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An examination of mathematical affordances available in grade 2 teachers' guide and learners' textbook on addition of whole numbers

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This paper present findings from a textbook analysis which examined the structure and patterns of variation of addition examples presented in a grade 2 mathematics teachers' guide (TG) and learners' textbook (LT). Classification scheme for addition problems was used to analyse the structure of the examples, and Mathematics Discourse in Instruction framework for Textbooks was used to analyse their patterns of variation. The findings revealed that the example sets in the TG and LT afford development of additive reasoning as they contain different patterns of variation that might lead to higher levels of generality if teachers focus learners' attention to these. The examples however constrain development of additive reasoning as they comprise of only one structure with both addends given and require calculating the result. To decide a better professional development for teachers, it is necessary to study how teachers use these resources to plan and deliver mathematics lessons.

Keywords: Examples, mathematics teachers' guide, mathematics leaners' textbook.

Background

Malawian learners, on average, have been performing poorly in mathematics in both national and regional assessments (Ravishankar et. al., 2016). Despite Malawi's overall improvement on national primary school examination pass rates, performance in mathematics is still very low (Ministry of Education Science and Technology [MoEST], 2020). At international level, the Southern African Consortium for Monitoring Education Quality (SACMEQ) results show that Malawian learners perform extremely low on number and operation in mathematics (Milner et al., 2011). This is very worrisome considering that number concept and operations define numeracy, which is an indicator of educational quality (Ravishankar et al., 2016). The problem of learners' low performance in mathematics in Malawi has persisted even after government's implementation of several educational reforms to improve education quality. Some of these reforms include revising of primary education curriculum and curriculum materials such as teacher guides (TG) and learners' textbooks (LT) for all subjects. Despite the crucial role that mathematics TGs and LTs play in the planning, teaching and learning of mathematics in developing countries (Leshota, 2020), little research has been conducted to examine how the content of the revised TGs and LTs afford or constrain learners understanding of mathematics. In mathematics education, "mathematics textbooks play a particularly prominent role in guiding teachers on specific materials to teach (Chang & Salalahi, 2017, p. 236). Thus many mathematics teachers use textbooks such as TGs and LTs to decide the type of tasks to implement in their classrooms and how to engage students in such tasks (Leshota, 2020; Stylianides, 2014).

The findings presented in this paper are part of an ongoing study which aims at examining the mathematical affordances presented in Malawian primary mathematics textbooks. In this paper, we pursue the following research question; what mathematical opportunities are made available in

addition of whole number examples presented in mathematics grade 2 TG and LT? For every primary education subject in each grade in Malawi, there is one TG and one LT. The TG contains instructions for teachers in terms of the tasks, examples, and resources to be used for teaching while the LT contains learners worked and exercise examples. In most cases, these two books are the only curriculum resources used by Malawian primary school teachers to plan and teach their lessons. Therefore this study is useful in informing policy during revision of the books as well as informing educators on necessary professional development needs for primary school mathematics teachers not only in Malawi but also in other countries. As noted in literature, mathematics textbooks such as TGs are the mostly used resource in mathematics teaching and learning not only in developing countries but in developed countries (Stylianides, 2014). We specifically focus on the quality of addition examples by examining their structure as we agree with Olteanu (2018) that mathematics teaching and learning is mainly done using examples. Suggesting that the quality of mathematical examples presented in textbooks determine the quality of mathematics teaching in the classroom (Leshota, 2020; Ronda & Adler, 2016).

Analytical frameworks

Two frameworks were used to analyse the examples in the TG and LT: the classification scheme for addition problems developed by Carpenter, Fennema, Franke, Levi and Empson (2015), and the Mathematics Discourse in Instructional framework for Textbook analysis (MDITx) developed by Ronda and Adler (2016). Carpenter et al. (2015) describe 2 classes of addition problems (which are called examples in MDITx) according to the kinds of action or relationships described in the problems; join and part-part-whole problems. “Join problems involve a direct or implied action in which a set is increased by a particular amount and part-part-whole problems involve adding or subtracting 2 disjoint subsets” (Carpenter et al., 2015, p. 8). These two types of addition problems can contain different structures in which either the result/whole is unknown but the start or change/parts are known, or one start/change/part is unknown, but the result/whole is known. When children engage with addition problems that require them to find the result/whole, they use a joining all basic counting strategies. In joining all (counting all) strategy, children use objects to first count each addend separately (numbers of object being added), put the objects together and counting them all again to find the sum (Carpenter et al., 2015). For children to use complicated counting strategies like joining to and trial and error, they need to be given either a join problem in which the start or change is unknown or a part-part-whole addition problems in which either the first or second part is unknown (Carpenter, et al., 2015). In joining to counting strategy, children do not count each start/change/part, but they start counting on from a predetermined number like either the given start/change/part (Carpenter, et al., 2015). As such joining to counting strategy is complex counting strategy and it enhances conceptual understanding and additive reasoning. Carpenter et al. (2015) therefore suggest that to increase number sense when learning addition of numbers, children should be given a variety of addition problems that offer them opportunities to use both simple and complicated counting strategies. We therefore used this classification scheme to examine the addition examples in the TG and LT to find out if they are varied in a way that promote learners’ number sense.

The MDITx framework describes the quality of mathematics made available to learn in a textbook (Ronda & Adler, 2016). MDITx comprises of five key elements aimed and achieving generality and structure; object of learning, examples, tasks, naming/word use and legitimations. The object of learning is what learners need to know and be able to do at the end of the lesson, as such, it is the goal of the lesson. Opportunities for learning mathematics are either afforded or constrained by the way author(s) use examples, tasks, words and legitimations (Ronda & Adler, 2016). Due to space limitations, we only give a brief description of what examples entail in MDITx as these are in focus in this paper. Examples are a particular case of a larger class used for drawing reasoning and generalisations (Ronda & Adler, 2016). They are what teachers and learners mainly work on during mathematics instruction. Carpenter et al. (2015) refer to examples as problems. MDITx draws from key principles of Variation Theory which emphasize on paying attention to variation amidst invariance when selecting examples (Marton & Pang, 2006). This means that textbooks must contain examples which are deliberately sequenced to enable learners to understand a particular object of learning in a coherent manner through noticing aspects that remain the same and those that change. Ronda and Adler (2016) therefore suggests that to analyse and determine variation in an example set, three categories of variation must be used. These are; contrast (C) (when differences are noticed), generalization (G) when similarity is noticed and fusion (F) (when at least 2 different objects of learning are in focus). Thus they describe a set of three progressive indicators for analysing and coding example spaces in a textbook lesson as follows: Level 1, if only one pattern of variation is used throughout the textbook lesson, Level 2, if two different patterns of variation are used in the textbook lesson, and Level 3, if all three patterns of variation are used. A fourth code called NONE is used to code example spaces in which no pattern of variation is detected. We used these descriptions to code the example sets during data analysis.

Methodology

Analysis of the structure of addition problems in grade 2 mathematics TG and LT (Kachisa, Mphando, Mwale, Soko, & Toto, 2012a; 2012b) involved examining what is given and what is required to be calculated in each problem using Carpenter et al.'s (2015) classification scheme. Thus, we examined whether a problem contained both addend and required calculating the sum, or whether it contained the sum and one addend and required calculating the other addend. To examine the variation in the examples, we regarded examples under each activity in both the TG and the LT as an example space and examined the type of variation available using MDITx framework by Ronda and Adler (2016). Where one type of variation was used throughout, we coded the example space as level 1. If two types of variation were used, we coded the example space as Level 2, and Level 3 if all three types of variation were used.

Results

The findings show that Unit 2 of both the TG and LT contain three main activities with specific object of learning for each activity. In the TG, each activity has several tasks and each task is accompanied with a set of examples. We begin by presenting findings on the nature of the structure of addition examples/problems, followed by findings on the nature of variation afforded by the examples using Figures 1, 2 and 3.

The structure of the addition problems

As it can be seen in Figures 1, 2 and 3, both the TG and LT have provided addition problems containing both addends and requiring finding of sums. Only activity 1 of the TG contain word problems as well as non-word problems, but the other activities only contain word problems. All LT activities do not contain word problems. The word problems in activity 1 of the TG belong to two types of addition problems. Problem 2a *Chifundo has 2 mangoes and Paul gives her 3 mangoes, how many mangoes does Chifundo have altogether?* is a join problem because as it implies action of giving mangoes, hence causing an increase in Chifundo’s total number of mangoes. The problem has initial number of mangoes that Chifundo had (2 mangoes) and then the change or increase she is given by Paul (3 mangoes), and the requirement is to find resulting amount (altogether). This problem can be represented as $2 + 3 = \square$. Addition problems 2b and 4 are part-part-whole problems because they do not imply action but require finding the sum of two disjoint sets of objects like sweets, sticks and stones. For example, problem 2b *Mphatso has 9 sweets and Tamanda has 4 sweets, how many sweets do they have altogether?* The structure of this part-part-whole problem is similar to that of problem 2a and can also be presented as $9 + 4 = \square$. As such learners might count the two sets of sweets separately and then count the total by starting from first set and continuing with the other set. The same structure of providing two addends that require finding sum is also observed in problem 4 and the other non-word problems in activity in all activities in both the TG and LT. This implies that both the TG and the LT do not contain addition problems with a structure that contains the sum but require finding of one of the addends like $9 + \square = 13$ or $\square + 4 = 13$.

Variation afforded by the examples

The examples under each task were regarded as an example set while all examples under each activity were regarded as an example space for a particular object of learning. Each activity in the LT mainly contains one task with several examples under each activity and these were also regarded as an example space. Table 1 presents a summary of the findings on the nature of variation afforded by the example set under each task.

Table 1: Nature of variation of examples

Activity 1 : Adding numbers horizontally	Activity 2: Adding numbers vertically	Activity 3:Mastering addition facts
Example set 1 (TG): Level 1 (C).	Example Set 1 (TG): Level 0	Example set 1 (TG):Level 3 (G, C, F)
Example set 2 (TG): Level 1 (C)	Example Set 2 (TG): Level 2 (C)	Example space from LT: Level 3 (G, C, F)
Example Set 3 (TG): Level 0	Example space from LT: Level 3 (G, C, F)	

Example Set 4 (TG): Level 1 (C)		
Example Set 5 (TG): Level 1 (C)		
Example space from LT: Level 3 (G, C, F)		

As Table 1 shows, in Activities 1 and 2 in the TG, one example set is in level 0 because it only had one example so it was not possible to generate pattern of variation. The other example sets in Activity 1 and 2 of the TG are in level 1 as they only enhance contrasting pattern of variation. Only examples of activity 3 in the TG enhance all patterns of variation. The LT examples for all activities enhance all patterns of variation. I clarify these findings using Figures 1 and 2.

Figure 1: Activity 1 from TG
(Kachisa et al., 2012a, 13)

Figure 2: Activity 1 from LT
(Kachisa et al., 2012b, 23)

As it can be noticed in Figures 1, 2, the number of addends and the position of the = sign remain the same (invariant) in all examples, what changes are the values of the addends. As shown in Figure 1, the examples from each example set contain different addends that generate different sums. We therefore coded each example set as contrast (C) as at it might enable generalising that different sets of addends generate different sums, hence level 1 of generalisation. In the LT, the example space under activity 1 is in Level 3 of generalisation because it contains examples which enhance all patterns of variation as shown in Figure 2. Several examples can be combined from the example space in Figure 2 to form example sets that might enhance generalisation through noticing different patterns of variation. Examples 1-2, 2-4, and 16-18 can be used to generalise that different sets of addends can generate similar sums. As such we coded these example sets as G. Since these examples also enhance development of number facts, we also coded them as F. Similar patterns of variation and fusion can also be drawn through combination of examples 9, 11 and 14, as well as examples 10, 12 and 15. However, if the learners solve the examples in a manner that is presented in the LT, then examples 6-15 would enhance generalisation through contrast (C) pattern of variation as they addends and sums are different. As such we coded this part of example space as C. Since the example space

contain examples which might help understanding of two objects of learning such as adding numbers and number bases, then we also coded it F.

Discussion of the results

In this paper, we examined the mathematical opportunities made available in grade 2 TG and LT by analysing the structure and nature of variation of examples/problems on addition. As the findings have revealed, the TG has provided more Level 1 example sets that enhance noticing of only one pattern of variation than Level 2 and Level 3 example sets that enhance noticing more than one pattern of variation. All level 1 example sets in the TG are enhanced contrast pattern of variation, meaning that the TG has provided more opportunities for only noticing how different set of addends can generate different sums, but not to notice how the different sets of addends can generate similar sums. Contrast is a first step or low level type of generalisation and is supposed to be followed by similarity to enable children to move into deeper levels of generalization and understanding through identification of more patterns and justifications (Watson & Mason, 2006). This suggests that to enhance development of additive reasoning, the TG is supposed to deliberately contain examples that afford them opportunities to recognise aspects that change within aspects that do not change (Ronda & Adler, 2016; Watson & Mason, 2006). Provision of example sets that help learners to see not only contrast but also similarity signals a move to higher level of generality (Ronda & Adler, 2016) and it can engage learners with understanding of mathematical structure (Watson & Mason, 2006). The findings however show that the examples in the TG activity of number bases might enhance learners' understanding of the commutative property that the change in the order of the addends does not lead to change in the sum of the addends (Carpenter et al., 2015; Hunter, 2010). Understanding of the commutative principle helps learners to develop generalisations that help develop algebraic reasoning which is the difficult part of later mathematics (Hunter, 2010). Activity 3 of the TG and other example sets in the LT might also enhance learners' development of additive reasoning through enhancing understanding of number facts (Carpenter et al., 2015). Although the LT contains example sets that enhance noticing of both similarity and contrast patterns of generalisation, the instructions from the TG only require the teacher to ask the learners to do the tasks but not to focus on noticing any pattern of variation. This implies that depending on their knowledge, some teachers may help the learners to pay attention to these patterns of variation while others may not. As Brown (2009) noted, teacher knowledge greatly influences how teachers adapt textbook content.

As the findings reveal, the TG and LT have only provided joint and part-part-whole addition problems which contain two addends and require learners to find the sum, but they do not provide problems requiring finding an addend. According to Carpenter et al. (2015), when children are given joint problems to find the result, and part-part-whole problems to find the whole, they only use simple counting strategies like joining all modelling strategy and counting on modelling strategy. Thus children need to be given problems whose either start, change, or part are unknown in order for them to use more sophisticated modelling and counting strategies like joining to, separating from, separating to, counting on to and counting (Carpenter et al., 2015). Providing different unknowns in addition problems enable children enhance their additive reasoning through promotion of use of different counting strategies (Carpenter et al., (2015). The findings imply that both the TG and LT

place more demand on the teacher especially during planning of lessons for delivering the curriculum (Brown, 2009). Thus implying that learners' full development of additive reasoning relies on teachers' ability to notice the shortfalls of the examples in both the TG and LT and deciding how to adapt them to increase their level of generalization and enhancement of development of additive reasoning. One of the factors that influence teachers' ability to recognise what the textbook affords and constraints is teacher knowledge (Brown, 2009; Leshota, 2020). Considering that like most Sub-Saharan countries, teacher knowledge is one of the challenges constraining Malawi from achieving the SDG 4 of improving education quality (MoEST, 2020), then it is unlikely that most Malawian teachers might notice the gaps in these mathematics textbooks. These findings confirm Milner et al.'s (2011) suggestion that the low quality of Malawian mathematics textbook content might be one of the causes of persistence of learners' low performance in mathematics. We concur with Leshota (2020) that countries with high textbook compliance policies need to ensure that they develop high quality textbooks and train their teachers on how to use these materials when planning lessons for delivering curriculum.

Conclusion

This study investigated mathematical opportunities available in addition examples/problems for addition topic in Malawian TG and LT of grade 2. The findings revealed that both the TG and LT contain addition examples comprising of only one structure whereby both addends are given and a sum is to be calculated, hence limiting learners' development of additive reasoning through use of complex counting strategies that develop when learners practice calculating an addend when given sum and one of the addends. Regarding variation of examples, the findings revealed that the TG has more example sets that can enhance achieving low levels of generality and few example sets that can enable achieving higher levels of generality. The LT contains more example sets that can enhance noticing of more than one pattern of variation, hence capable of enabling learners to move to higher level of generality of addition. However, the teacher might not focus learners' attention to notice the different variations because the TG which is the main resource that the teacher uses to plan lessons does not contain any instruction about noticing similarity or difference. Thus the structure and patterns of variation of the addition examples available in the TG and LT place much demand on the teacher as they require much adaptation to enhance additive reasoning through changing of structure and variation. This might be possible if teachers have adequate content knowledge to adapt these materials. These findings suggest that there is a need for professional development of teachers on how they can use these resources as suggested by other researchers. However, for an effective professional development of teachers, it is necessary to investigate how teachers use these resources to plan and teach mathematics lessons to find out if they do notice the gaps in the examples and make necessary adaptations.

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