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Problematizing the notion of problem posing expertise

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Problem posing, which we view as a form of task design performed by the learner, is an important mathematical activity. Several studies have investigated differences between expert and novice problem posers, but no clear definition of problem posing expertise has generally been agreed upon. In a study involving 66 masters and 60 sixth-grade students we identified expert problem posers based on their performance on problem posing tasks (with or without numbers/context) rather than as in prior research presupposing who expert problem posers were based on their mathematical experience or backgrounds. The results showed that masters students had a significantly lower proportion of the Top-level (expert) problem posers and a significantly higher proportion of the Bottom-level (novice) problem posers than sixth graders, demonstrating that presupposing problem posing expertise based on mathematical experience or backgrounds would have been misleading.

Keywords: Mathematical problem posing, experts, novices, task design.

Introduction

Problem posing has recently attracted much attention among researchers and educators, including curriculum standards (e.g., NCTM, 2000). As a form of a curricular opportunity (embedded in curriculum resources) in which teachers and students can engage, problem posing is an important mathematical activity like the well-known problem solving. Several types of problem posing tasks have been used in research and teaching practice (Lee, 2021). The aim of having problem posing tasks in curriculum resources could be not only for assessing students' mathematical understanding or mathematical learning, but also for supporting students to become competent problem posers and problem solvers (Cai et al., 2015). One research strand in this area views problem posing as a goal of mathematics instruction and focuses on how one develops problem posing capacity (Cai & Leikin, 2020), showing that students and teachers are capable to pose problems (Cai et al., 2015) and that training to problem pose is feasible (Cai et al., 2015; Koichu & Kontorovich, 2013). However, little attention has been paid thus far to the meaning of problem posing expertise. This is problematic because without a good understanding of what constitutes an expert problem poser, efforts to develop problem posing capacity among students and teachers are lacking important conceptual foundation.

In this study, we aimed to identify expert problem posers by examining our participants' performance on problem posing tasks rather than as in prior research presupposing who expert and novice problem posers were based on their mathematical experience or backgrounds. Specifically, we addressed the following research question: *How can we distinguish between expert and novice problem posers based on their performance on problem posing tasks, and how do the resulting groups of expert and novice problem posers compare with how these groups would normally be defined based on participants' mathematical experience or backgrounds?*

Theoretical Considerations

Experts in Mathematical Problem Posing

Several researchers explored differences between expert and novice problem posers by assuming that problem posing expertise was connected to a particular attribute of participants' mathematical experience or backgrounds – such as their problem solving experience (Pelczer & Gamboa, 2009), teaching experience (Voica & Pelczer, 2009), or mathematical maturity (Zhang et al, under review) – but the results showed that their “expert” problem posers were not always performing better than the “novices.” For example, Voica and Pelczer (2009) compared problems posed by pre-service and in-service teachers, considering the former group to be novices in problem posing and the latter group to be experts. They found that their perceived expert group did not perform better than their perceived novice group, presumably because in-service teachers' pedagogical knowledge and classroom experience constrained their views of the problems that could be posed. Also, our prior research (Zhang et al, under review) had examined problem posing by comparing mathematically more mature students (masters students majoring in pure mathematics or mathematics education) and mathematically less mature students (sixth-grade students). The surprising finding that the more mature participants did not outperform the less mature participants in problem posing further prompted us to problematize the notion of problem posing expertise and its presumed association with participants' prior mathematical experience or backgrounds.

Kontorovich and his colleagues conducted a series of studies to explore the possible characteristics of the problem posers for mathematical competitions, arguing that those who systematically create problems for high-level mathematical competitions may be considered as expert problem posers (Kontorovich, 2020). While this notion of expertise is still linked to participants' mathematical backgrounds, problem posers for mathematical competitions may be less controversial as an expert group than in-service teachers or masters students due to mathematical competitions often being reputed as treasures of “elegant,” “intriguing,” and “surprising” problems that reach the students after thorough committee discussions (Koichu & Andzans, 2019).

To conclude, no clear definition has generally been agreed upon for who problem-posing experts are though prior studies have provided some useful insights into possible characteristics of these experts. The participants of those studies identified as expert problem posers were presupposed to be experts based on their prior mathematical experience or backgrounds. Yet this approach to identifying experts yielded some surprising findings such as novice problem posers outperforming presumed experts.

Criteria for Identifying Expert Problem Posers

Most of the research on problem posing has involved the assessment of the types, quality, and quantity of the posed problems, often inferring from these the participants' problem posing ability. Silver and Cai (2005) proposed three criteria that might be selectively applied to most problem-posing tasks when used in assessment settings: quantity, originality, and complexity. Several researchers have adapted these criteria to assess participants' problem posing ability (Cai et al., 2020). In what follows, we discuss four common criteria used in prior studies for assessing the problems been posed. In the study we report herein, we used these criteria to operationally measure problem posing performance and thus as criteria for identifying expert problem posers.

The first criterion is *the number of mathematical problems posed* (Leung & Silver, 1997; Silver & Cai, 2005). Silver and Cai (1996) found that nearly 30% of the problems posed by middle school students were either nonmathematical problems or simply nonproblem statements (even though the directions clearly asked for mathematical problems). Crespo and Sinclair (2008) hypothesized that students' difficulties generating mathematical problems might relate to a lack of opportunity to explore a problem situation adequately before and during the posing process. Therefore, the number of mathematical problems been posed might reflect how adequately the posers explore the problem situation. The number of problems posed was also used as a measure of fluency in studies that assessed creativity using problem-posing as a test tool (Bicer et al., 2020). The more mathematical problems being posed, the more adequately posers might have explored the problem situation and the higher their anticipated level of mathematical fluency.

The second criterion is *the number of posed problems that are solvable* (Cai et al., 2015; Leung & Silver, 1997; Zhang et al., under review). Even though students and teachers were found to be able to pose mathematical problems, the posed problems were not always solvable or relevant (Silver & Cai, 1996), which suggests that the participants might not have selected enough elements or organized adaptive relations to construct the problems. The more solvable mathematical problems being posed, the more adaptive elements and relationships posers might have selected to formulate the problems or the better their understanding of the problem-posing tasks.

The third criterion is *the complexity of the posed problems*. Some research measured this criterion by the number of steps for solving the posed problem (Cai et al., 2020; Leung & Silver, 1997). According to the concept of problem space proposed by Milinkovic (2015), any problem can be described in terms of its context, elements, and the relationships between elements. Zhang et al. (under review) considered the sum of relationships and elements in the constructed problem space to reflect the complexity of the posed problems. The larger the sum of relationships and elements constructed in the problem, the more complicated a problem space has been formulated by the poser.

The fourth criterion is *the clarity of the posed problems*. According to NCTM (2000), using mathematical vocabulary, notation, and structure to clearly represent ideas, describe relationships, and model situations are important for students' ability to communicate mathematically. Zhang et al. (under review) found that nearly a third of the problems posed by students were expressed unclearly or partially clearly. The greater the clarity of the posed problem, the better the ability of the poser to communicate in problem posing.

Methodology

Participants

The participants were 66 masters students and 60 sixth-graders from 11 classes of a primary school, all in China. The masters students were recruited via an advertisement that we posed to call for volunteers among masters students majoring in pure mathematics or mathematics education in a university among the top 100 Chinese universities. Regarding the sixth-graders, 5 or 6 students from each class of the selected school, which was affiliated with the aforementioned university (ranked in the top 10 of the local city), volunteered to participate in this study. Each participant signed an informed agreement letter prior to the study. The sample was appropriate for our study because,

according to the conventional way of defining problem posing expertise based on participants' experience or backgrounds, the former group would be the experts and the latter the novices. Whether these expert/novice groups would be confirmed empirically was the subject of the research.

Data Collection

The study was conducted in a soundproof and uniform light laboratory with a voice recording. The participants were given a description of the research procedure along with brief instructions that they were expected to think aloud while completing the Problem-Posing Tasks (PPTs). Then they underwent a training phase to get familiar with problem posing via two simple tasks. Two kinds of test combination (1A-2B-3A-4B and 1B-2A-3B-4A) were provided on a daily rotating cycle. The participants chose their available time to get one of the test combinations, described in Table 1, and were tested individually. We considered two types of tasks (with/without number; with/without context) since several researchers (English, 1998; Leung & Silver, 1997) have predicted that task format affects subjects' problem posing performance. The target tasks were chosen from the PPT-number test comprising translated versions of tasks used by Leung and Silver (1997), which included the task situations of House Purchase and Pool Maintenance, and the PPT-context test comprising modified versions of tasks used by Cai et al. (2020), which included the task situations of Driving Home and Sporting Goods. No time limits were set on participants' work with the tasks.

Table 1: Test items and distribution of participants

		PPT-number				PPT-context			
		House Purchase		Pool Maintenance		Driving Home		Sporting Goods	
		1A ¹	1B	2A	2B	3A	3B	4A	4B
Masters	P ₁ =30	✓			✓	✓			✓
Students	P ₂ =36		✓	✓			✓	✓	
Sixth	N ₁ =30	✓			✓	✓			✓
Graders	N ₂ =30		✓	✓			✓	✓	

¹Format A in the PPT-number test is the task with numbers, Format B in the PPT-number test is the task without numbers; Format A in the PPT-context test is the task with context, Format B in the PPT-context is the task without context.

Data Analysis

We re-analyzed the data in our prior research (Zhang et al., under review), guided by our new research question. The responses of the 126 participants who completed the PPTs were recorded and coded with respect to six criteria as shown in Table 3. These included the four main criteria for identifying expert problem posers that we discussed earlier and two further sub-criteria in which “the complexity of the posed problems” was divided into “the largest sum of relations” and “the largest sum of elements.”

Principal Components Analysis (PCA) was conducted according to the four main criteria to select the Top 15% (19 participants, experts) and the Bottom 15% (19 participants, novices) from the overall

sample of 126. We chose 15% so as to be selective but also to have enough students in each group to conduct meaningful statistical analyses. For the correlation assumption test, the correlation between each index (criterion) ranged from 0.507 to 0.975 (>0.30), which meant that there was a linear correlation between each index; the KMO (Kaiser- Meyer- Olkin) value of 0.773 indicated that the sampling was adequate (a value between 0.7 and 0.8 was taken to be “middling”); the Bartlett’s test of sphericity with an associated p value of $<.001$ indicated that we could proceed to use PCA to analyze our data. The result of total variance explained showed that only the first component had eigenvalues over 1.00 (that is, 3.267) and that together they explained 81.669% of the total variability in the data. This led us to the conclusion that one factor solution was adequate.

Results

Top 15% and Bottom 15% participants’ distribution

We sorted the participants according to the main factor extracted by PCA, and we selected the top 15% (19/126) participants and the bottom 15% participants (19/126). The general information of both groups is shown in Table 2. Surprisingly, in the Top 15% group nearly 80% of the participants (N=15) were sixth graders; only 4 participants were masters students. A Z-Test showed that there is a significant difference (at 95% confidence interval) between masters students and sixth graders in the Top 15% group. In the Bottom 15% group, nearly 70% of the participants (N=13) were masters students; only 6 participants were sixth graders. There is a significant difference (at 80% confidence interval) between masters students and sixth graders in the Bottom 15% group. Table 2 also shows the distribution of participants in the two categories of tests.

Table 2: Top 15% and Bottom 15% participants’ distribution

	Categories of students			Categories of tests ²	
	Master students	Six graders	Z-Test ¹	1A-2B-3A-4B	1B-2A-3B-4A
Top 15% (total=19)	4	15	2.234(95%)	9	10
Bottom 15% (total=19)	13	6	1.492(80%)	13	6

¹95% (80%) refers to significant difference at 95% (80%) confidence interval (2-tailed). ²Specific information about the categories of tests was shown in Table 1.

Top 15% and Bottom 15% participants’ performance in each of the problem posing criteria

We selected the top-level and bottom-level students by ranking the *combined score* on the main factor, which was extracted by PCA. However, the top-level students’ performance in *each criterion* was not necessarily better than the bottom-level students’ performance. This made meaningful the comparison of the two groups across each criterion separately, which we present in Table 3.

Regardless of the task format (with or without context/numbers), the results of a multi-factor analysis of variance indicated that the mean performance of the Top 15% participants was higher than that of the Bottom 15% participants on all criteria. In Table 3 we present the findings for the two PPT-context tasks; the findings for the two PPT-number tasks were similar but are not reported here due to space constraints. Specifically, for the Driving Home task, the Top 15% participants posed more

mathematical problems ($F=83.97^{***}$, $\eta_p^2=.712$), posed more solvable problems ($F=102.4^{***}$, $\eta_p^2=.751$), posed problems with a larger sum of relationships and elements (more complicated problems) ($F=63.36^{***}$, $\eta_p^2=.651$), and posed more clearly-expressed problems ($F=90.28^{***}$, $\eta_p^2=.726$).

Table 3: Top 15% and Bottom 15% participants' performance on the PPT format with/without context

Criteria		Driving Home			Sporting Goods		
		With context	Without context	Group effect	With context	Without context	Group effect
Number of math problems posed	Top 15%	7.67(2.9)	5.30(1.7)	$F=83.97^{***}$	7.70(4.9)	3.11(1.1)	$F=23.74^{***}$
	Bottom 15%	1.77(0.7)	0.50(0.5)	$\eta_p^2=.712$	2.00(0.9)	0.31(0.5)	$\eta_p^2=.411$
<i>Task format effect</i>		$F=9.70^{**}$; $\eta_p^2=.222$			$F=12.95^{**}$; $\eta_p^2=.276$		
Number of solvable problems posed	Top 15%	7.33(2.4)	5.20(1.8)	$F=102.4^{***}$	5.30(2.3)	3.11(1.1)	$F=55.97^{***}$
	Bottom 15%	1.69(0.8)	0.17(0.4)	$\eta_p^2=.751$	1.33(0.8)	0.23(0.4)	$\eta_p^2=.622$
<i>Task format effect</i>		$F=12.03^{**}$; $\eta_p^2=.261$			$F=12.93^{**}$; $\eta_p^2=.276$		
Complexity of the posed problems	Top 15%	10.33(1.3)	7.00(0.0)	$F=63.36^{***}$	10.00(2.4)	7.00(0.0)	$F=37.18^{***}$
	Bottom 15%	5.23(2.7)	1.17(2.9)	$\eta_p^2=.651$	4.83(3.4)	1.62(3.1)	$\eta_p^2=.522$
<i>Task format effect</i>		$F=28.99^{***}$; $\eta_p^2=.460$			$F=12.91^{**}$; $\eta_p^2=.275$		
- The largest sum of relationships	Top 15%	3.78(0.4)	2.00(0.0)	$F=52.53^{***}$	3.00(0.8)	2.00(0.0)	$F=37.35^{***}$
	Bottom 15%	2.00(1.0)	0.33(0.8)	$\eta_p^2=.607$	1.33(1.0)	0.46(0.9)	$\eta_p^2=.524$
<i>Task format effect</i>		$F=52.53^{***}$; $\eta_p^2=.607$			$F=12.74^{**}$; $\eta_p^2=.273$		
- The largest sum of elements	Top 15%	6.56(0.9)	5.00(0.0)	$F=62.78^{***}$	7.00(1.6)	5.00(0.0)	$F=36.96^{***}$
	Bottom 15%	3.38(1.8)	0.83(2.0)	$\eta_p^2=.649$	3.50(2.3)	1.15(2.2)	$\eta_p^2=.521$
<i>Task format effect</i>		$F=19.67^{***}$; $\eta_p^2=.366$			$F=12.94^{**}$; $\eta_p^2=.276$		
Clarity of the posed problems	Top 15%	7.56(2.8)	5.20(1.8)	$F=90.28^{***}$	7.50(4.6)	3.11(1.1)	$F=26.45^{***}$
	Bottom 15%	1.77(0.7)	0.17(0.4)	$\eta_p^2=.726$	1.83(1.0)	0.23(0.4)	$\eta_p^2=.438$
<i>Task format effect</i>		$F=12.08^{**}$; $\eta_p^2=.262$			$F=13.00^{**}$; $\eta_p^2=.277$		

¹ According to Cohen (1988)'s partial eta squared, 0.01 is considered a small effect, 0.06 is considered a medium effect, and 0.14 is considered a large effect. ² $^{***}p < .001$, $^{**}p < .01$, $^{*}p < .05$.

Discussion

Prior research tended to presuppose problem posing expertise based on participants' mathematical experience or backgrounds (Koichu & Kontorovich, 2013; Kontorovich, 2020; Pelczer & Gamboa, 2009; Voica & Pelczer, 2000). In this paper, we problematized whether this way of identifying expert problem posers is indeed valid. Specifically, we followed a data driven approach to identify expert problem posers based on participants' problem posing performance rather than presupposing who the

expert problem posers were based on participants' prior mathematical experience or backgrounds. Had we followed prior practice, we would have considered the masters students to be the experts and the sixth graders to be the novices in problem posing. However, our PCA showed that the masters students had a significantly lower proportion of Top-level problem posers and a significantly higher proportion of Bottom-level problem posers than the sixth graders. This result indicated that it was reasonable to identify expert problem posers by their problem posing performance rather than by their mathematical experience or backgrounds. Also, we compared the Top-level and Bottom-level groups across each of the four criteria used in prior studies for assessing problem posing performance; in PCA the 126 participants were ranked by an extracted main factor with a combined score. The fact that experts outperformed novices in each theory-driven criterion complements the PCA findings in suggesting that the identification of problem posing expertise by the data driven approach as used in this study was indeed appropriate.

From interviews with masters students in our prior study (Zhang et al., under review), we can get some insights as to why the masters students as a group did not do as well as one might have expected in the problem posing tasks. When asked about what their biggest challenge was when posing the problems, the masters students indicated that the cues motivating them to construct new problems were those they had seen in their primary school textbook or relevant reading materials. They did not attempt to use higher level mathematical knowledge. Also, the masters students were more used to working on problem solving tasks rather than problem posing tasks. Conversely, over the last decades primary school students have had increasing opportunities to pose problems in their classes. This is supported by Cai et al.'s (2017) investigation of the problem posing tasks in Chinese textbooks from the 1990s to the 2010s, which found that the number of problem posing tasks significantly increased over the years. The aforementioned indicates that participants' problem posing performance is influenced by many factors, and so it is not reasonable to assume in advance that one group of participants will be the expert group simply from their level of mathematical knowledge/backgrounds.

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