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► **To cite this version:**

David Urman, Mathias Legrand. Nonsmooth Modal Analysis of a Rectangular Plate in Unilateral Contact. 10th European Nonlinear Dynamics Conference, Jul 2022, Lyon, France. hal-03753823

HAL Id: hal-03753823

<https://hal.science/hal-03753823v1>

Submitted on 18 Aug 2022

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Nonsmooth Modal Analysis of a Rectangular Plate in Unilateral Contact

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Summary. A scheme for the non-smooth modal analysis of two-dimensional objects in unilateral contact is demonstrated for the case of a rectangular plate in frictionless unilateral contact with a rigid foundation. The unilateral contact conditions are described via the Signorini complementarity conditions. Here, application of nonsmooth modal analysis requires finding periodic solutions to the Signorini problem. To treat the Signorini boundary conditions, the nodal boundary method in the framework of the finite element method is used. The nodal boundary method allows to approximate the solution to the Signorini problem via a system of nonsmooth ordinary differential equations in the internal displacements. The resulting system exhibits periodic solutions without non-physical chattering. These periodic solutions are found via the harmonic balance method and sequential continuation is used for detection of nonsmooth modes.

Introduction

Nonsmooth modal analysis refers to application of nonlinear modal analysis to structures governed by nonsmooth equations [7]. Here, the subject of analysis is the rectangular plate in unilateral contact with a rigid obstacle. The unilateral contact boundary is treated via the Signorini complementarity conditions [1]. In essence, nonsmooth modal analysis requires the detection of families of periodic motions to the autonomous plate in frictionless unilateral contact [5, 7, 9]. Generally, nonsmooth modal analysis requires use of numerical techniques as closed-form solutions to the Signorini problem do not generally exist [7, 9]. In [9], the wave finite element method (WFEM) was used for nonsmooth modal analysis of the bar and has proven to provide accurate result for the case of the uniform-area bar. However, extension of this methodology to the two-dimensional case has failed due to high energy dissipation effectively annihilating periodic solutions [9]. Otherwise, the finite element method (FEM) has been commonly used to solve the Signorini problem numerically [3]. In the FEM framework, there exist several numerical schemes for treatment of the Signorini conditions. Although, those were not used for nonsmooth modal analysis for various reasons. For example, schemes utilizing fully elastic Newtonian impact law exhibit non-physical chattering [3, 7] while the penalty [3] and Nitsche [2] methods do not exactly enforce impenetrability with the obstacle. In turn, the mass redistribution method (MRM) [4], while allowing for energy conservation and elimination of chattering, requires solving a constrained optimization problem in many variables to form the reduced mass matrix. However, the *Nodal Boundary Method* (NBM) was shown to eliminate chattering by reducing the mass and stiffness matrices via a series of linear operations on the mass matrix's rows and columns. The NBM-ODE admits periodic solutions which can be found via the harmonic balance method [5]. In this manuscript, families of periodic solutions are found via sequential continuation along different periods of the motion.

Boundary Value Problem for Periodic Motions with Unilateral Contact

We investigate the problem of a thin and isotropic plate prone to unilateral contact with a rigid foundation. The equations governing the motion of the plate away from the boundaries are

$$\rho \bar{\mathbf{u}}_{tt}(\mathbf{x}, t) - \operatorname{div}(\boldsymbol{\sigma}(\bar{\mathbf{u}}(\mathbf{x}, t))) = \mathbf{0}, \quad (\mathbf{x}, t) \in \Omega \times [0, \infty) \quad (1)$$

where $\bar{\mathbf{u}}(\mathbf{x}, t) : \mathbb{R}^2 \times \mathbb{R}^+ \rightarrow \mathbb{R}$ and ρ describe the displacement field and density, respectively. The stress tensor $\boldsymbol{\sigma}(\bar{\mathbf{u}}(\mathbf{x}, t))$ is related to the displacements via plane-stress assumptions. Other than the classic Neumann and Dirichlet boundary conditions, a portion of the plate's boundary, Γ_C , is prone to unilateral contact which are treated via the Signorini complementarity conditions

$$0 \leq g_0 - \bar{\mathbf{u}}(\mathbf{x}, t) \cdot \mathbf{n}, \quad \sigma_n \leq 0, \quad (g_0 - \bar{\mathbf{u}}(\mathbf{x}, t) \cdot \mathbf{n})\sigma_n(\bar{\mathbf{u}}(\mathbf{x}, t)) = 0, \quad \mathbf{x} \in \Gamma_C \quad (2)$$

where g_0 defines the distance between the non-deformed boundary and the rigid obstacle. The contact pressure is captured by $\sigma_n(\bar{\mathbf{u}}(\mathbf{x}, t)) = \mathbf{n}^\top \boldsymbol{\sigma}(\bar{\mathbf{u}}(\mathbf{x}, t))\mathbf{n}$ where \mathbf{n} is the outward normal to the contact boundary.

At last, for nonsmooth modal analysis, we require solutions of period T such that $\bar{\mathbf{u}}(\mathbf{x}, 0) = \bar{\mathbf{u}}(\mathbf{x}, T)$ and $\bar{\mathbf{u}}_t(\mathbf{x}, 0) = \bar{\mathbf{u}}_t(\mathbf{x}, T)$.

Nodal Boundary Method

The NBM applies on the FEM formulation of the Signorini problem Equations (1) and (2)

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{G}^\top \boldsymbol{\lambda} \quad (3)$$

$$\boldsymbol{\lambda} + \mathbf{max}(\mathbf{0}, g\mathbf{1} - \mathbf{u}_C - \boldsymbol{\lambda}) = \mathbf{0} \quad (4)$$

where $\mathbf{u}(t) \in \mathbb{R}^N$ constitute the N nodal displacements approximating the displacement field $\bar{\mathbf{u}}(\mathbf{x}, t) \approx \mathbf{P}(\mathbf{x})\mathbf{u}(t)$ with $\mathbf{P}(\mathbf{x})$ denoting piecewise-Lagrangian shape functions. In the NBM, we distinguish between nodal displacements $\mathbf{u}_C(t)$

on Γ_C and the remainder of the nodal displacements $\mathbf{u}_O(t)$ such that $\mathbf{u}(t) = (\mathbf{u}_O(t) \ \mathbf{u}_C(t))^\top$. It is well known that formulation (3) and (4) form an ill-posed problem due to the existence of infinitely many values for λ at the moment of contact [1]. In the NBM, we attempt to resolve this ill-posedness by assigning λ a relationship with $\mathbf{u}(t)$ via the FEM approximation of the boundary integral on Γ_C :

$$\mathbf{G}^\top \boldsymbol{\lambda} \leftarrow \int_{\Gamma_C} \mathbf{P}^\top(\mathbf{x}) \sigma_n(\mathbf{P}(\mathbf{x}) \mathbf{u}(t)) d\mathbf{x} = \mathbf{G}^\top (\mathbf{N}_C \mathbf{u}_C(t) + \mathbf{N}_O \mathbf{u}_O(t)) \quad (5)$$

where \mathbf{N}_C and \mathbf{N}_O are constant matrices. Substitution of (5) into (4) results in an LCP in $\mathbf{u}_C(t)$ admitting a unique solution for given $\mathbf{u}_O(t)$ and g . This unique solution consists of piecewise constant matrix $\mathbf{A}(\mathbf{u}_O(t), g)$ and vector $\mathbf{d}(\mathbf{u}_O(t), g)$ such that $\mathbf{u}_C(t) = \mathbf{A}(\mathbf{u}_O(t), g) \mathbf{u}_O(t) + g \mathbf{d}(\mathbf{u}_O(t), g)$. The values of \mathbf{A} and \mathbf{d} can be found numerically via solution of the LCP. From this expression, we construct the NBM approximation of the displacement which always answers (4) and (5) and effectively always answers the Signorini conditions

$$\bar{\mathbf{u}}(\mathbf{x}, t) \approx \mathbf{P}(\mathbf{x}) (\mathbf{A}^*(\mathbf{u}_O, g) \mathbf{u}_O(t) + \mathbf{d}^*(\mathbf{u}_O, g)), \quad \mathbf{A}^*(\mathbf{u}_O, g) = \begin{bmatrix} \mathbf{I} \\ \mathbf{A}(\mathbf{u}_O, g) \end{bmatrix}, \quad \mathbf{d}^*(\mathbf{u}_O, g) = \begin{bmatrix} \mathbf{0} \\ \mathbf{d}(\mathbf{u}_O, g) \end{bmatrix} \quad (6)$$

In the FEM framework, substitution of approximation (6) into (3) admits the system of ODEs

$$(\mathbf{A}^*(\mathbf{u}_O, g))^\top (\mathbf{M} \mathbf{A}^*(\mathbf{u}_O, g) \ddot{\mathbf{u}}_O(t) + \mathbf{K} \mathbf{A}^*(\mathbf{u}_O, g) \mathbf{u}_O(t) + \mathbf{K} \mathbf{d}^*(\mathbf{u}_O, g)) = \mathbf{0} \quad (7)$$

which are non-smooth ODEs (due to \mathbf{A}^* and \mathbf{d}^*) in $\mathbf{u}_O(t)$. The resulting non-smooth ODEs can be solved using explicit or implicit schemes, similar to the modified mass scheme in [4]. Here, periodic solutions of the ODE can be found via the Harmonic Balance method [5]. Then, sequential continuation is used to determine families of periodic solutions of the plate in unilateral contact, in a similar fashion described in [5].

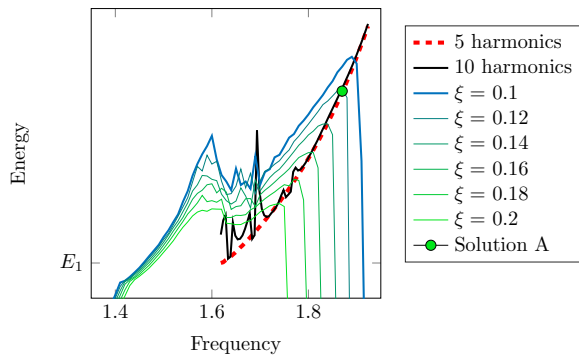


Figure 1: Forced Response of Plate for various damping coefficients ξ and Backbone curves generated via NBM-HBM with varying number of harmonics (i.e. cosine functions multiples of the corresponding frequency)

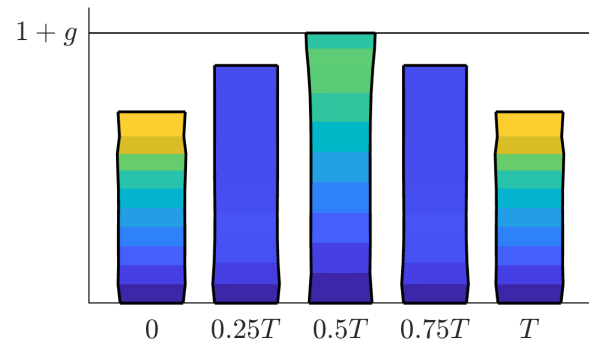


Figure 2: Depiction of Solution A in Figure 1 at different times fractions of the period $T = 2\pi/1.87$. Color gradient represents $\|\mathbf{u}(\mathbf{x}, t)\|_2$

Conclusions

Under the NBM's formulation of the Signorini problem, nonsmooth modal analysis of the plate was done successfully. Indeed, this formulation may apply to more intricate structures prone to unilateral contact. Therefore, future work shall include expanding the NBM formulation to practical engineering cases involving the Signorini problem.

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