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### ► To cite this version:

Rosmarie Junker, Susanne Spies. Initiate reflection by original sources – Conception and empirical evaluation of a teaching project in calculus class. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03754200

**HAL Id: hal-03754200**

**<https://hal.science/hal-03754200v1>**

Submitted on 19 Aug 2022

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# Initiate reflection by original sources – Conception and empirical evaluation of a teaching project in calculus class

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*One effect of working with original sources in mathematics classes that is repeatedly mentioned is the stimulation of reflection on mathematics and, as a result, a change in the students' mathematical beliefs. In the context of the teaching project presented here on working with Bernoulli's first textbook on calculus, this could be impressively confirmed empirically by means of a qualitative study. The article is dedicated to the conception of the teaching sequence, as well as first results of the empirical evaluation, and also poses the question of the conditions for dealing with original sources in a digital setting in a reflection-stimulating way.*

*Keywords: Beliefs, Calculus, Original sources, Reflection.*

## Working with original sources as a unique tool

The integration of the history of mathematics can not only contribute to the acquisition of process-related competencies and deeper understanding of the role of mathematics in the world as required by the curriculum, but also promises meaningful experiences with mathematics. Potential opportunities as well as difficulties depend strongly on the type of historical references used and also on the method chosen. Therefore, with regard to the teaching project discussed here, we first take a special focus on the motivations and problems specific for working with historical sources. Working with historical sources not only makes it possible to learn something about the history of the subject, but, if the source is chosen appropriately, it also offers the rare opportunity to gain insights into real (historical) research work in mathematics lessons. Thus, authentic insights into the thinking and working of researching mathematicians become possible. The ICME Study (2000) in this context lists three motivational fields: "replacement," "reorientation," and "cultural understanding" (Jahnke et al. 2000, p. 292). Thus, especially the work with historical sources is said to have the possibility to complement the learners' beliefs of mathematics by the aspect of mathematics as a process and human cultural achievement and thus to contribute to a more comprehensive picture of mathematics among the students. This initially theoretically expected outcome could also already be confirmed in various empirical studies and traced back to experiences within the history of mathematics (cf. e.g. Jankvist, 2015, or the approaches referred to in Bütüner, 2015). Furinghetti et al. (2006) implicitly take up this and add two further aspects to the three prominent points mentioned above: In addition to the opportunities for deeper understanding they also address potential problem areas. On the one hand, a historical text cannot be completely grasped on the first reading, and the interpretation is not clearly right or wrong, unlike what we are used to in mathematics classes. On the contrary, a hermeneutic, discursive approach like known from humanities is required here, which is clearly different from the ordinary experience in

mathematics classroom and to which both the learners and the teachers must be prepared. This is also true for the different levels of language, which have to be switched between, if mathematics from the source is to be connected with today's school mathematics. The differences are to be made fruitful as an irritation that promotes understanding. The language competence necessary for this is certainly a high hurdle for many students, to which appropriate assistance must be provided. However, if these hurdles, created by the difference of mathematical representation in historical texts, are taken up and (at least partially) overcome, it is precisely these differences that offer the opportunity to reflect on the mathematics we have known so far and to enter into discussion:

"Sources from mathematics history that defamiliarize the familiar arouse epistemic curiosity and trigger reflections in the classroom in a natural (intrinsic), effective, and performance-enhancing way." (translated from Glaubitz, 2010, p. 30)

This thesis particularly emphasizes the role of the source as an occasion for reflection on mathematics at various levels (see below).<sup>1</sup> To summarize the motives outlined here, mathematics history especially by working with original source-texts does not function first as a "tool" (among others) for learning mathematics (cf. Jankvist, 2009), but rather as a (possibly unique) tool on the way to a valid image of mathematics and to meaningful experiences in mathematics education.

With regard to teaching practice, at least two research questions are following: Is there empirical evidence for the claim that working with historical sources leads to increased reflective competence? And if so, which conditions for success can be named? In order to answer these questions, we will first present a project in calculus classes in a digital setting and after that discuss the empirical evaluation of student products with regard to different levels of reflection. Finally, first conditions for success for a reflection-oriented (digital) use of historical sources in mathematics lessons will be derived from this project.

### **A teaching project on Bernoulli's differential calculus**

We are convinced that the opportunities described will not happen by themselves. Rather, good methodological framing is necessary.<sup>2</sup> This is especially true if the use of sources does not take place in face-to-face lessons, but must take place in a digital setting. How this can be done successfully is shown by the project presented below, in which the source work was rather closely guided and methodologically accompanied. As meant by the hermeneutic approach to original sources (cf. Jahnke, 1991) the project was organized as the ending of the "normal" calculus series in Q1 (Grade 11 in German Gymnasium) in an mathematics course of a manageable size with 18 students. The course is characterized in particular by the fact that almost all students have at least average mathematical competencies. Only three students deviate downwards in this respect. In addition, the course as a whole shows a high willingness to perform and a readiness to engage with unusual content and methods. In contrast to the teaching project on Bernoulli's differential calculus

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1 Reflection on mathematics and its relationship to environment, and the individual is repeatedly seen as a contribution to general mathematics education (cf. Bauer, 1990, Skovsmose, 1998 or Lengnink, 2006).

2 For a discussion on possible methods for introducing and reading historical texts in general see Junker & Spies (2020).

presented by Jahnke (1995), our project started by a specific introduction to the historical context of the source dealt with. In addition, hermeneutic reading of the source was guided to a greater extent than was intended by Jahnke. Due to the corona-related school closure, the project ultimately took the form of a digital guided source study in which the focus was on a sort of “interrupted reading” (cf. Metz et al., 2007) of the source excerpts flanked by work assignments.<sup>3</sup> These consisted of reproducing the newly learned historical procedures and tasks aimed at tracing Bernoulli's argumentation in detail and clarifying the unfamiliar historical terminology. This was accompanied by writing assignments. Due to the fact that there was no possibility for direct interaction while working, the formulations of the assignments were very small-guiding and pointing the students to the crucial points explicitly. Students were required to submit their assignments and each received individual feedback on their results. Questions regarding the content could also be directed to the teacher at any time via e-mail. At the end of each work block, explanations and answers were provided for everyone in various digital formats in response to questions asked and uncertainties observed. In this way, questions and uncertainties of individuals could be answered and at the same time productively turned around for the other students. By this at least indirectly the group was addressed despite the isolated way of working at home.

The project was introduced with a podcast that was supposed to introduce the students to the mathematical-historical situation around 1700 related to the beginnings of differential calculus and the scientific debate between Johann Bernoulli and his colleague Marquis de l'Hospital. The podcast did not yet contain mathematically relevant details, but served mainly to give the students an insight into the possibilities of scientific exchange and to point out the special situation between Johann Bernoulli and l'Hospital (cf. Jahnke, 1999). The format of a podcast was chosen in order to preserve, in the spirit of “StoryTelling”, the character of oral storytelling and its advantages over a read story (cf. Alchin, 2011 or Heering, 2016), even in digital format. Furthermore, the constellation described in the podcast could be taken up again at the end of the series by asking the students themselves to enter into a scientific exchange with Johann Bernoulli as l'Hospital. In order to prepare the students for the unusual fact that Johann Bernoulli did not use the representation of functions in a coordinate system to introduce the differential, an excursus was made on the possibility of constructing parabolas without a coordinate system. In several explanatory videos, the students were given the opportunity to reproduce such a parabola construction using only compass and ruler and to repeat relevant mathematical content (Thales' theorem and the geometric mean theorem). This was followed by an initial view in Johann Bernoulli's Text - an excerpt in German translation from the work "Die Differenzialrechnung" (Schäftlin, 1924), which dealt with the introduction of the postulates fundamental to his work as well as the derivation of the differential of the quadratic function. In a small-step, task-guided analysis of the short source excerpt, the students should interpret the derivation of the "rule of derivation" for parabolas geometrically. On the one hand, this rather small-step approach to Bernoulli's geometrical way of reasoning is due to the experience that even advanced mathematics students showed great problems in leaving the context

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<sup>3</sup> Due to limitation of space we can't present the tasks and materials in detail here. An english translation of some of the used texted in and classassignments is available online via <https://www.uni-siegen.de/fb6/phima/mu/schulprojekte.html>

of 'function, coordinate system, function graph' and following the geometrical arguments when working with this source (cf. Spies & Witzke, 2018). On the other hand, even in high school students usually have little experience in approaching a mathematical text independently, which is especially true for historical mathematical texts. The questions posed in the tasks thus exemplify to the learners how such a text can be approached, how deep one has to turn to each character with 'pen and paper' in order to really understand the mathematical argument. Through the questions asked, the repeated reading and the repeated comparison of what is read with one's own - in this case elementary geometric - (pre-)understanding, i.e. the reading in the hermeneutic circle is instructed.<sup>4</sup> This example shows, however, that the difference between "mathematical reading" and the reading of texts in the humanities or fiction, for example, can be experienced at least implicitly in the intensive examination of historical texts and thus a reading competence is trained that is indispensable for later (mathematics) studies. Because for Bernoulli's work, the notion of differential is of central importance, this initial source work was concluded with the task of writing a fictional entry for Diderot and d'Alembert's Encyclopedia of Knowledge on the meaning of the term differential. The genre of the encyclopedia article calls for the formulation of thoughts in an educational language, thus leaving behind both the level of (historical) technical mathematical language and individual linguisticization in the process of understanding. The change of language level not only initiates reflection on content, but is also considered an educational goal in mathematics education (c.f. Jahnke, 1991, p. 8). Besides, this task also points to the encyclopedia as a work that shaped the science of the Enlightenment and, with Diderot and d'Alembert, at least to two other greats of the time. In order to bring the students to a common level of learning for the following tasks, an explanatory video was created afterwards, in which the problems of understanding that had become apparent in the processing of the tasks, especially with regard to the concept of the differential, were taken up and attempted to be solved. Following the derivation of the differential, the first task from Bernoulli's textbook was to be reproduced, namely the construction of the tangent line to a parabola point. Here, too, we decided on a small-step, question-guided analysis of the source aimed the independent construction of a tangent to any parabola point. The tasks in this block are intended to sharpen the eye for the peculiarities of a geometrically reasoning calculus: While in school lessons, in which the slope of a tangent at a given point  $x_0$  is searched for or the derivative function is to be determined, Bernoulli solves the classical ancient, geometrically formulated tangent problem for a conic section. Thus, a second point is to be constructed for an arbitrary curve point in order to be able to construct the tangent line in this point on the basis of Euclidean geometry. The term "subtangent" mentioned Bernoulli's solution is an example of a mathematical term that was mathematics-common at the time and in the context of Bernoulli's argumentation, but which is unknown in (school) mathematics today. Here the students can also experience a development of mathematical terminology. By asking the students to construct the tangent line on their own at the end, it was at least possible to ensure that those students who were able to do so had a rough understanding of the theory. Just as after the first

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4 For the students in the project it was the first time to deal with historical sources in mathematics classroom. Perhaps there is no such small-stepped guidance necessary if there is more experience in hermeneutic reading and if the teacher has the possibility to interact directly with her students.

source work, an explanatory video was after this block, too, which was supposed to answer questions of the students that arose during the processing of the source work and which tried to solve problems of understanding that appeared in the students' results.

To conclude the project, the students should now take on the role of l'Hospital themselves and enter into a fictive scientific exchange with Johann Bernoulli. In a letter, they were to address their questions regarding Bernoulli's procedure for the determination of differentials and the construction of tangents to Bernoulli, by comparing their newly acquired knowledge with their previous ideas of differential calculus.<sup>5</sup> This task also served the purpose to ask last open questions for understanding. It is noticeable that the letter form was well received. Most of the students kept the letter form given at the beginning in their own texts and obviously enjoyed writing down their thoughts in this style. The questions and reflections touched on a very wide range of topics: They ranged from questions about concrete conclusions in the source excerpt read, to astonishment about the geometric method of constructing and discussing curves and the validity of Bernoulli's postulates, to the question of why this type of calculus is not part of the curriculum. At the end a response from the teachers again as a letter on behalf of Johann Bernoulli addressed all of these issues and collected questions from the students to address and clarify. Obvious misconceptions were addressed and could be cleared.

### **Levels of reflection within the students letters – Qualitative content analysis**

The students' letters to Johann Bernoulli are written testimonies that in a certain sense document first effects of the source project. The "scientific exchange" in the letter format also leads to relatively open and personal formulations of the thoughts. The students' documents thus allow conclusions about the extent to which the source work actually led to reflections on mathematics. In order to examine the students' letters in a structured way the documents were read following the method of qualitative content analysis (Mayring, 2002). The units of analysis thereby resulted from the sections of meaning within the letters. Guided by the expectation to find testimonies of personal reflection on mathematics, the levels of reflection also formulated by Glaubitz (2010) following Bauer (1990) were first used as deductive categories: the object level, which contains both the reflection in the mathematical work and the reflection on the object, directed "to the essence of the discipline". Furthermore, there is the level of meaning, which includes the reflection on the meaning of working with mathematics as well as the reflection on the meaning and limits of mathematical thinking itself. And finally, the level of self-reflection, which contains the reflection about the effects of the occupation with mathematical contents on the possibilities of the own thinking and acting as well as the reflection about the role of mathematics for the own person and the own self-conception. As a result of a first review of the material, the object level was again divided by us into three levels and thus the category catalog was inductively expanded. On the one hand, there is the level in which mainly a comparison of different mathematical methods and approaches is made (level I). Furthermore, we distinguish here a level in which thinking about a transfer of mathematical approaches and methods to obvious questions and problems beyond the treated mathematical problem takes place (level II). And as a third level we have singled out the level that

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<sup>5</sup> While the previous are past-related tasks, this writing task is a present-related task in the sense of Schorcht (2018).

involves reflection on the question of the genesis of mathematical content and problems (level III). These sub-areas of the subject level correspond at the same time to different requirement areas of mathematical tasks. With this subdivision, we thus map the very different levels of object reflection that are visible in the letters. The analysis of the letters based on these categories show that each category can be found in several examples<sup>6</sup>. So one can be justified in hoping that the treatment of also historical perspectives and contents in mathematics lessons can reach the most diverse levels of reflection. Almost all of the students' letters contain expressions such as "exciting," "surprised," "impressed," or "interesting.". This shows that the confrontation with historical mathematics also had an emotional effect and that, similar to the reflection on aesthetic experiences with mathematics (cf. Müller-Hill & Spies, 2015), meaningful experiences are documented here as well. Furthermore, these are all clear indications that most of the students were able to use the series of lessons to restructure their own understanding of mathematics and its meaning, to add new aspects, and to expand their repertoire of ways of thinking about mathematical questions. Through the analysis of the letters to Bernoulli, we can thus assume that the students' engagement with the mathematical content was stimulated at various levels, some of which were high, and also at structurally different levels. This provides further empirical evidence for the theoretically assumed effectiveness of historical sources in mathematics education.

### **Suggesting conditions of success (not only) in a digital setting**

The evaluation of the final letters shows that the students were stimulated to reflect deeply on a wide variety of levels through their intensive engagement with the historical source material. This finding is nevertheless astonishing, since often in particular the direct exchange about the readings of the learners among themselves and with the teacher as well as the openness of the task in the context of the hermeneutic method is described as a special moment, which stimulates the reflection in the first place. However, precisely these aspects of the source work had to be omitted in the digital implementation of the project. Therefore, in conclusion, some methodological conditions will now be discussed here, which allow historical source texts to become an occasion for reflection in mathematics lessons even in the digital setting.

Working with original sources requires a framework that can both convey a sense of the historical context and motivate work with the historical text (which is usually outside the curriculum). In a digital setting, this might be done with the help of a podcast, a historical documentary, or an introductory text. Depending on the format chosen, a more or less differentiated picture of the historical context is drawn. A deeper understanding of the source continues to be a necessary basis for successful reflection. This in turn requires that on the one hand the professional as well as the historical preconditions are given in order to be able to do something with the historical mathematics. In the project described, a first important step was to introduce the students to the geometric construction of the parabola in order to be able to recognize the geometric-analytical arguments in the source text as such. So the alienation effect of the source does not become a barrier to understanding. In a digital setting, such an introduction can take place, for example, via an

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6 Cf. Junker & Spies (2020) for exemplary quotations for each category

explanatory video or a text flanked by tasks. On the other hand, a more in-depth individual engagement with the text is necessary. As described above, we believe that this cannot be initiated by a simple reading assignment, at least in a digital setting, but requires questions that guide the reading, encourage repeated reading, and draw attention to the central parts of the argument. Although such guidance in face-to-face classes can, of course, happen in individual exchanges, one advantage of the digital course emerged here: All students could (and had to) work intensively on the sources on their own. Thus, the work was done at the individual pace and each individual had the chance to experience that he or she is able to work out a completely unknown content and to comprehend the unfamiliar mathematical way of reasoning, since there was simply no possibility to have a contact person immediately in case of difficulties in understanding or to wait until the issue is clarified in the plenary. Even if the experience of competence and deeper understanding is certainly an important basis for a successful reflection, the direct exchange is naturally missing in a purely digital setting, which in turn would have led to fruitful discussions and reflection about the subject matter. In order to move from an in-depth reading of the source excerpt to reflection on what has been read, the mathematical argumentation must be left behind and the meta-level must be taken up. This also needs to be instructed. In the digital setting, this step can be stimulated, for example, by writing assignments, as it was done in the project presented here by the encyclopedia entry on the differential and the concluding letter. It is important to choose a form that stimulates the change of language levels in a natural way and also encourages the students to connect the historical argumentation with their previous knowledge. Again, the advantage of the purely digital setting is that own thoughts have to be formulated individually. On the other hand, there was no need for communication about the individual results, which would certainly have led to fruitful discussions, since ideas can be taken up directly, thought through further and then formulated together. The experiences in the presented project show that source texts can also become an occasion for reflection in a purely digital format if the discussed conditions for success are met. Above all, the experience of (guided) individual engagement with the text and writing about mathematics turned out to be important factors for successful reflection. The missing exchange about what was read could at least in part be taken up by direct feedback and interposed explanatory videos, even if here of course the potential can only be exhausted in real communication about historical mathematics. It can therefore be assumed that a combination of digitally guided intensive individual source work and the exchange about it in face-to-face lessons with a subsequent writing task to be worked on individually would be a format that could strengthen the observed effects and that the "foreign and bulky" of the source texts could be used productively in class.

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