



HAL
open science

Learning to teach statistics through study and research paths

Janielly Taila Verbisck, Marilena Bittar, Marianna Bosch

► **To cite this version:**

Janielly Taila Verbisck, Marilena Bittar, Marianna Bosch. Learning to teach statistics through study and research paths. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03754718v1

HAL Id: hal-03754718

<https://hal.science/hal-03754718v1>

Submitted on 19 Aug 2022 (v1), last revised 25 Jan 2023 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Learning to teach statistics through study and research paths

Janielly Taila Verbisck¹, Marilena Bittar² and Marianna Bosch³

¹Universitat de Barcelona, Spain, Universidade Federal de Mato Grosso do Sul, Brazil, janielly.verbisck@gmail.com;

²Universidade Federal de Mato Grosso do Sul, Brazil, marilenabittar@gmail.com;

³Universitat de Barcelona, Spain, marianna.bosch@ub.edu

In Brazil, as in many other countries, there is a gap between the type of statistics preservice teachers learn in graduate programmes and what they will teach at compulsory education. This paper presents the design of a study and research path for teacher education (SRP-TE) as a tool to reflect, develop, analyse, and experience teaching proposals in statistics for the final grades of the lower secondary school. The investigation is based on some principles of the Anthropological Theory of the Didactic and its proposals to locate teacher education within the paradigm of questioning the world. The proposed SRP-TE starts from a school activity about the water distribution in Brazil and its extension to incorporate dimensions of the statistical work that tend to be absent from secondary education, such as the search, collection, cleansing, and representation of data.

Keywords: Anthropological Theory of the Didactic, study and research path for teacher education, statistics teaching, secondary school, working with data.

Introduction – Research context and aim

In this paper, we present a PhD investigation developed at the *Universidade Federal de Mato Grosso do Sul* (Brazil) in partnership with the *Universitat de Barcelona* (Spain). The question motivating the research project came from a perceived limitation in the preservice teacher education process followed by the first author during her degree in mathematics education. In the case of statistics, it seems that the double discontinuity described by Klein (Isaev & Eichler, 2017) appeared there with even greater prominence. The subject *Probability and Statistics* did not provide future mathematics teachers with a practical vision of the area. It is then common to hear reports from new mathematics teachers expressing difficulty working with this theme in Brazilian compulsory education¹ when they take up a classroom, as happened with the first author of this article. A significant gap is perceived between the education received and the teaching practice, and the situation is not limited to Brazil. According to Martignon (Batanero et al., 2011):

[...] the gap between disciplinary statistics and school statistics has to be taken seriously into account when preparing future school teachers of statistics who have to be aware that they will be providing future citizens with “statistical literacy”. In other words, future citizens need to be endowed with tools for interpreting statistical information in the media, for dealing with relative and absolute risks, and for understanding the effect of base rates on the predictive accuracy of medical tests” (Batanero et al., 2011, p. 34)

¹ In Brazil, the compulsory education is composed by preschool (for children between four and five years old); primary and lower secondary education (for children between six and fourteen years old), that is also known as elementary education; and upper secondary education or high school (for teenagers between fifteen and seventeen years old).

In her master's thesis, Verbisck (2019) focused on the case of probability and the teaching proposals of different textbook collections from grade 1 to grade 12. In all cases, only a small part is devoted to probability and statistics, with a tendency to locate it in the last chapters of each book. More than ten years ago, Lopes and Ferreira (2004, p.12, our translation) stated that “until the implementation of [...] PCNs² (MEC, 1998), the teaching of statistics at elementary and high school grades was very restricted and marginal. The topics covered were included in the mathematics discipline, in the most advanced grades and, generally, it was one of the last topics in the textbook [...]”. However, this persists today, at least in more than half of the textbooks approved by the Brazilian ministry of education in 2017.

There is an agreement in the international research community on the importance of adopting a broad view of statistics that includes aspects such as searching for and collecting data, cleaning and organising data, visualising data, using specific software, preparing reports, all within a perspective of solving open questions and studying variability. These perspectives are described in terms of: “statistical reasoning” (Garfield & Ben-Zvi, 2008), “statistical literacy” (Watson, 2006), “statistical thinking” (Wild & Pfannkuch, 1999), “informal inferential reasoning” (IIR) (Leavy, 2010) and “informal statistical inference (ISI)” (de Vetten et al., 2019).

How to transpose this perspective in teacher education is still an open problem. Statistical investigations are often taken as a basis for enlarging teachers content knowledge while providing them with new pedagogical resources (Makar & Fielding, 2011, Pfannkuchm & Ben-Zvi, 2011, Santos & da Ponte, 2014). Our PhD research proposal follows this line of research, focusing on lower secondary school teacher education. We aim to investigate the possibilities and contributions of implementing a teacher education proposal to focus their reflections on current statistical practice and literacy. The proposal should also provide teachers with experience in carrying out statistical inquiries. Thus, the broad research question framing the development of the thesis is: *What educational proposal is it possible to implement with a group of preservice mathematics teachers in Brazil and how this proposal contributes to providing future teachers with tools to design, analyse and implement new didactic processes for the teaching of statistics in lower secondary school?* This question is approached within the Anthropological Theory of the Didactic (ATD) and, more particularly, a specific teacher education proposal called “study and research paths for teacher education” (SRP-TE). After presenting the theoretical approach, we will describe the design of an SRP-TE that will be implemented during this academic year 2022, with a pilot version during the last term of 2021. The SRP-TE starts from an open question taken from a Brazilian textbook. We will illustrate in this paper how the inquiry promoted by an SRP-TE can highlight the different dimensions of the statistical work and also be used as a tool to analyse curriculum and textbook proposals.

Theoretical framework: Study and research paths for teacher education

Chevallard (2015) states that, in our societies, teaching mathematics—and teaching in general—participates from what he calls the *paradigm of visiting works*. In this paradigm, the role of students is to study ready and finished knowledge organisations built in topics, areas, domains, and disciplines. It is up to the students to “look and admire” these works without necessarily questioning their validity or value. So, in this paradigm, topics and subjects are like monuments: students cannot change them; they do not need to know their *raison d'être*; they just have to study them. At most, questions about knowledge value and validity are posed by the teacher, characterised as “the one who knows”.

² *Parâmetros Curriculares Nacionais* (National Curriculum Parameters), official documents used in Brazil as a curricular tool for educators and educational institutions. These curriculum documents were published in 1997 and 1998 and have recently been replaced by the document: BNCC, *Base Nacional Comum Curricular* (Common National Curriculum Base). The BNCC was approved by the Brazilian Education Minister on December 20, 2017. With this document, “school systems and public and private educational institutions now have a mandatory national reference for the preparation or adaptation of their curricula and pedagogical proposals” (Brasil, 2017, p. 5, our translation).

In contrast to this first paradigm, Chevallard describes the *paradigm of questioning the world* in which knowledge organisations change into *questions*. Teaching and learning processes become inquiry processes aiming at answering the questions. In other words, students are the inquirers of the generating questions Q proposed by the teachers (or by the students themselves). There may be times when visiting works is required in the search for answers to a question, but with a specific *raison d'être* (answering Q). They also have to search for answers in medias (as the internet, books, experts, etc.), and they have to validate these answers and check their utility to answer Q . This paradigm creates significant changes in the teachers' and students' roles. The first ones are no longer the "holders of knowledge", and the students raise questions, investigate, search or elaborate answers and even validate them. To study the conditions needed to transition to this second paradigm, Chevallard (2015) proposes a general inquiry format called *study and research paths* (SRP). The interplay between questions and answers plays a crucial role in the dynamics of SRPs. Students, helped by teachers, address an initial question Q , display Q into derived questions Q_i , search or elaborate answers A_i , find new questions during the process which, in turn, call for new answers, etc. Bosch and Winsløw (2015) point at the importance of such a dialectic between questions and answers to ensure the dynamics of SRPs, which is usually represented using *questions-answers maps* (Figure 1).

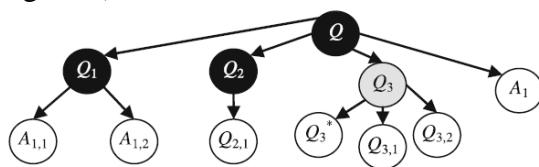


Figure 1: Example of A) (Winsløw et al., 2013,

Questions-Answers maps (Q-p. 271)

Barquero, Bosch and Romo (2015) argue that teacher education proposals also need to be conceived within the new paradigm of *questioning the world*. Thus, they consider implementing *study and research paths for teacher education* (SRP-TE) "as a way to provide teachers with pertinent (theoretical and practical) tools to nourish and sustain their professional development" (Barquero et al., 2015, p. 810). An SRP-TE consists of five modules, which will be described below.

Module 0 is to propose the open question (generating question Q_{0-TE}) related to the subject to be discussed. In our case, our Q_{0-TE} is: "How to teach statistics in the final grades of lower secondary school?". Other derived questions will appear, such as: What statistics should we teach at these levels according to the curriculum? What kind of activities are proposed by official textbooks?

What other proposals can exist? Partial answers to Q_{0-TE} will appear during the entire SRP-TE (and hopefully also afterwards), making it a transversal module of the educational process. *Module 1*

consists of proposing the group of teachers an SRP with a relevant question that could be

approached in a real classroom as students. With this, the group of teachers will be introduced to an inquiry process, a strategy also suggested by Makar and Fielding (2011). *Module 2* is the moment

for teachers to analyse the experienced SRP using epistemological and didactic tools spontaneously accessed by the students or provided by the educators. In *Module 3*, teachers design and implement

(if possible) an SRP for the schooling level under discussion. The new design is carried out based on the analyses made in the previous module, adapting, adding, or excluding (if necessary) didactic tools that seem necessary – or at least useful – for the implementation of this new SRP. Finally,

Module 4 consists of the teachers sharing their experiences through an a posteriori analysis. Again, the mathematical and didactic tools used in modules 2 and 3 are present here and have great

importance, "not only to provide some provisional answers to the question that was at the origin of the whole process (e.g., "How to teach ...?"), but also as a means to analyse other possible alternative answers" (Barquero et al., 2015, pp. 810-811), as those answers found in Module 0.

These modules structure the SRP-TE. We present below an exercise identified in a Brazilian textbook of the eighth grade of secondary school that gave us the initial idea of a generating question to be studied in the SRP when developing module 1 with the group of teachers.

Designing a teacher education proposal

The proposed SRP for the teachers starts from a school activity about “the water distribution in Brazil”. Looking at the approach to statistics in some textbooks from the eighth and ninth grades of secondary school, we find the exercise below:

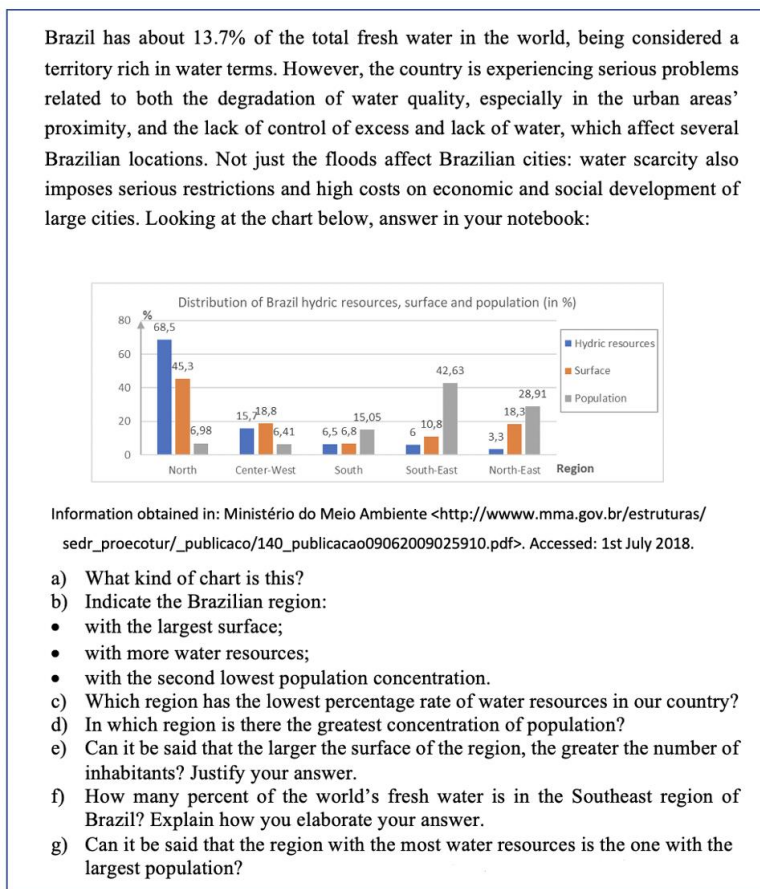


Figure 2: Brazilian exercise (Giovanni Júnior & Castrucci, 2018, p. 26, our translation)

This type of exercise shown in Figure 2, and which is relatively common in the textbooks, is clearly located in the paradigm of visit works. Although it presents a relevant topic to be discussed (and even includes a link to the source of the data), it does not pose any open question, nor does it encourage students to search the data, organise and summarise them, and investigate other issues related to the topic. The questions raised in the textbook only require students to look at the graph, identify the largest or smallest bar, and perform some simple percentage calculations. If we look at the official curriculum guidelines, we can observe that this is not the kind of activity that encourages students to develop the skills assigned to this level of education, which are:

- Assess the suitability of different types of charts to represent a survey dataset.
- Classify the frequencies of a continuous variable of a survey into classes so that they summarise the data in a way that is suitable for decision making.
- Obtain the values of measures of central tendency of a statistical survey (mean, mode and median) with an understanding of their meanings and relate them to the data dispersion, indicated by the range.
- Select reasons of different natures (physical, ethical, or economic) that justify conducting sample and non-census surveys and recognise that sample selection can be done in different ways (simple random, systematic, and stratified sampling).

- Plan and execute a sample survey, selecting an appropriate sampling technique, and write a report that contains the appropriate graphs to represent the data sets, highlighting aspects such as measures of central tendency, range, and conclusions. (Brasil, 2017, p. 315, our translation)

Despite the limitations of the exercise proposed, the topic presents many interesting questions that could generate a potential SRP within and SRP-TE. For this, we need to move from the paradigm of visiting works to the paradigm of questioning the world and *question* the piece of information given. At the same time, we will see how the inquiry process it generates can incorporate dimensions of the statistical work that tend to be absent from secondary education, like the search, collection, cleansing and representation of data, together with a critical reading of quantitative information. Thus, for the SRP to be developed with the group of preservice teachers in Module 1, we propose to start with Q0 and initiate a questioning that can lead to the following derived questions (starting from the same text and graphical information as before):

Q0: How can we explain the contradiction between the abundance of water resources and the water problems (scarcity, quality degradation, lack of control, etc.) of Brazilian locations? How are hydric resources distributed in Brazil compared to the population and surface?

Q1_Water: What do we know about water distribution in Brazil?

Q1.1. Are there studies about the water problem in Brazil? Where can we find them?

Q1.2. What disciplines are involved in the studies: geography, politics, geology?

...

Q2_Graph: What information can we draw from the graph?

Q2.1. What variables appear in the graph? What others could it be interesting to consider?

Q2.2: What are hydric resources? How are they measured?

Q2.3: Why are the variables in percentages? How are these percentages calculated?

Q2.4: How will be the graph if we use units instead of percentages?

Q2.5: Can we improve the graph using another type or adding/omitting information?

...

Q3_Data: What data is used to make the graph?

Q3.1 Is it available? Where? [The presented link does not work.]

Q3.2. Is the data available also reliable? How is it obtained?

Q3.3. What are the units of the different variables in the data source?

Q3.4. Can we use the available data to reproduce or update the graph?

Q3.5. Are there other interesting variables with available data to consider?

...

Q4_Working with data: How to download the data to start working with it?

Q4.1. How to clean the table of data to make it ready to use?

Q4.2. What tools are available for data processing, and which ones can we use?

Q4.3. What types of numerical and graphical summaries are appropriate?

...

We started from the main subject of the text presented in the textbook exercise and elaborated a relevant open question (Q0) that could be worked on in a classroom. The exercise presented a graph with the distribution of water in Brazil and the population and surface area by region. This graph will also serve as a possible answer for a derived question since its source is the Ministry of the Environment. In this initial work to list our generating question and possible derived questions, we identified two large groups of questions: those related to the topic “water” (Q1, Q1.1, Q1.2, ...) and those related to the topic “graph” (Q2, Q2.1, Q2.2, ...). Still, within the topic “graph”, another large

group of questions appears, which is the topic related to “data” (Q3, Q3.1, Q3.2, ...). By raising the questions related to “data”, we entered the topic “working with data” (Q4), which also made us think of derived questions (Q4.1, Q4.2, ...). With this, we obtain an a priori questions-answers map (Figure 3) that is part of SRP-TE.

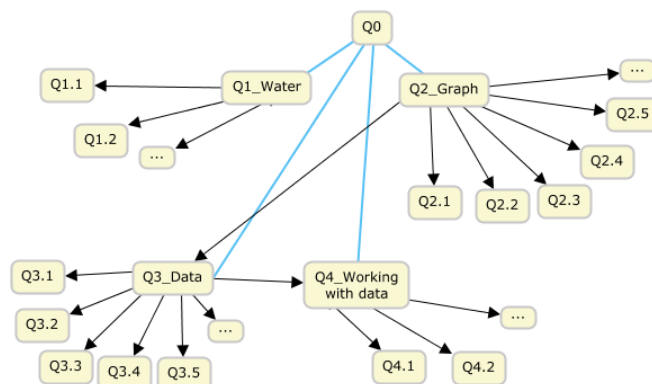


Figure 3: Questions-answers map about water distribution in Brazil

We started from an exercise proposed in a Brazilian textbook and reworked it to display the initial and derived questions that may arise during the inquiry process. This questions-answers map serves as a possible SRP to be developed with the group of preservice mathematics teachers. The answers are still being developed since we have not yet worked directly with the group. It is an a priori study about the proposed theme: distribution of water in Brazil and, with this SRP in development, we believe it will be possible to work with dimensions of the statistical work that tend to be absent from the final grades of the lower secondary education, as the search, collection, cleansing and representation of data. The inclusion of provisional answers will likely enrich the map with more questions and potential maps to follow. It will also introduce more connections between the derived questions and the different branches they form.

Conclusions

The a priori questions-answers map shows the productivity of changing attitude from *visiting works* to *questioning the world*. It first gives visibility to many interrogations that are usually hidden in the way school tends to present (quantitative and qualitative) information, giving no room for doubts or questioning. It also shows how these interrogations go beyond the strict statistical reading and analysis of data to merge with other disciplines or areas concerned by the question addressed. This is not specific to statistics, even if it plays a crucial role there. Teaching modelling also suffers from the same phenomenon of disciplinary confinement. It makes many connections between different topics of statistics that tend to be presented separately even if they nourish each other (like the definition and measurement of variables, the reliability of data, the ambiguity of percentages, the pertinence of the type of graphs chosen, etc.). Finally, it provides future teachers with a broader and more realistic vision of statistics that can help them detach from the narrow perspective proposed by textbook exercises and facilitate the introduction of engaging statistical activities in the classrooms. We hypothesise that the design and implementation of an SRP-TE in statistics will provide students with tools to reflect, develop, analyse, and implement new statistics teaching proposals when working in lower secondary school and develop a critical posture when acting in their profession. The change of paradigms proposed by Chevillard (2015) seems particularly appropriate in the case of statistical inquiries for the specific descriptive tools it provides (Markulin et al., 2021). It also makes us, researchers, adopt a critical stance towards the social, educational, political and epistemological dimensions that involve the themes of teacher education and statistics teaching.

Acknowledgments

This study is financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brazil (CAPES) – Finance Code 001 and by the Spanish project RTI2018-101153-B-C21, Programa Estatal de I+D+i orientado a los Retos de la Sociedad (MCIU/AEI/FEDER, UE).

References

- Barquero, B., Bosch, M., Romo, A. (2015). A Study and Research Path on mathematical modelling for teacher education. In K. Krainer & N. Vondrova (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 809-815). Charles University, Faculty of Education.
- Bosch, M. & Winsløw, C. (2015). Linking problem solving and learning contents: the challenge of self-sustained study and research processes. *Recherches en Didactique des Mathématiques*, 35(3), 357-401.
- Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: a case for an oncoming counter paradigm. In S. J. Cho (Ed.), *The Proceedings of the 12th International Congress on Mathematical Education: Intellectual and attitudinal challenges* (pp. 173–187). Springer. https://doi.org/10.1007/978-3-319-12688-3_13
- De Vetten, A., Schoonenboom, J., Keijzer, R., & Van Oers, B. (2019). Preservice teachers and informal statistical inference: Exploring their reasoning during a growing samples activity. In G. Burrill & D. Ben-Zvi (Eds.), *Topics and trends in current statistics education research: International perspectives* (pp. 199-224). Springer. https://doi.org/10.1007/978-3-030-03472-6_9
- Garfield, J., & Ben-Zvi, D. (2008). Preparing school teachers to develop students' statistical reasoning. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (2008). (Eds.), *Teaching Statistics in School-Mathematics-Challenges for Teaching and Teacher Education: A Joint ICMI/IASE Study*. Springer.
- Giovanni Júnior, J. R. & Castrucci, B. (2018). *A conquista da matemática. 8o ano ensino fundamental: anos finais* (4e), FTD.
- Isaev, V., & Eichler, A. (2017). Measuring beliefs concerning the double discontinuity in secondary teacher education. In T. Dooley, G. Gueudet (Eds.), *Proceedings of the 10th Conference of the European Society for Research in Mathematics Education* (pp. 2916-2923). Dublin City University and ERME.
- Leavy, A. M. (2010). The challenge of preparing preservice teachers to teach informal inferential reasoning. *Statistics Education Research Journal*, 9(1), 46-67. doi.org/10.52041/serj.v9i1.387
- Lopes, C. E. & Ferreira, A. (2004). Texto nº 1: a estatística e a probabilidade no currículo de matemática da escola básica. In *Anais do VIII ENEM (Encontro Nacional de Educação Matemática)*.
- Makar, K., & Fielding-Wells, J. (2011). Teaching Teachers to Teach Statistical Investigations. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching Statistics in School-Mathematics-Challenges for Teaching and Teacher Education: A Joint ICMI/IASE Study* (pp. 347–358). Springer. doi.org/10.1007/978-94-007-1131-0_33
- Markulin, K., Bosch, M., & Florensa, I. (2021a). Project-based learning in Statistics: A critical analysis. *Caminhos da Educação Matemática em Revista (Online)*, 11(1), 200–220.

- Martignon, L. (2011). Future Teachers' training in statistics: the situation in Germany. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching Statistics in School-Mathematics-Challenges for Teaching and Teacher Education: A Joint ICMI/IASE Study* (pp. 33–36). Springer. doi.org/10.1007/978-94-007-1131-0_6
- Ministério da Educação do Brasil (2017). *Base Nacional Comum Curricular*. Brasília: MEC.
- Pfannkuch, M. & Ben-Zvi, D. (2011). Developing Teachers' Statistical Thinking. In C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching Statistics in School-Mathematics-Challenges for Teaching and Teacher Education: A Joint ICMI/IASE Study* (pp. 323–333). Springer. doi.org/10.1007/978-94-007-1131-0_31
- Santos, R., & da Ponte, J. P. (2014), Learning and Teaching Statistical Investigations: A Case Study of Prospective Teacher. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in Statistics Education. Proceedings of the Ninth International Conference on Teaching Statistics*, International Statistical Institute.
- Verbisck, J.T.S. (2019). *Uma análise praxeológica da proposta de ensino de probabilidade em livros didáticos da educação básica* (Master thesis). Universidade Federal de Mato Grosso do Sul, Brazil.
- Watson, J. (2006). *Statistical literacy at school: Growths and goals*. Lawrence Erlbaum Ass.
- Wild, C., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *Int. Stat. Rev.* 67(3), 223-265. <https://doi.org/10.1111/j.1751-5823.1999.tb00442.x>
- Winsløw, C. Matheron, Y. Mercier, A. (2013). Study and research courses as an epistemological model for didactics. *Educational Studies in Mathematics*, 83, 2, 267–284. <https://doi.org/10.1007/s10649-012-9453-3>