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Investigating forms of relation between teaching and research in the case of graph theory

Theresia Tabchi¹ and Hussein Sabra²

¹University of Reims Champagne Ardenne, France; theresia.tabchi@gmail.com

²University of Reims Champagne Ardenne, France; houssein.sabra@univ-reims.fr

This paper deals with university teachers' practices. It seeks to characterize the forms of relation that can take place between their teaching practices and their research activity in the domain of graph theory. Our exploratory study draws on an interview with a university teacher and observations of her classes that focus on her use of generic examples in the classroom. Generic examples are particularly important in proofs in graph theory, and we therefore estimate that their use by teachers can shed light on the potential influence of their research activity on their teaching.

Keywords: Generic examples, Graph theory, Relation between research and teaching, Resources, University teachers' practices.

Introduction, context, and research question

Our work deals with the practices of university mathematics teachers. It is a contribution to the growing body of research on university teachers' practices (Biza et al., 2016). This same study raised a main issue as a field that requires exploration: the relation between evolution in practices, evolution in resources, and evolution of teachers' professional knowledge. This paper is a contribution to the work around the mentioned issue. We are particularly interested in the impact of research on teaching in mathematics. We suggest seeking this issue through the lens of teachers' interaction with resources.

Some studies tried to characterize the influence of the research activity of university teachers on their teaching practices (Neumann, 1992; Madsen & Winsløw, 2009). These studies consider research to be potentially important in university education where teaching practices can be informed by research practices. Neumann (1992) suggested criteria to describe the connections between teaching and research. She underlined the need for further research on the topic that considers disciplinary specificities. Madsen and Winsløw (2009) proposed a theoretical model based on specific research tasks to describe the teaching-research nexus that takes disciplinary specificities into account.

We will be using, throughout the paper, "teaching practices" and "research activity" to refer respectively to the "teaching" and "research" aspects of the university teachers' work. We designate by "university teachers" the university teachers who occupy teaching/research positions.

In a previous research we presented in this Thematic Working Group (Sabra, 2019), we identified, from case studies, factors related to research activity that influence teaching practices. We mentioned the need to characterize the relation between the research activity and teaching practices of mathematics teachers to better understand some of the choices made in a given teaching context. In a related perspective, Speer et al. (2010) highlighted that the rationales behind some teacher choices are largely covert. They underlined that teachers' choice of examples seems to be an important dimension of their teaching practices; Speer et al. (2010) specified that the selection of examples by teachers can be influenced by their pedagogical knowledge and their knowledge of the mathematical

content that can evolve due to their interaction with this content in their activity of research. Indeed, the forms of relation between teaching and research depend closely on the profile of the university teachers. Several characteristics of the profile enhance those forms. Among them, if the teacher is an active mathematician or a trained mathematician (simply having a PhD in mathematics), their research field, the taught courses (the mathematics taught to engineers would differ from the mathematics taught to future mathematicians), the software used in their research, etc.

Therefore, we consider that examining teachers' use of examples can reveal elements about the knowledge that impacted their choices of examples, which can contribute to our understanding of their teaching practices and the possible influence of their research on these practices. We hypothesize that characterizing the influence of research on teaching practices provides a deeper understanding of these practices. We will further underpin our claim at a later stage in this section.

We consider in our study a specific type of examples: the generic examples. A generic example is an example presented in a way that focuses on key features and ignores others to bring out its role as carrier of the general (Mason & Pimm, 1984). In a study that seeks to characterize university teaching, Mali et al. (2014) studied the influence of research on teaching through focusing on a teacher's use of generic examples in tutorials. They found that the teacher relates her use of generic examples to the practice of "decoding and encoding" that mathematicians use in research, where decoding refers to understanding all parts of a problem and encoding means writing it down formally.

To study the teaching-research relation, we choose to consider generic examples in the case of graph theory. Many reasons motivate our choice. Graphs are useful tools for modeling real-life situations and play a significant role in the mathematical training of undergraduate students in engineering, economics, and mathematics (González et al., 2021). Rosen (2012) points out that graph theory trains students to construct proofs, by developing their abilities of abstraction and reasoning. In this context, generic examples allow chains of reasoning about particular objects that make explicit the reasons for the truth of a statement (Rowland, 2009) making them an important aspect of producing proofs. Indeed, in this paper, we will try to bring elements of answer to the following research question: how can we characterize, through the use of generic examples in proofs, the forms of relation between the teaching practices of graph theory and the research activities in the same field?

We expect that studying the relation between research and teaching at university level can help gain a deeper understanding of teachers' choices, in and out of classroom. It helps identifying reasons that can enhance – or reduce – the dissemination of research contents and skills in the context of university teaching. Hence, it can offer university students better learning opportunities by fostering a learning environment, connected - as much as possible - to the reality of the development of mathematics and its applications. In what follows, we will present our theoretical framework followed by the methodology and the collection of data along with the results obtained and some perspectives.

Theoretical Framework

We consider the model developed by Neumann (1992) to describe the teaching-research nexus. In addition, we draw on the Documentational Approach of Didactics (DAD) (Gueudet et al., 2012) with

a focus on operational invariants to study teachers' interactions with resources in their teaching practices. We also rely on Chevallard (1998) to define the two institutions, teaching and research.

Teaching-research nexus: forms of relation between teaching and research

Some researchers in science education attempted to find evidence of “positive” or “negative” correlations between research and teaching without considering a specific discipline (Neumann, 1992). They tried to characterize possible relations between teaching and research activities (symbiosis, conflict, tension, etc.). Neumann (1992) presents three aspects of what she calls “teaching-research nexus”: tangible, linked to a transfer of knowledge from research into teaching; intangible, related to the actions of a researcher in the teaching activity and vice versa; and global, referring to the nexus between teaching and research at departmental level. She underlines that these three types of nexus interrelate and intermingle, they are not necessarily separate or distinguishable.

In a study based on interviews focusing on the specificities of physical geography and mathematics, Madsen and Winsløw (2009) explore the nature and conditions of a positive “link” between teaching and research. They distinguish between two institutions, teaching and research. An institution is defined by Chevallard (1998) as being a “total” social entity, which allows and imposes on its subjects - people who occupy different positions offered within it - ways of doing and thinking. Chevallard sheds light on the cognitive dimension of institutions and on the relations of a subject (teacher or student) and of objects (such as mathematical knowledge) to institutions. He underlines that the objects of knowledge and the tools to produce them are considered differently when moving from an institution to another, from the research institution to the teaching institution in our case.

In what follows, we consider the tangible, intangible and global links between research and teaching as defined by Neumann (1992). We investigate the potential of this model as a lens to study the teaching-research nexus in the specific domain of graph theory. We rely on this model that was developed in the context of educational sciences; we investigate, in the specific domain of graph theory, its potential as a lens to study the teaching-research nexus.

Documentational Approach of Didactics: a framework to study interactions with resources

The DAD considers the activity of a teacher as a continuous process. It distinguishes between resources and documents (Gueudet, 2017), where resources are defined as all the things that could re-source a teacher's activity and documents are the result of the association of resources and schemes of use of these resources.

A scheme is defined by Vergnaud (1998) as the invariant organization of conduct for a class of situations which is a set of situations having the same aim. It is a dynamic structure with four interacting components: An *aim* that indicates intentionality in the organization of the activity; *Rules of actions* which are the ways of acting generated by the scheme to achieve a specific aim; *Operational invariants* that are the reasons behind the rules of action and the underlying justifications; and possibilities of *inferences* which are the adaptations brought by a subject to his activity to respond to the specificities of a situation corresponding to an aim. In a study based on interviews with university teachers, Gueudet (2017) constructed “documents tables” where each row comprises a class of situations, and the corresponding, resources, rules of action and operational invariants.

In our work, we seek to characterize teachers' interactions with resources in the teaching institution. We draw on the DAD to study teachers' interactions with resources in teaching graph theory. Hence, we specifically consider teaching aims and the corresponding situations classes. We hypothesize that characterizing teachers' interactions with resources and the operational invariants that drive and justify them interactions can provide us with elements about the research-teaching relation. This relation could take the form of a migration of the resources between the teaching and research institutions and the adaptations that follow, or of a dissemination by a teacher of the professional knowledge and mode of teaching (the operational invariants). We seek to identify operational invariants that guide teachers' use of generic examples which seem to have an important role in proofs in graph theory. Their use by teachers can shed light on their underlying mathematical and pedagogical knowledge, hence provide elements to characterize the nature of the teaching-research nexus (tangible, intangible, global) as defined by Neumann (1992).

Methodology

To identify forms of relation between teaching and research, we consider the case of a teacher, who we are going to call Chiraz. She is a trained mathematician; whose research was on applications of graph theory in telecommunication. We consider her interactions with a resource she uses to teach graph theory for second year students majoring in mathematics at the Lebanese University ([Click here to view the resource](#)). The resource, designed collectively by Chiraz and two colleagues, focuses on the theoretical aspect of graph theory. We consider her interactions with the resource in her teaching (planning and implementation), with a focus on the use of generic examples. We expect that her choice of examples can inform us about her relation to graph theory in the teaching and research institutions, hence about the possible influence of her research on her teaching. Our methodology is based on an interview ([Click here to view the grid](#)) with Chiraz and observation of her classes.

In the first stage, we transcribed the interview and constructed a documents-table. For that, we identified classes of situations in Chiraz's teaching practices from the analysis of the interview. We particularly considered classes of situations related to Chiraz's use of generic examples in her teaching. For each class of situations, we identified the corresponding rules of action and operational invariants. We highlight that a contrast can exist between Chiraz's relation to graph theory in the teaching and research institutions, where her teaching focuses on the theoretical aspect while her research targets the applied aspects of the domain. Identifying Chiraz's rules of action and operational invariants in her teaching can help us gain a deeper understanding of the choices she made in the design of the resource and its use in the classroom. Therefore, it can help us detect the influence, if it exists, of her research activity on her teaching, and the institutional conditions that impact her choices.

The second stage consists in classroom observations which can allow us to characterize forms of use of the resource in class, hence complete and specify the results obtained in the first stage. We chose to attend three sessions, consisting of lectures and tutorials, that deal with the concepts of "paths" and "cycles". Several theorems involving these concepts are presented in the resource with their proofs. We selected episodes where Chiraz makes use of generic examples. We were particularly interested in her approach when writing an argument to prove a statement or when solving an exercise such as reasoning on small objects that have a characteristic representative of a broader class of objects before

considering larger cases. These lines of reasoning are closely related to the epistemology of graph theory and can therefore indicate an influence of Chiraz’s research activity on her teaching in the classroom. Our role was limited to observing the progress of the sessions (with video recording) and the implementation of the resource, taking into account the interactions with students.

Data Analysis

Analysis of the interview

Chiraz claims that her research activity does not influence her teaching because her research was on applied mathematics while she teaches in a mathematics major. In her PhD work, she modeled telecommunications systems using graph theory, which assumes the optimization of costs and the number of operators to be used. This work requires to juggle between particular and general cases in connection with the phenomena to be modeled. She says: “in teaching graph theory, we start with the simple concepts, and then bit by bit, we increase the level”. Her perception of graph theory in the teaching institution as content where knowledge and skills are built in a cumulative way allows us to hypothesize the absence of a tangible teaching-research nexus in her teaching practices.

The analysis of the interview with Chiraz shows that most of the identified operational invariants in teaching situations classes are related to didactical knowledge and institutional conditions. Presenting contents in the resource in a theoretical manner with no links to other domains and disciplines seems influenced by institutional constraints related to the training course. Chiraz underlines that in graph theory, students have difficulty with proofs. Hence, she divides the proofs of the theorems into several parts to facilitate their understanding, which seems due to her personal experience with graph theory in the teaching institution. She points out that examples help students get to the proof of a theorem, she says “graph theory is more about sketching to be able to analyze the ideas, specially that it is the first introduction of students to graph theory, they cannot see it directly, they have to sketch”.

Eventhough we did not identify a tangible teaching-research nexus in the analysis of the interview with Chiraz, we estimate that an intangible nexus may exist between her research activity and her in-class teaching practices. This nexus can be identified in her use of the resource during class, her use of generic examples when reasoning on proofs, the attitude she tries to develop in students.

Analysis of classroom observations

In the first session we observed, Chiraz gave propositions and worked on their proofs with the students. We take as an example one proposition as presented in the resource (cf. figure 1).

<p>Proposition 21. <i>Let P and Q be two paths of a connected graph G of maximal length. Then, $P \cap Q \neq \emptyset$ ($V(P) \cap V(Q) \neq \emptyset$).</i></p> <p>Proof. Let us suppose that $V(P) \cap V(Q) = \emptyset$. Let $x \in P, y \in Q$. Let u, v be the extremities of P and u', v' the extremities of Q. Since G is connected, it contains an xy-path R. According to the previous proposition, since $P \cap Q = \emptyset$, then $\exists \alpha \in R \cap P, \beta \in R \cap Q$ such that $R_{[\alpha, \beta]} \cap P = \{\alpha\}$ and $R_{[\alpha, \beta]} \cap Q = \{\beta\}$. We can suppose, without loss of generality, that $l(P_{[u, \alpha]}) \geq \frac{l(P)}{2}$ and $l(Q_{[u', \beta]}) \geq \frac{l(Q)}{2}$. Then, $R' = P_{[u, \alpha]} \cup R_{[\alpha, \beta]} \cup Q_{[\beta, u']}$ is a path.</p> $l(R') = l(P_{[u, \alpha]}) + l(R_{[\alpha, \beta]}) + l(Q_{[\beta, u']}) \geq \frac{l(P)}{2} + 1 + \frac{l(Q)}{2}$ $= l(P) + 1 \text{ (Since } l(P) = l(Q)\text{)}$ <p>Then P is not a path of maximal length. Contradiction.</p>
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Figure 1: Our translation of proposition 21 and its proof, as shown in the collectively designed resource

The proof of the proposition consists in showing that if two longest paths of a graph have no intersection, then there will be a path longer than these two paths. Chiraz co-constructed the proof with the students, by asking questions about the steps to follow such as “what do we need to prove? What if it makes plenty of intersections?” (cf. figure 2).

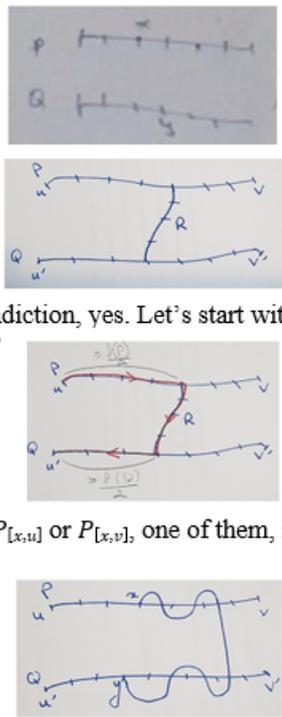
<p>Chiraz: What do you need to prove?</p> <p>Student: We need to prove that there is one common vertex, in P and Q.</p> <p>Chiraz: Yes, so P and Q intersect at some vertices, at least at one vertex, ok? So, there should be an intersection between any two longest paths in the graph if the graph is connected. So, the proof of this will be done by contradiction. Suppose that $P \cap Q$ is empty. Can you illustrate the paths P and Q?</p> <p>Students: They don't touch each other.</p> <p>Chiraz: So, I am in the case where P and Q are disjoint paths.</p> <p>Student: The graph is connected.</p> <p>Chiraz: Connected. This means what? That there is a path between any two vertices.</p> <p>Students: We take vertices from here and from there.</p> <p>Chiraz: Ok, very good. I will take two arbitrary vertices, x on P and an y on Q, (...), then G contains an xy-path, I will call it R. In your opinion, how will you reach the contradiction?</p> <p>Student: That P is not the longest path anymore?</p> <p>Chiraz: So, I will be able to find a path longer than P and Q, which will give the contradiction, yes. Let's start with the simple case (...) the simple case will make it closer to your understanding. If path R was like this, we should be able to find a path longer than both P and Q. Will you find it in this case?</p> <p>Student: Yes. Because G is connected, then y and v, there is a path between them... because it is connected, xy is equal to xu' plus $u'y$ for example. xv will be smaller than xy plus yv.</p> <p>Chiraz: Why? We have no information. Ok, here, x is on the path P, ok, one of these, $P_{[x,u]}$ or $P_{[x,v]}$, one of them, its length will be ... how much?</p> <p>Student: Smaller than the second one.</p> <p>Chiraz: Yes, so how much would it be? At least half of the length of P. One of them will be longer than the other one. The longer will have a length that is equal to what? To the length of P over 2, at least.</p> <p>So, this is the simple case. What if it makes plenty of intersections?</p>	
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Figure 2: Extract of the transcription of Chiraz's explanation of proposition 21 in class

She started by reasoning on particular cases. She first considered a case where the path between two vertices taken respectively on the two longest paths does not intersect with any of these two paths (except in the two vertices) (cf. figure 2). Then she considered a more general case where the path between the two vertices has several intersections with each of the two longest paths. Focusing on the key features of these two cases allowed Chiraz to establish the truth of the proposition. Taking small cases as a starting point before working on general cases can be justified by Chiraz's comment during the interview: “Particular cases, simpler cases, are closer to students' logical thinking, and once they understand the particular cases, they can generalize”. Reasoning on smaller/particular cases and moving to larger/general cases is a ‘classic’ research approach in graph theory (Goldin, 2004) and can therefore indicate the presence of an intangible nexus between Chiraz's activity of research and her teaching practices in class.

Results and discussion

In our attempt to characterize the forms of relations between research and teaching in graph theory, we conducted an interview with a graph theory teacher followed by classroom observations. The analysis of the interview did not reveal the presence of a teaching-research nexus. However, the in-class teaching, particularly the use of generic examples when reasoning on proofs by the teacher, showed an intangible nexus between the teacher's research activity and her teaching practices. It seems to us that the nature of the teaching-research nexus detected using Neumann's model (1992) depended on whether the teacher was describing her activity globally, or whether she was involved in a particular activity. It was also affected by the specificity of the discipline involved.

The choice of graph theory as a mathematical domain seems relevant to identify the possible forms of relation between the teacher's research activity and her teaching in the classroom. We were able to identify moments when the teacher in the classroom tried to develop in students' ways of reasoning that are used by mathematicians when working on research problems in graph theory which indicates the presence of an intangible teaching-research nexus. This nexus is specially identified in the teacher's use of generic examples. We highlight the fact that this study focuses on the case of a teacher with a profile characterized by a contrast between her PhD work (applied mathematics) and the mathematics she teaches (rather theoretical). Observing a teacher with a different profile in the same institutional context can shed more light on the observed phenomena.

The interview, as methodological tool, has limits in elucidating the intangible teaching-research nexus. There might be sometimes a disparity between the declared practices and the effective practices in the classroom. Despite the teacher's perception of no existing relation between her research and her teaching, and despite the choices made in the collective design to present contents (high level of formalism, and no mention of applications), an intangible teaching-research nexus was identified in her implementation of the resource in the classroom.

Considering the resources of a university teacher opens up new perspectives to improve our understanding of the relation between research activity and teachers' practices. Considering the interactions with students can be crucial to understand the relation between teaching and research. This consideration, in the particular case of intangible teaching-research nexus, may offer the possibility for identifying correlations between research activity and the expected activity of students.

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