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# Learning to teach statistics through study and research paths

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*In Brazil as in many other countries, there is a gap between the type of statistics preservice teacher learn in graduate programmes and the statistics they will teach at compulsory education. This paper presents the design of a study and research path for teacher education (SRP-TE), as a tool to reflect, develop, analyse, and experience teaching proposals in statistics for the final grades of the lower secondary school. The investigation is based on some principles of the Anthropological Theory of the Didactic and its proposals to locate teacher education within the paradigm of questioning the world. The proposed SRP-TE starts from a school activity about the water distribution in Brazil and its extension to incorporate dimensions of the statistical work that use to be absent from secondary education, like the search, collection, cleansing and representation of data.*

*Keywords: Anthropological Theory of the Didactic, study and research path for teacher education, statistics teaching, secondary school, working with data.*

## Introduction – research context

In this paper we present our PhD research proposal that is being developed at the *Universidade Federal de Mato Grosso do Sul* (Brazil) in partnership with the *Universitat de Barcelona* (Spain). The questions for the elaboration of the research project came from the initial education process followed by the first author during her degree in mathematics education. In the case of statistics, it seems that the double discontinuity described by Klein (Isaev & Eichler, 2017) appears with even greater prominence. The subject *Probability and Statistics* does not provide future mathematics teachers with a practical vision of the area. It is then common to hear reports from new mathematics teachers expressing difficulty in working with this theme in Brazilian compulsory education<sup>1</sup> when they take up a classroom, as happened with the first author of this article. An important gap is perceived between the education received and the teaching practice. In her master's thesis, Verbisck (2019) focused on the case of probability and the teaching proposals of different textbook collections from grade 1 to grade 12. In all the cases, only a small part of the books is devoted to probability and statistics, with a tendency to locate it in the last chapters of each book. More than ten years ago, Lopes and Ferreira (2004, p.12, our translation) stated that “until the implementation

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<sup>1</sup> In Brazil, the compulsory education is composed by preschool (for children between three and five years old); primary school (for children between six and ten years old); secondary school (for children between eleven and fourteen years old); and high school (for teenagers between fifteen and seventeen years old).

of [...] PCNs<sup>2</sup> (MEC, 1998), the teaching of statistics at elementary and high school grades was very restricted and marginal. The topics covered were included in the mathematics discipline, in the most advanced grades and, generally, it was one of the last topics in the textbook [...]"'. However, this is a provision that persists today, at least in more than half of the textbooks approved by the Brazilian ministry of education in 2017.

We believe that this and other results observed in this analysis reflect on the practice of mathematics teachers, who need to look for other sources to overcome the difficulties found when teaching probability and statistics. Therefore, for the PhD research proposal, we decided to focus only on statistics and restrict it to the final two grades of lower secondary school (students between 12 and 14 years old).

Thus, our PhD research objective is to investigate possibilities and contributions of implementing a teacher education proposal for pre-service mathematics teachers as a way to focus their reflections on these and other issues surrounding statistics teaching. It also provides teachers with experience in carrying out an inquiry, with the search and construction of means to overcome difficulties that future teachers may face when teaching statistics at these levels of schooling. Thus, the research question that we will seek to answer with the development of this thesis is: *What educational proposal is it possible to implement with a group of pre-service mathematics teachers in Brazil and in what aspects this proposal contributes to provide future teachers with tools to design, analyse and implement new didactic processes for the teaching of statistics in the final grades of the lower secondary school?* This investigation is based on the Anthropological Theory of the Didactic (ATD), within the paradigm of questioning the world (Chevallard, 2015), which will be detailed in the next section.

### **Theoretical framework: study and research paths for teacher education**

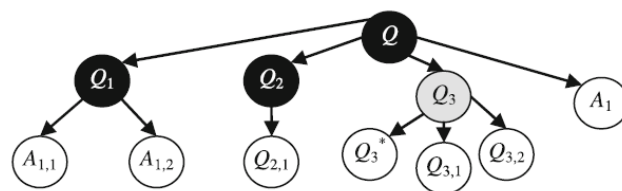
Chevallard (2015) states that, in our societies, teaching mathematics – and teaching in general – participates from what he calls the *paradigm of visiting works*. In this paradigm, the role of students is to study ready and finished knowledge organisations built in topics, areas, domains, and disciplines. It is up to the students to “look and admire” these works without necessarily questioning their validity nor their value. So, in this paradigm, topics and subjects are like monuments: students cannot change them, they do not need to know their *raison d'être*, they just have to study them. Questions about the knowledge value and validity are, in any case, posed by the teacher, characterized as “the one who has the knowledge”.

In contrast to this first paradigm, Chevallard describes the *paradigm of questioning the world* in which knowledge organisations change into *questions*. Teaching and learning processes become inquiry processes aiming at answering the questions. In other words, students are the inquirers of the generating questions *Q* proposed by the teachers (or by the students themselves). In the search

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<sup>2</sup> *Parâmetros Curriculares Nacionais* (National Curriculum Parameters), official documents used in Brazil as a curricular tool for educators and educational institutions. These curriculum documents were published in 1997 and 1998 and have recently been replaced by the document: BNCC, *Base Nacional Comum Curricular* (Common National Curriculum Base). The BNCC was approved by the Brazilian Education Minister on December 20, 2017. With this document, "school systems and public and private educational institutions now have a mandatory national reference for the preparation or adaptation of their curricula and pedagogical proposals" (Brasil, 2017, p. 5, our translation).

for answers to a question, there may be times when *visiting works* is required, but with a specific *raison d'être* (answering  $Q$ ). They also have to search for answers in medias (as internet, books, experts, etc.), and they have to validate these answers and check their utility to answer  $Q$ . This paradigm creates strong changes in the teachers' and students' role. The first are no longer the "holders of knowledge" and the students raise questions, investigate, search or elaborate answers and even validate them. To study the conditions needed for the transition to this second paradigm, Chevallard (2015) proposes a general inquiry format called *study and research paths* (SRP). The interplay between questions and answers plays a crucial role in the dynamics of SRPs. Students, helped by teachers, address an initial question  $Q$ , display  $Q$  into derived questions  $Q_i$ , search or elaborate answers  $A_i$ , find new questions during the process which, in turn, call for new answers, etc. Bosch and Winsløw (2015) point at the importance of such dialectic between questions and answers to ensure the dynamics of SRPs, which is usually represented using *questions-answers maps* (Figure 1).



**Figure 1: Example of Questions-Answers maps (Q-A) (Winsløw et al., 2013, p. 271)**

Barquero, Bosch and Romo (2015) argue that teacher education proposals also need to be conceived within the new paradigm of *questioning the world*. Thus, they consider implementing *study and research paths for teacher education* (SRP-TE) "as a way to provide teachers with pertinent (theoretical and practical) tools to nourish and sustain their professional development" (Barquero et al., 2015, p. 810). An SRP-TE consists in five modules, which will be described below.

*Module 0:* This stage is to propose the open question (generating question  $Q_{0-TE}$ ) related to the subject to be discussed. In our case, our  $Q_{0-TE}$  is: "How to teach statistics in the final grades of lower secondary school?". Other derived questions will appear as search elements to answer the generating question, such as: Why is there so little attention in statistics at school? What are the conditions and constraints<sup>3</sup> for teaching statistics in the final grades of the lower secondary school? What statistics should we teach at these levels according to the curriculum? What kind of activities are proposed by official textbooks? What other proposals exist? Partial answers to  $Q_{0-TE}$  will appear during the entire SRP-TE (and hopefully also afterwards), making it a transversal module of the educational process.

*Module 1:* This module consists of proposing the group of teachers an SRP with a relevant question that could be approached in a real classroom, as if they were the "students" of the final grades of the lower secondary school. "The SRP can actually have been implemented in previous investigations

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<sup>3</sup>Conditions can be understood as limitations (or rules) that can be overcome and constraints are those that cannot be overcome. The school imposes conditions and constraints on those who occupy some function in this institution, such as the teacher and the student. For example, the time/class is a condition that the teacher cannot change, so it is characterized as a restriction (Chevallard, 2007).

or may simply have been designed by researchers for this purpose” (Barquero et al., 2015, p.810). With this, the group of teachers will be introduced to an inquiry process.

*Module 2:* This stage is the moment for teachers to analyse the experienced SRP:

Three main phases are distinguished: (a) the mathematical analysis of the work done, including the elaboration of a reference epistemological model describing the modelling process involved (Bosch & Gascón, 2006); (b) a didactic analysis of the process, including a description of the differences between the contract established during the SRP to manage the modelling process, compared to the usual school didactic contract centred on the transmission of contents; (c) a more general study of the viability of the SRP, including the identification of the institutional conditions and constraints affecting the development of modelling practices in school settings. (Barquero et al., 2015, p. 810)

*Module 3:* At this stage, teachers design and implement (if possible) an SRP for the schooling level under discussion. The new design is carried out based on the analyses made in the previous module, adapting, adding, or excluding (if necessary) didactic tools that seem necessary – or at least useful – for the implementation of this new SRP.

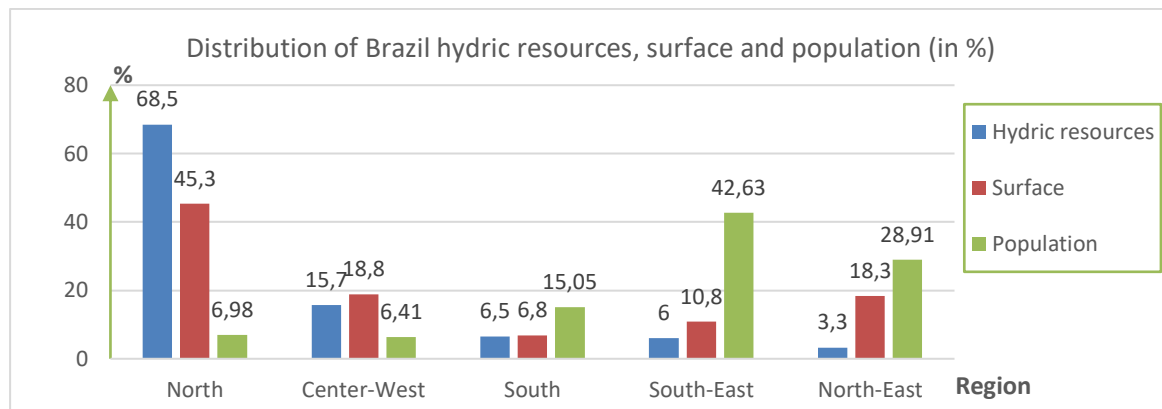
*Module 4:* This last module consists in the teachers sharing their experiences during all the implementations of the SRP to carry out an a posteriori analysis. Again, the mathematical and didactic tools used in modules 2 and 3 are present here and have great importance, “not only to provide some provisional answers to the question that was at the origin of the whole process ('How to teach ...?'), but also as a means to analyse other possible alternative answers” (Barquero et al., 2015, pp. 810-811), as those answers found in Module 0.

These are the modules that structure the SRP-TE. We present below an exercise identified in a Brazilian textbook of the eighth grade of secondary school that gave us the initial idea of a generating question to be studied in the SRP when we are developing module 1 with the group of teachers.

### **From a school exercise to a study and research path**

The proposed SRP for the teachers starts from a school activity about “the water distribution in Brazil”. Looking at the approach to statistics in some textbooks from the eighth and ninth grades of secondary school, we find the exercise below:

Brazil has about 13.7% of the total fresh water in the world, being considered a territory rich in water terms. However, the country is experiencing serious problems, related to both the degradation of water quality, especially in the urban areas' proximity, and the lack of control of excess and insufficiency of water, which affect several Brazilian locations. It is not just the floods that affect Brazilian cities: water scarcity also imposes serious restrictions and high costs on economic and social development of large cities in Brazil. Looking at the chart below, answer in your notebook:



Information obtained in: MINISTÉRIO DO MEIO AMBIENTE. Available in: <[http://www.mma.gov.br/estruturas/sedr\\_proecotur/\\_publicaco/140\\_publicacao09062009025910.pdf](http://www.mma.gov.br/estruturas/sedr_proecotur/_publicaco/140_publicacao09062009025910.pdf)>. Accessed: 1st July 2018.

- What kind of chart is this?
- Indicate the Brazilian region:
  - with the largest surface;
  - with more water resources;
  - with the second lowest population concentration.
- Which region has the lowest percentage rate of water resources in our country?
- In which region is there the greatest concentration of population?
- Can it be said that the larger the surface of the region, the greater the number of inhabitants? Justify your answer.
- How many percent of the world's fresh water is in the Southeast region of Brazil? Explain how you elaborate your answer.
- Can it be said that the region with the most water resources is the one with the largest population? (Giovanni Júnior, 2018, p. 26, our translation).

This type of exercise, which is rather common in the textbooks, is clearly located in the paradigm of visit works. Although it presents a relevant topic to be discussed (and even includes a link to the source of the data), it does not clearly pose an open question, nor does it encourage students to search the data, organise and summarise them, and investigate other issues related to the topic. The questions raised in the textbook only require students to look at the graph, identify the largest or smallest bar, and perform some simple percentage calculations. If we look at the official curriculum guidelines, we can observe that this is not the kind of activity that encourages students to develop the skills assigned to this level of education, which are:

- Assess the suitability of different types of charts to represent a survey dataset.
- Classify the frequencies of a continuous variable of a survey into classes so that they summarise the data in a way that is suitable for decision making.
- Obtain the values of measures of central tendency of a statistical survey (mean, mode and median) with an understanding of their meanings and relate them to the data dispersion, indicated by the range.
- Select reasons of different natures (physical, ethical, or economic) that justify conducting sample and non-census surveys and recognise that sample selection can be done in different ways (simple random, systematic, and stratified sampling).
- Plan and execute a sample survey, selecting an appropriate sampling technique, and write a report that contains the appropriate graphs to represent the data sets, highlighting aspects such as measures of central tendency, range, and conclusions. (Brasil, 2017, p. 315, our translation)

Despite the limitations of the exercise proposed, the topic presents many interesting questions that could generate a potential SRP within and SRP-TE. For this, we need to move from the paradigm of visiting works to the one of questioning the world and *question* the piece of information given. We will see, at the same time, how the inquiry process it generates can incorporate dimensions of the statistical work that use to be absent from secondary education, like the search, collection, cleansing and representation of data, together with a critical reading of quantitative information. Thus, for the SRP to be developed with the group of pre-service teachers in Module 1, we propose to start with Q0 and initiate a questioning that can led to following derived questions (starting from the same text and graphical information as before):

Q0: How can we explain the contradiction between the abundance of water resources and the water problems (scarcity, quality degradation, lack of control, etc.) of Brazilian locations? How are hydric resources distributed in Brazil, compared to the population and surface?

Q1\_Water: What do we know about water distribution in Brazil?

Q1.1. Are there studies about the water problem in Brazil? Where can we find them?

Q1.2. What disciplines are involved in the studies: geography, politics, geology?

...

Q2\_Graph: What information can we draw from the graph?

Q2.1. What variables appear in the graph? What others could it be interesting to consider?

Q2.2: What are hydric resources? How are they measured?

Q2.3: Why are the variables in percentages? How are these percentages calculated?

Q2.4: How would be the graph if we use units instead of percentages?

Q2.5: Can we improve the graph using another type or adding/omitting information?

...

Q3\_Data: What data is used to make the graph?

Q3.1 Is it available? Where? [The presented link does not work.]

Q3.2. Is the data available also reliable? How is it obtained?

Q3.3. What are the units of the different variables in the data source?

Q3.4. Can we use the available data to reproduce or update the graph?

Q3.5. Are there other interesting variables with available data to consider?

...

Q4\_Working with data: How to download the data to start working with it?

Q4.1. How to clean the table of data to make it ready-to-use?

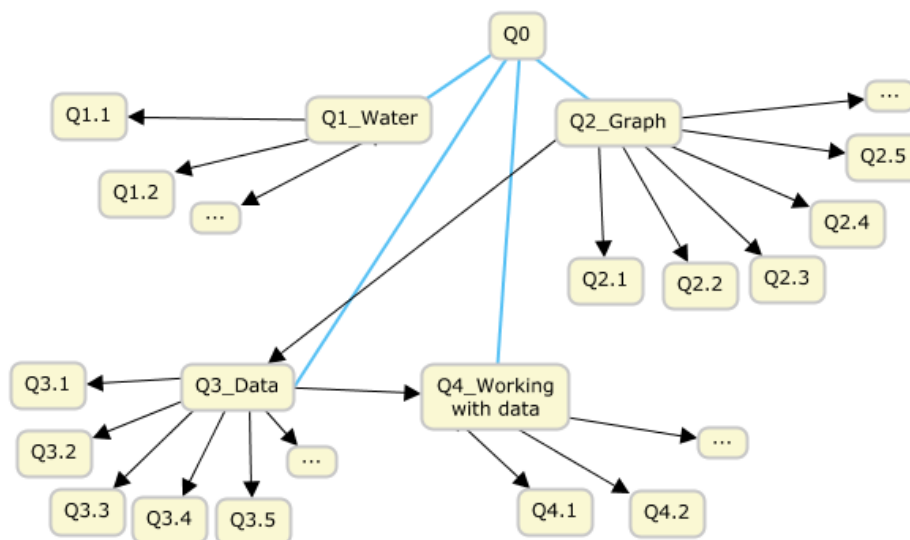
Q4.2. What tools are available for data processing and which ones can we use?

Q4.3. What types of numerical and graphical summaries are appropriate?

...

We started from the main subject of the text presented in the textbook exercise and elaborated a relevant open question (Q0) that could be worked on in a classroom. As the exercise presented a graph with the distribution of water in Brazil, as well as the population and surface area by region, this graph will also serve as a possible answer for a derived question, since its source is the Ministry of the Environment. In this initial work to list our generating question and possible derived questions, we identified two large groups of questions: those related to the topic "water" (Q1, Q1.1, Q1.2, ...) and those related to the topic "graph" (Q2, Q2.1, Q2.2, ...). Still, within the topic "graph",

another large group of questions appears, which is the topic related to "data" (Q3, Q3.1, Q3.2, ...). By raising the questions related to "data", we entered the topic "working with data" (Q4), which also made us think of derived questions (Q4.1, Q4.2, ...). With this, we obtain an a priori questions-answers map (Figure 2) that is part of SRP-TE.



**Figure 2. Questions-answers maps about water distribution in Brazil**

We started from an exercise proposed in a Brazilian textbook and reworked it to display the initial and derived questions that may arise during the inquiry process. This questions-answers map serves as a possible SRP to be developed with the group of pre-service mathematics teachers. The answers are still being developed, since we have not yet worked directly with the group. It is an a priori study about the proposed theme: distribution of water in Brazil and, with this SRP in development, we believe it will be possible to work with dimensions of the statistical work that use to be absent from the final grades of the lower secondary education, like the search, collection, cleansing and representation of data. It is likely that the inclusion of provisional answers will enrich the map with more questions and potential maps to follow. It will also introduce more connections between the derived questions and the different branches they form.

The a priori questions-answers map shows the productivity of the change of attitude from *visiting works* to *questioning the world*. It first gives visibility to many interrogations that are usually hidden in the way school tends to present (quantitative and also qualitative) information, giving no room for doubts or questioning. It also shows how these interrogations go beyond the strict statistical reading and analysis of data to merge with other disciplines or areas concerned by the question addressed. This is not specific to statistics, even if it plays a crucial role there. Teaching modelling also suffers from the same phenomenon of disciplinary confinement. It makes many connections between different topics of statistics that tend to be presented separately even if they nourish each other (like the definition and measurement of variables, the reliability of data, the ambiguity of percentages, the pertinence of the type of graphs chosen, etc.). Finally, it provides future teachers with a broaden – and more real – vision of statistics that can help them detach from the narrow vision proposed by textbooks exercises and might facilitate the introduction of richer statistical activities in the classrooms.



## Conclusions

Our hypothesis is that the design and implementation of an SRP-TE in statistics with a group of pre-service mathematics teachers will provide them with tools to reflect, develop, analyse and implement new statistics teaching proposals when working in the final grades of the lower secondary school, as well as developing a critical posture when acting in their profession. We emphasize that the theoretical (ATD) and methodological (SRP-TE) frameworks adopted are essential for this purpose. The change of paradigms proposed by Chevallard (2015) makes us, researchers, also adopt a critical stance towards the social, educational, political and epistemological dimensions that involve the themes of teacher education and statistics teaching.

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