (see Figure 4). The user can follow the graph and therefore the development directly using the preview on the right side of the screen.

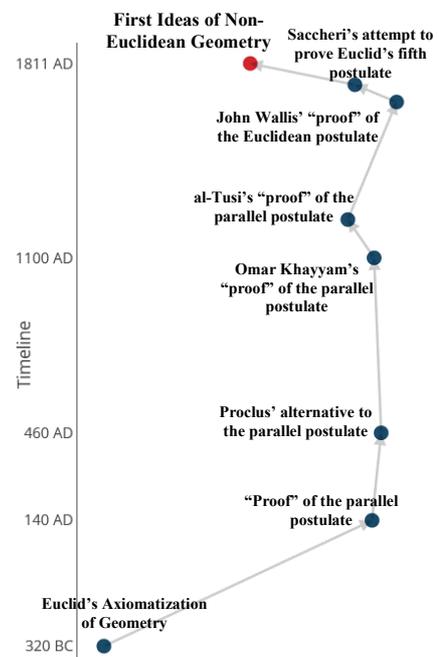


**Figure 3: Screenshot of the threedimensional mathematical map for geometry (Status: 05.07.2021)**

In addition, a file containing an overview of all nodes occurring in the evolutionary line can be created and downloaded via one click, so that it can be opened offline and be shared with others. In addition to a better understanding of the mathematical content, the main goal of the vertical cut is to promote an accurate picture of mathematics as an *emerging science*. It is a consequence of its very nature that mistakes and misunderstandings can and probably will occur when *doing* or *creating* mathematics. A good example from the history of mathematics is the centuries-long discussion about Euclid's parallel postulate (see Figure 4), which was also treated in the given geometry course. It also connects school mathematics (Euclidean geometry) with university mathematics (non-Euclidean geometry).

### Implementation

In the current research cycle, the mathematical maps have been used as part of a geometry course at Karlstad University. The course is anchored in the second semester of the teacher training program and can also be attended by teachers who would like to receive further mathematical training. The individual components of the course were designed cooperatively by university mathematics teachers and researchers from mathematics education from the University of Karlstad and the University of Passau. The content-related goal of the course is to cover central contents from Euclidean geometry and to introduce key concepts from non-Euclidean geometry. In addition, the module catalog requires the development of competencies for the evaluation of digital learning tools.



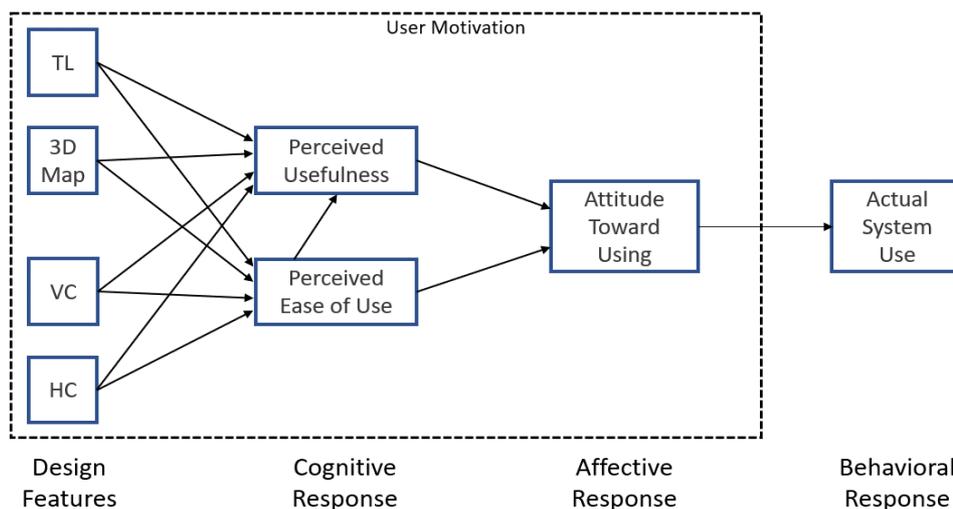
**Figure 4: Vertical Cut fort he node 'First Ideas on Non-euclidean Geometry' (Labels shifted, Status: 05.07.2021)**

In the semester under consideration, 44 (of in total 64) students (27 teachers, 17 prospective teachers) actively participated in the course. The weekly lectures were supplemented by contents and work assignments dealing with the digital didactical tool so that an evaluation of the individual components of the mathematical map could take place. Due to time constraints in the study plan, this was broken up into weekly short parts. The weekly work assignments were arranged chronologically as follows: 1. Timeline (*TL*), 2. Narrative didactics as an area of application of the contents in the mathematical map, 3. three-dimensional mathematical map (*3D Map*), 4. Vertical Cut (*VC*), 5. Horizontal Cut (*HC*), 6. The discussion around the parallel postulates 7. General evaluation of the didactical tool. For each of the components of the map considered in this article, a short explanatory video of no more than 10 minutes was created. Afterward, the participants were supposed to try out the respective functionality themselves on a work assignment.

### Evaluation

The weekly assignments<sup>2</sup> concluded with a brief technical and content evaluation of the individual component examined, organized in a quiz with both, single choice questions, which are laid out in a 5-point Likert scale, and free text questions. In addition, students and teachers had the opportunity to provide criticism and suggestions on how the learning tool should be improved in text fields. At the end of the course, an overall evaluation was carried out, in which learning gains through the use of the map and expected applicability for teaching with the map in schools were to be assessed. Results from this part are not part of this paper.

As the foundation for the technical evaluation of the digital tool, we use the Technology Acceptance Model of Davis (TAM, 1985; see Figure 5), which is widely used in the community and well supported both theoretically and empirically (cf. Scherer et al. 2018).



**Figure 5: Illustration of the Technology Acceptance Model for evaluating the mathematical maps (Davis, 1985)**

<sup>2</sup> The weekly assignments are freely available at the web address [https://docs.google.com/document/d/e/2PACX-1vTq\\_w6A2MqresPr5PQLGujDt3NIS6i7aiL7ShsmaR841TtI-hrsyTM2zLY6UEhbULBumIx0zEtTOz6V/pub](https://docs.google.com/document/d/e/2PACX-1vTq_w6A2MqresPr5PQLGujDt3NIS6i7aiL7ShsmaR841TtI-hrsyTM2zLY6UEhbULBumIx0zEtTOz6V/pub)

According to the model, a potential user's overall attitude toward using a given system is a major determinant of whether or not he actually uses it. Attitude toward using, in turn, is a function of two major beliefs: perceived usefulness and perceived ease of use. Perceived ease of use has a causal effect on perceived usefulness. Design features directly influence perceived usefulness and perceived ease of use. (Davis, 1985, p. 24) Therefore, the factors that determine whether a digital tool is used or not are perceived usefulness and perceived ease of use. This in turn depends crucially on the properties of the design features. In our case, we consider as design features the individual components and functionalities of the map (see Figure 5). In the quizzes, we ask about the perceived ease of use and perceived usefulness (for student learning and possible usage at school) of the individual components of the map. Therefore, we are able to draw conclusions about ease of use and usefulness of the entire digital tool.

### **Insights from the current evaluation and consequences for future research cycles**

In general, there were numerous suggestions to improve the didactical tool, which in their frequency and depth show that the students have intensively dealt with the map and its usefulness for teaching and learning processes. Examples of these are the request for a user manual for pupils, several design suggestions, improvement of the zoom function in the Horizontal Cut, and the general improvement of the functionalities for touch devices (especially of the 3d map). Thus, the goal of the course that students shall evaluate a digital didactical tool to build up competencies in this area was realized.

At this point, some results will be highlighted exemplary, with a focus on technical elements as hints for adjustments in the next research cycle. The timeline was found by over 80% of the students to be useful for presenting historical events in a clear and structured way. A content table or a search function within the timeline was requested several times to provide a clear overview of the included content. This could increase the given percentage. In the 3-dimensional map, both the search and the filter functions were found helpful by 89% of the active students. The preview within the map was also perceived as helpful by almost 80%. A large portion of the remaining 20% was bothered by a technical detail that has since been adjusted. While the timeline was perceived as difficult to use by only one person, there were more difficulties with the 3d map and a short training was necessary for fluent handling the 3D Map. This must be considered when using the map in another course. The statement "I see the 'Vertical Cut' functionality as a useful and clear way to show historical developments" was agreed to by 95% of the participants. In this functionality, the students and teachers saw the greatest opportunity for the use of the didactical tool in schools. The Horizontal Cut was seen by 93% as an appropriate way to show thematic connections of mathematical contents.

From a didactical perspective and concerning our research question, the evaluation results seem to indicate that the use of the map (within this course) contributes to favorable beliefs concerning the nature of mathematics, which "are a crucial part of the professional competence of mathematics teachers" (Felbrich et al., 2008, p. 763). In this context, a distinction is made between principal orientations: "the *formalism*-related orientation", "the *scheme*-related orientation", "the *process*-related orientation" and "the *application*-related orientation" (cf. *ibid.*, p. 764). The first two present a static picture and thus non-accurate picture of mathematics, which is unfavorable for teachers and learners (cf. *ibid.*). The last two reflect a dynamic understanding of mathematics and therefore

emphasize mathematics as an *emerging science* in which failures belong. It promotes an accurate view of *doing mathematics*, where an open error culture can be established. About 70% of the student responses reveal signs of a *process*-related and *application*-related orientation, which is a clear indicator that our research question can be answered with “yes”. One student, for example, wrote that the presentation of the history of mathematics as it is done in the map, shows “that mathematics is a subject that has been evolved over a long time and in close proximity to practical problems”. Signs of how the map may open up for dynamic understanding of mathematics can be seen in, for instance, students’ reasoning about the struggle when developing mathematical concepts. One student describes it as follows:

Knowing that learning is a process and that even the geniuses who discovered or invented the math they learn now were struggling as well and had the help of all the work that has been done before. This can create a sort of solidarity and lessen the anxiety of not immediately knowing what the teacher is talking about.

Many students’ show an application-related orientation in relation to their future role as teachers’ when using the map as a tool to motivate their future students “to make the content more interesting, and spice up the teaching”; their future students need to see the relevance of the maths. As one student put it: “it is unlikely that students in difficulties will use this tool by themselves”.

Overall, the current research cycle shows that the implemented parts of the map are perceived as useful and mostly easy to use. However, minor shortcomings in the functionalities and especially in the design became apparent, which will have to be adjusted in the next design cycle. In addition, thanks to the proposals of the participants, the map will be enhanced by some helpful functionalities.

## Acknowledgment

This project is part of the “Qualitätsoffensive Lehrerbildung”, a joint initiative of the Federal Government and the Länder which aims to improve the quality of teacher training. The programme is funded by the Federal Ministry of Education and Research (funding reference 01JA1924). This project was also supported by the research centre Science, Mathematics and Engineering Educational Research (SMEER) at Karlstad University. The authors are responsible for the content of this publication.

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