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# **Standing on the shoulders of Giants – looking at the teaching subject “conics”**

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*Why do we teach parabolas? One possible answer is because they are in the curriculum. How does content get into the curriculum, whose responsibility was and is the design of curricula and textbooks in Germany? The history of conic sections as a subject reveals a rich and varied teaching tradition that not only calls into question ideas about “invariability” of teaching subjects, but also enables greater openness and joy in the design of mathematics curricula.*

*Keywords: Conic sections, curriculum development, Neuere Geometrie, Meraner Reform, history of mathematics education*

## **Why should math teachers in Germany study the history of German math curricula?**

How did the current curriculum of secondary school mathematics come about? Over the past millennia, a large number of (still-valid) mathematical truths have been discovered, discussed, and proven. Who chooses what to teach in class and on what grounds is such a decision taken? To what extent does the development of modern mathematics effect curriculum development? Students obtaining a teaching profession should ask themselves these questions also in order to recognize their own prospective responsibility in mathematics curriculum development.

However, the study of the historical development of curricula in Germany is complicated, as these differ in the different federal states. The pronounced federalism in the German cultural and educational system has a long tradition. Until 1871 Germany consisted of many independent feudal states and free commercial towns. These small states had their own cultural and educational policies. Even with the establishment of the German Reich in 1871, the competencies were not centralized, the states remained responsible for education and culture. The promotional principle continued during the Weimar Republic. Under the rule of the National Socialist regime, education and culture were centralized. The accompanying ideologization of science and culture through racial theories, as well as the indoctrination of young people in schools, can also be demonstrated in the maths curriculum and maths textbooks (Mehrtens, 1989). The ease with which the Nazi regime was able to influence schools and universities in the centralized system led to a special appreciation of the federal division of responsibilities after the end of the Second World War. This principle was also used by the allied victorious powers of the Second World War when developing administrative structures in the western zones of occupation. In the Soviet occupation zone, however, after the founding of the GDR in the 1950s, the federal structures were dissolved and centrally standardized curricula and textbooks were developed. As early as 1956, the guidelines for the mathematics curricula for the upper level in the GDR show a strong reduction in the treatment of conic sections

to spheres and calculations using methods of analytical geometry and analysis<sup>1</sup>, while the mathematics textbooks for the the upper secondary schools in the FRG still were based on the methodology to the Weimar Republic. The *EOS* (extended general education polytechnic high school) in the GDR began with the 9th grade, the upper level had only 2 years compared to the 3 years of upper secondary classes of the *Gymnasium* in the FRG. Accordingly, in the justification of the guidelines for the teaching of mathematics in the *EOS*, content reduction and pre-employment focus on calculation methods are in the foreground. After reunification, federal principles were also established in education policy in the new federal states. Today, according to the federal principle, educational policy in Germany is a matter of the federal state. Accordingly, curricula, textbooks, school types differ in the different states. Standardizations and coordination are planned and implemented by the *KMK* (Conference of Ministers of Education) and relate to specific structures, such as Exam formats. The educational standards set nationwide for Germany in 2011 provide guidelines on what students should be able to do and when; these requirements are based less on content than on skills. The shift from input-oriented content requirements for the main subjects and foreign languages in the form of educational plans and curricula to output-oriented standards is one of the measures of a general turn towards output orientation and the definition of central standards in Germany. Conic sections are no longer obligatory. The introduction of central exams had a strong influence on the content of mathematics education at the *Gymnasium*. After 2000, a new, nationwide trend towards the *Zentralabitur* (centralised A-level-examination) began, partly with reference to the PISA studies and the unexpectedly poor performance of German students. The ideologised and scandalized debate about central exams did not refer to existing experiences with central exams in the federal states of the former French occupation zone, nor to the central examinations of the GDR and the majority of the new federal states.

The discussion of educational policy, educational institutions, mathematics curricula and exam formats in Germany, including the time of the Nazi regime, the post-war period and the special features of the GDR's educational system, is of particular importance with regard to right-wing populist developments. The growing centralization efforts in the context of the digitization of schools, which are presented exclusively as an answer to new global challenges, also require cultural and historical contextualization. For the latter, the study of mathematics education is particularly suitable due to its algorithmic subjects (Weiss, 2019). The preoccupation of mathematics teachers with the history of mathematics teaching and the development of selected content as a subject can also be motivated pragmatically in Germany at present. In some federal states there are opportunities for schools to have more autonomy, e.g. through internal school curricula that are developed by the school's teaching staff. The content-related discourses are often restricted by current pedagogical trends, such as discovery learning or project teaching. Dealing with historical teaching materials gives one the opportunity to be inspired by alternative presentations and task formats without coming into conflict with current educational theories.

1 a collection of digitized GDR curricula can be found at <https://bbf.dipf.de/de/sammeln-entdecken/besondere-bestaende-sammlungen/lehrplaene#0>

## **Why should math teacher students in Germany study the history of conic sections as a teaching subject in German mathematics curricula?**

During their university studies, the student teachers are confronted with the values of various communities of interest and common practices and thus various reasons for different selections of mathematical subjects and its concept developments. Most of the time, however, this happens implicitly through the enculturation in various communities of practice. The appearance and the disappearance of conic sections in mathematics teaching offer ample opportunities to prospective mathematics teachers to look into their own traditions. Dealing with different value systems from the historical perspective enables the prospective mathematics teachers to develop their own points of view and to structure and classify different value and norm systems without personal conflicts with actors of the different communities in their own environment.

In the following considerations we limit ourselves to the teaching practices of three communities that are or were important for the university education of German *Gymnasium* mathematics teachers: the community of researching mathematicians, the community of mathematics educators and educational scientists and that of historians and philosophers of mathematics. We also limit ourselves to the teaching of conic sections. Of course, there are lecturers and teachers who belong to several communities. Fundamental principles such as the historically determined close connection between research and teaching and the freedom of research and teaching in German university education are represented by the norms and values of all three communities. In the present article, however, the main focus will be on tensions and contradictions between these communities and their historical contextualization. For this reason, we also put contrasting values of the different communities in the foreground.

### **The teaching subject “conics” in the teaching practice of researching mathematicians**

The education of *Gymnasium* math teachers in Germany is closely linked to the education of mathematicians. The mathematics diploma was only introduced in Nazi Germany.<sup>2</sup> To contrast the different communities, we refer in the following to university locations in which only teachers for upper secondary schools like the *Gymnasium* are educated (e.g. the universities of Bonn, Göttingen and Mainz). The separation of teacher education for *Gymnasium* teachers from that for the primary, elementary and secondary schools<sup>3</sup>, corresponds also to the historical development. In the university education of mathematics teachers for *Gymnasium*, math courses are held by researching mathematicians and are usually shaped as common lectures with students studying only mathematics. The teaching activities of the community of researching mathematicians and mathematics lecturers are very much motivated by the care for the next generation of mathematicians. The value system of this community is characterized by the promotion of mathematical achievement. In many ways, values and norms result from the role of mathematics as

<sup>2</sup> Information about the historical development of German mathematical institutes one can find in (Schubring, 1985)

<sup>3</sup> Grundschule (from school year 1 to 4), Hauptschule (from school year 5 to 9) and Realschulen (from school year 5 to 10)

a competitive sport. The mathematics lectures focus on learning modern mathematical language and modern methods to solve open and important problems.

Conic sections appear in the mathematical lectures of math teacher students, if mentioned at all, in group-theoretical or projective contexts, but not associated with the elementary mathematical object “conic”. Analytical geometry is taught in the form of linear algebra in which conic sections are used for classification implicitly. Mathematical research is highly specialized; the recognition of what counts as important knowledge is regulated by the appreciation of the international community. The predominant teaching method so far is the deductive, axiomatic. In the last few decades there have been efforts to introduce additional mathematical courses for student teachers that deal with elementary mathematical content. Such courses are usually designed by researching mathematicians.

### **The teaching subject “conics” in the teaching practice of historians of mathematics**

The teaching and research area History of Mathematics has in Germany a long and worldwide recognized tradition (Purkert & Scholz, 2009, Weiss, 2020). In the GDR, courses on history, philosophical aspects, and the logical foundations of mathematics were compulsory in the curriculum for all students mathematics teachers (Schreiber, 1996). The extent to which mathematical-historical contexts are addressed in today's university education for student teachers depends on local conditions. In the community of mathematical historians, however, there is agreement that conic sections are a fundamental topic in the history of ideas in mathematics (see e.g. Struik, 2013). Both the Greek origins of the conceptual development of conic sections (e.g. Coolidge, 1968) and their outstanding role as a problem-solving method (e.g. Renn, Damerow & Rieger, 2002) were and are main topics of lectures on the cultural history of mathematics. However, there are also delimitations. The values and norms of teaching history of mathematics are linked to the use of historical scientific methods. The latter can differ from the use of the history of mathematics by mathematicians or math educators (Fried, 2001). Michael N. Fried and Sabetai Unguru demonstrate the explication of these contradictions using the theory of conics (Fried & Unguru, 2017). The possibility to limit oneself to conic sections when considering fundamental philosophical differences becomes apparent in Évelyne Barbin's research on the philosophies or theories behind history and education (Barbin, 2015). The main examples for comparing the different methods and perspectives are conic sections (see also Barbin, 2012, Bartolini Bussi, 2015).

### **The teaching subject conic sections in the teaching practice of mathematics educators**

In the context of subject matter didactics, the parabola, the hyperbola as examples of functions and the area and volume calculation of conics are topics of teaching. Here, too, it depends on local conditions whether connections to the content of higher mathematics or to topics of the history of mathematics are shown. As we shall see, the teaching history of conic sections is particularly suitable not only to show aspects of the historical development of the values and norms of the community of mathematics educators, but also their relationships to the other communities. In the 19th century, math teachers of the Gymnasium were simultaneously researching mathematicians and very often also historians. They developed the teaching of conics taking all three perspectives into account.

## **The history of teaching conics as a path to common traditions and values**

### **Scientific development and the elementarization of mathematics**

This contradiction between mathematics as a science and the exemplified elementary school mathematics was the basis of different fruitful discourses in the 19<sup>th</sup> century, which led to a transformation of the pre-university teaching that was to great extent shaped by Euclid's elements until then. The focus on the development of geometry teaching in the 19<sup>th</sup> century is especially illuminative since here the scientific development and the elementarization of mathematics happened parallel to each other at frequent intervals and often by the same people. The historian and maths teacher Max Simon for instance notes: "When you look at the elementary geometry of the 19<sup>th</sup> century, it is especially worth mentioning, how the great developments of science also come to light in elementary geometry." (Simon, 1906, translation by the author). These developments are among others descriptive geometry (Monge), Analysis situs (Carnot, von Staudt), geometrical constructions (Steiner), projective properties (Poncelet), barycentric coordinates (Möbius), linear algebra and algebra (Graßmann, Plücker), analytical geometry (Gergonne).

### **Flourishing teaching culture in *Neuere Geometrie***

Under the banner of *Neuere Geometrie* (Newer Geometry), the research in geometry as a science and the educational reforms in mathematics teaching merge. "The New Geometry, seen from its genesis, is not as much in contrast to the geometry of the ancient than it is in contrast to analytical geometry... Analytical geometry as a subject is a continuation to the elements, but as a method, it is in contrast to the elements"(Pasch, 1882, S.1, translation by the author). The immediate junction of new developments in mathematics with teaching reforms is also fostered by the professionalization of the teachers, restructuring of the school system, development of new curricula as well as changes in the university system. In 1810, for instance, the examination of teachers for *higher schools* was introduced, which did not only require decent knowledge in philosophy and history but also in mathematics. In 1812, the deep knowledge of Euclid's books 1-6, 11 (spacial geometry) and 12 became a general requirement for the final examination (*Abiturprüfung*) at school.

With the so called *Süvernscher Normalplan* (1816) and a renewed lesson scheme for maths education, for instance the analytical approach to conic sections became a teaching subject in grade 10 and 11 (*Sekunda*, 16-17 years old) at the *Gymnasium*. While the conic sections were taken up in the curriculum, text books with different approaches to the subject appeared. For an impression about these different presentations, we recommend a look at the antiquarian or digitally available text books of this time. The theologist Johann Andreas Matthias (1813) for instance, chose the approach to conic sections along the Apollonian way. The mathematician Johann August Grunert (1824) however, used the analytical method to deal with conic sections in his teaching script with exercises and their demonstrated solutions. Also, the mathematician, philosopher, reform educator (*Reformpädagoge*), politician, school teacher (*Schulmann*) and founder of the Berlin Pedagogical Seminar, Karl Heinrich Schellbach, composed in 1843 a text book about conic sections (Schellbach, 1843). An impression of later teaching texts on the subject, which even took projective approaches into account, as well as a detailed analysis of the presentation of *Neuere Geometrie* is provided by

Sebastian Kitz in his dissertation on *Neuere Geometrie* as teaching subject for higher teaching institutions (*höhere Lehranstalten*) between 1870 and 1920 (Kitz, 2015). Examples of the appearance of modern mathematical developments in elementary geometry, as it was described by Max Simon, are also Hermann Hankel's (Hankel, 1875) and Jakob Steiner's (Steiner, 1876) synthetical treatises on conic sections.

### **The royal road to geometry**

The expectations regarding the reforming power of *Neuere Geometrie* become apparent in Hankel's way to rephrase the well-known ancient anecdote: "There is no royal road to geometry. We, however, can add: The *Neuere Geometrie* is this royal road." (Hankel 1875, S.33, translation by the author). Despite these high expectations in the *Neuere Geometrie* and its rapid development as scientific discipline, the school reform initially experienced setbacks. The choice between synthetic and analytic geometry, between Euclidean and *Neuere Geometrie* was at first in the *Gymnasium* decided in favor of Euclidean geometry without the treatment of conic sections. Consequently in 1837, by a Prussian circular directive (*Preußisches Zirkularreskript, i.e. Runderlass*) of Johannes Schulze, the successor of Süvern, disposed a reduction of scheduled mathematics lessons and the removal of conic sections of the curriculum at the *Gymnasium* (Treutlein, 1911, pp. 37- 45).

### ***Meraner Reform and Anschauungslehre***

Only during the gathering of philologists in 1864 in Jena, it came to the foundation of a mathematical-pedagogical section and to the revival of the discussion on conic sections for the teaching at secondary school. In these discussions, the treatment of conic sections in analytical form was linked with the notions of variable and function and hence with the intentions of the *Meraner Reform* for the introduction of differential- and integral calculus (Schimmack, 1911). The proposals of the *Meraner Reform* did not only take those parts of the theory of conic sections with a direct relation to the notion of function into account, but also recommended to deal with conic sections in analytical and synthetical form – even with application to the elements of astronomy, albeit without exemplification of its implementation. Another source of the reformation of the Euclidean tradition of geometry teaching is the development of the *Anschauungslehre*, an education to an inner intuition and view. The geometry book in three volumes of Henrici and Treutlein (Henrici & Treutlein, 1981-1983) as well as Treutlein's *Anschauungslehre* (Treutlein, 1911) – called by Felix Klein "exceptionally noteworthy book" (Klein, 1925, p. 261) – give a good impression of a versatile pedagogically rich treatment of conic sections respecting the different approaches. Accordingly, Treutlein connects plane geometry with spatial geometry by geometrical transformations as reflections, creates references to applications and uses folding and models for the education of internal intuition and view (*Anschauung*) (Weiss, 2016). Also, descriptive geometry, that was only taught at *Realgymnasium* and *Oberrealschule* (secondary schools with a focus on science) can be found in the appendix of the third volume of the geometry text book of Henrici and Treutlein. Here we find (without exercises) an introduction in different projection methods and hence an integration of this approach. An other interesting textbook on the theory of conics, which implemented the



perspective of geometric algebra, was the book by Hieronymus Georg Zeuthen (Zeuthen, 1882), a Danish historian and mathematician.

### **The dawn of conic sections**

From the beginning of the twentieth century until to the New Math in the Seventies, one can find planimetric, stereometric, analytical, affine, perspective, projective up to group theoretical conceptions of conic sections, mostly close to the treatise of Walter Lietzmann's *Elementare Kegelschnittlehre* (Lietzmann, 1949). Until the end of the sixties, one can speak of a bloom of conic sections. The New Math brought conic sections in relation with differential and integral calculus as well as considerations with set-theory and geometrical transformations. Spherical geometry served as contextualization of contents and methods that were acquired in the theory of conic sections (Athen et al., 1967). Not well-known are the international endeavors in the New Math reform (De Bock & Zwaneveld, 2017) to strengthen the application side of New Math. Also, in general secondary schools of the GDR the basics of descriptive geometry were taken up in grade 7 and 8 when the school subject Technical Drawing was introduced. With the reform of the upper school in 1975 and unified examination requirements the conic sections were more and more reduced to linear structures in analytic geometry and in the analysis to the investigation of function graphs of parabolas (Schupp, 1988) and are nowadays reduced to the context of functions. The introduction of dynamic geometry software, in particular with the possibility of 3-dimensional dynamic representations, has not yet led to the high expectations placed on it with regard to the revival of traditional geometrical content. On the other hand, the new technical possibilities for visualizations and animations also arouse the interest of mathematicians who are teaching higher mathematics and perhaps open new doors for common practice.

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