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A suitcase full of logs – an experiment with the use of history of mathematics in teacher training

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A group of students in a Dutch teacher-training-program were given a collection of fragments from different textbooks, to devise a lesson-plan for the introduction of the concept logarithm. Some of the fragments in the collection contained information on the history of mathematics. The lesson-plans were analyzed to determine how the history of mathematics was incorporated. Some students only used biographical information on the inventor of logarithms, others used exercises and a logarithmic table to enhance the comprehension of the concept.

Keywords: History of mathematics, logarithms, lesson design, empirical study, teacher training.

Introduction.

This paper describes an empirical research activity that I carried out as part of a larger study. In this study the way history of mathematics can be used by teachers to improve mathematical cognitive demand is investigated. In this paper I will focus on an assignment on the history of logarithms. The research question is: in what ways do in-service teachers use the history of mathematics in a lesson design for the introduction of the concept of logarithms?

The structure of the paper is as follows. First, I will shortly present formats for presenting history of mathematics in curriculum materials. Then I will describe the design and results of a teaching experiment at a teacher training program at the Amsterdam University of Applied Sciences (AUAS), where participants were asked to design a lesson plan for the introduction of logarithms, using a collection of preselected fragments. Finally, I will discuss the results of the teaching experiment and make some remarks on how this is embedded in the larger study.

The 4S formats.

The history of mathematics (HoM) can be used in mathematics education in many ways. Both Tzanakis et al. (2000) and Jankvist (2009) have written important texts on how this can be done and reasons for doing so. In a recent article (Agterberg et al., 2021) we describe a categorization of ways how HoM can be presented in curriculum materials or classroom activities. This categorization is meant to be used as a tool in what we call a design-oriented approach. With the help of this tool, teachers should be enabled to select and adjust curriculum materials that use HoM or even design classroom activities themselves. We distinguish between four formats for presenting HoM in curriculum materials. We chose format names that all start with the letter S: *speck*, *stamp*, *snippet* and *story*. The term *speck* was chosen to indicate the smallest format. The word *stamp* refers to the notion of a small, separate box of information, alongside the main text. The term *snippet* is used for the average size piece of historical information with substantial relation to the text in which it is incorporated. The fourth and last format *story* relates to larger historical packages that transfer a story of mathematics. Table 1 summarizes the main characteristics of the formats. A complete description of the formats can be found in (Agterberg et al., 2021).

Table 1: Four formats for presenting HoM in terms of size, content, location and functions

Format	Speck	Stamp	Snippet	Story
Size	Very small	Small	Medium	Large
Content	Text only	Text with or without illustrations	Text with or without illustrations, related to instruction or task	Text with or without illustrations, related to instruction or task
Location	Arbitrary location in curriculum material	Arbitrary location in curriculum material	Particular location in curriculum material	Particular location in curriculum material
Goal	Transfers historical aspects of the discipline	Transfers historical aspects of the discipline	Can transfer historical aspects of the discipline	Can transfer historical aspects of the discipline
Tool	Primarily affective, motivational	Primarily affective, motivational	Primarily support of learning of mathematics itself	Primarily support of learning of mathematics itself

In the next section I will show some concrete examples of these formats from Dutch mathematics textbooks. These examples were used in the teaching experiment that is mentioned earlier and will also be described in the next sections.

A suitcase full of fragments.

In February 2021 I taught a course on history of mathematics at the mathematics teacher training program at the AUAS, together with two colleagues. The students that participated in this course are all in-service teachers. They had already finished a teacher training program for lower secondary education and are working as mathematics teachers. The course on history of mathematics is part of a part-time teacher training program for higher secondary education. The goals of the course are to increase the participants' knowledge on history of mathematics and to teach them how to use history of mathematics in the classroom. Due to Covid-19 the class was taught completely online.

In the first session the students were instructed to do an assignment we called 'suitcase assignment'. For this assignment we used a virtual suitcase, containing fragments from three chapters from different Dutch mathematics textbook series, in which the concept of logarithms was introduced. Figure 1 shows one of the pictures that was in the suitcase (coded P2). It is a photo of some handwritten long multiplications, used in the original chapter to illustrate how calculations were done back in the 16th century. No wonder that Napier wanted to create a shortcut, seems to be the message of this picture. In didactical terms one could say this picture can be used to demonstrate the usefulness of logarithms, so the history of mathematics serves as a tool for understanding why a certain concept is developed. In terms of the formats described in the previous section, this picture can be regarded as a stamp: a small piece of historical information that can be placed at relatively arbitrary position in a chapter on logarithms.

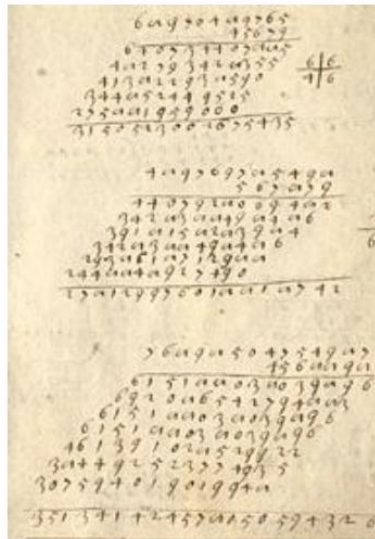


Figure 1: handwritten multiplications (cTWO, 2011, p. 21)

Figure 2 is a translated text from another chapter, stating some information on John Napier, the inventor of logarithms. In the original chapter the text is accompanied by a picture of a statue of Napier, which was left out in the suitcase assignment. This fragment contains only biographical information and does not give any mathematical information on why or how the logarithms were invented. This fragment can also be categorized as a stamp.

John Napier, also known as John Neper (Edinburgh, 1550-1617), was a Scottish mathematician who made his name mainly with his invention of logarithms. John studied for some time at St. Andrews University but also spent considerable time in other European countries. He was a convinced Protestant and particularly passionate about theology. In 1593, he published a religious work entitled Plaine Discovery of the Whole Revelation of St. John, which was translated into Dutch, French and German, so that he also became known on the continent. He practised mathematics mainly as a hobby.

Figure 2: translated text on John Napier (Wageningse Methode, 2015, §7.2)

Figure 3 is a translation from an exercise from the same chapter as picture P2 (figure 2). In this exercise the pupil learns by some examples how to use a logarithmic table to calculate specific multiplications, through some examples. The pieces of the logarithmic table that are necessary for performing this calculation are also shown on the right-hand side. In the original chapter, there is a full logarithmic table available as well. This table was also put in the suitcase, as picture P4.

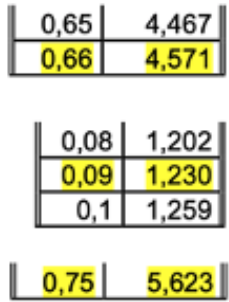
<p>How can you use these tables to approximate $456 \times 12,3$?</p> <ol style="list-style-type: none"> 1. Search the table for the number closest to 4,56 $\rightarrow 4,574$ 2. Find out which power of 10 equals 4,56 $\rightarrow 10^{0,66}$ 3. Find out which power of 10 equals 456 $\rightarrow 10^{2,66}$ 4. Search the table for the number closest to 1,23 $\rightarrow 1,230$ 5. Find out which power of 10 equals 1,23 $\rightarrow 10^{0,09}$ 6. Find out which power of 10 equals 12,3 $\rightarrow 10^{1,09}$ 7. Find out which power of 10 is the answer: $10^{2,66} \times 10^{1,09} = 10^{3,75}$ 8. Find out which number this is: $5,623 \times 1000 = 5623$ 	
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Figure 3: translated exercise on the use of the logarithmic table (cTWO, 2011, p. 22)

Table 2 lists the complete contents of the suitcase and specifies the contents of all historical fragments. All pictures, texts and exercise were separated from the original structure in the chapter and put together randomly in a table. The texts were retyped and renumbered, with identical fonts, colors and layout, so the fragments that belonged together in the same chapter could not be recognized on external features.

Table 2 Contents of virtual suitcase with fragments of chapters on logarithms

Type of fragment	Number of fragments	Historical fragments	Name of fragment	Contents of fragment
picture	4	3	P1	face of Napier
			P2	handwritten long multiplication
			P4	logarithmic table
text	8	3	T2	short biography of Napier
			T3	reason for developing logarithms
			T5	explanation of logarithmic table
			T8	long biography of Napier, including mathematical content
exercise	10	3	E5	use of logarithmic table to find specific powers of 10
			E8	use of logarithmic table to convert numbers into powers of 10
			E10	use of logarithmic table for multiplication and division

The students were given time to read all the fragments in the suitcase, then select the fragments they wanted to use in their lesson plan and adjust them if they wanted. Finally, they arranged the fragments in an order of their choice and wrote an explanation of their choices and purposes. They

were instructed to select at least one fragment that contained, in their opinion, something related to the history of mathematics.

Results.

Twenty students handed in a complete lesson plan after the session. Their designs were scored on what historical fragments they used. The results are in table 3.

Table 3 Amounts and combination of selected historical fragments in suitcase assignment

(P=pictures, T=texts, E=exercises, #=frequency)

#P	#	selection	#	#T	#	selection	#	#E	#	selection	#
0	1	-	-	0	1	-	-	0	8	-	-
1	10	P1	5	1	9	T2	4	1	8	E5	4
		P2	1			T3	1			E8	4
		P4	4			T8	4	2	4	E5 & E8	3
2	7	P1 & P4	5	2	8	T2 & T8	2			E8 & E10	1
		P2 & P4	2			T3 & T8	3				
3	2	P1, P2 & P4	2			T5 & T8	3				
				3	5	T2, T5 & T8	2				
						T3, T5 & T8	3				

Almost all students chose to incorporate at least one piece of biographical information. Three students did not include it at all (and four students chose both texts T2 and T8 that contained biographical information). The logarithmic table P4 was chosen thirteen times, but two of those lesson plans did not contain an explanation and/or exercises to accompany it. This way it was merely used as an illustration, a bit like illustration P2: an ancient long multiplication, showing how arithmetic was a tedious activity back in those days. The reason for developing logarithms is mentioned in fragments T3 and T8. Six students chose neither of them (among them two of the students that also did not incorporate any biographical information).

Twelve students selected one or more historical exercise. One of these students did not include the logarithmic table (P4) in his lesson plan, so it is questionable if this student really understood the exercises himself. From the eight students that did not select any historical exercise, four only selected fragments that contained biographical information (only T2 and/or P1) and the other four also incorporated some historical mathematical information (at least one of T3, T8, P2 or P4).

The way the students incorporated the fragments in their lesson plan was also categorized according to the 4S formats. The smallest format speck was not found amongst the lesson plans. The fragments in the suitcase were not that small and no student deliberately adjusted a chosen fragment in a way that reduced it to a speck. The eight lesson plans that did not incorporate exercises were all

categorized as stamps. The remaining twelve lesson plans were mostly categorized as using history of mathematics in one or more snippets. Two students arranged their chosen fragments with history of mathematics to a complete story. The results of all twenty lesson plans are shown in table 4.

Table 4 Lesson plans categorized according to 4S formats

Format	Frequency
speck	0
stamp	8
snippet	10
story	2

Discussion.

In the previous section the research question “in what ways do in-service teachers use the history of mathematics in a lesson design for the introduction of the concept of logarithms” was answered with regards to type of fragment (picture, text, exercise), contents (biographical, mathematical) and finally format (speck, stamp, snippet, story).

The choice of fragments and design of lessons by the students will certainly be affected by the constraints of the suitcase assignments: limited time for reading, selecting, arranging and reporting, the assignment was completed during a (virtual) classroom session instead of a homework assignment, unfamiliarity of students with the fragments from different series of textbooks. The concept of logarithms can be presented in different ways (Kuper & Carlson, 2020). The students could possibly have had problems with the concept of logarithms themselves.

The historical development of logarithms is complex, but also provides promising opportunities to use it in class (Panagiotou, 2011). Even if the explicit historical information is left out, the geometric and arithmetic sequences from Stifel, Napier and others can be used for constructing logarithms through covariational reasoning (Ferrari-Escolá et al., 2016). The teacher can use this as an approach for constructing the concept during instruction, or have the pupils develop it themselves through a form of guided reinvention.

The students that did the suitcase assignment had received some instruction on how to use history of mathematics in their classrooms prior to this course, but this depended on the classes they took in previous years of teacher training. The fact that most of the students had little prior knowledge of the history of logarithms has probably also affected their choices considerably. Finally, knowing that the lessons were only planned and did not have to be enacted in class, could also have impacted the design. If the lessons had to be performed with a group of pupils, the students might have altered their plans.

The course on history of mathematics will be taught again in the beginning of 2022. The suitcase assignment with fragments from textbook chapters on logarithms will remain in the course, as a starting point for discussing the possibilities of using history of mathematics in the classroom. We intend to have the students enact the lessons they designed and reflect on this during the course.

This will surely provide us with a lot of interesting information from the classroom perspective, that we can learn from, to improve the course on history of mathematics itself and incite new research activities.

Acknowledgments.

I would like to thank my AUAS-colleagues Peter Lanser and Lidy Wesker-Elzinga. We taught the history of mathematics course, in which the suitcase assignment took place, together. They are very supportive to my research activities and it is inspiring to work with them.

The suitcase assignment is one of three assignments that were carried out by our students during the course on history of mathematics. In each assignment the students designed a lesson plan and they also filled out questionnaires on what they think about using history of mathematics for classroom activities. All lesson plans and questionnaires are collected as data and I am analyzing them as part of a larger study. I want to thank my PhD-supervisors Ron Oostdam, Fred Janssen and Bonne Zijlstra, who guide me in this research project.

References.

- Agterberg, D.A., Oostdam, R.J. & Janssen, F.J.J.M. (2021). From speck to story: relating history of mathematics to the cognitive demand level of tasks. *Educational Studies in Mathematics*. <https://doi.org/10.1007/s10649-021-10093-6>
- cTWO (2011). *(ont)Wikkelen*. commissie Toekomst WiskundeOnderwijs.
- Ferrari-Escolá, M., Martínez-Sierra, G., & Méndez-Guevara, M. (2016). “Multiply by adding”: Development of logarithmic-exponential covariational reasoning in high school students. *Journal of Mathematic Behaviour*, 42, 92–108. <https://doi.org/10.1016/j.jmathb.2016.03.003>
- Jankvist, U.T. (2009). A categorization of the “whys” and “hows” of using history in mathematics education. *Educational Studies in Mathematics*, 71, 235–261. <https://doi.org/10.1007/s10649-008-9174-9>
- Kuper, E., Carlson, M. (2020). Foundational ways of thinking for understanding the idea of logarithm. *Journal of Mathematical Behavior*, 57. <https://doi.org/10.1016/j.jmathb.2019.100740>
- Panagiotou, E. (2011). Using history to teach mathematics: the case of logarithms. *Science & Education*, 20, 1–35. <https://doi.org/10.1007/s11191-010-9276-5>.
- Tzanakis, C., Arcavi, A., Correia de Sa, C., Isoda, M., Lit, C.-K., Niss, M., et al. (2000). Integrating

history of mathematics in the classroom: An analytic survey. In J. Fauvel & J. van Maanen (Eds.), *History in mathematics education: The ICMI study*. New ICMI Study Series (Vol. 6, pp. 201–240). Kluwer.

Wageningse Methode (2015). *Wageningse Methode 5VWO*. Stichting de Wageningse Methode.