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Reading comprehension and modelling problems: Does it matter where the question is placed?

Valentin Böswald and Stanislaw Schukajlow

University of Münster, Department of Mathematics, Germany;

vboeswald@uni-muenster.de, schukajlow@uni-muenster.de

Text comprehension is a key aspect to consider when designing modelling problems. One important feature of mathematical problems is where the question is placed in the text. We present a theoretical background on text comprehension and modelling problems, and we discuss the pros and cons of placing the question before the text rather than placing it after the text. A review of the research revealed that placing the question before the text is more likely to result in improved comprehension. Further, we propose consequences for task design and future research.

Keywords: Reading comprehension, eye movements, modelling, mathematical materials.

Introduction

Prior research has indicated that mathematical problems are very important for learning mathematics. For example, in the TIMSS study, 80% of the time spent in mathematics classes was spent working on mathematical problems (Hiebert et al., 2003). Thus, in order to appropriately foster students' learning, thorough research must be conducted on task design. Niss et al. (2007) distinguished mathematical problems on the basis of their connection to reality, resulting in three different types of mathematical problems: modelling problems, word problems, and intramathematical problems. Whereas the third type have no connection to reality at all and can therefore be solved by applying heuristics and mathematical procedures, word problems are moderately connected to reality, meaning that the intended use of mathematics is embedded in an extramathematical context. Word problems differ from modelling problems in the strength of their connection to reality and in the cognitive processes necessary to solve the problem (Niss et al., 2007). Modelling problems require the problem solver to make assumptions about missing data and to structure and simplify the given context, whereas word problems are already prestructured and simplified. Usually, modelling problems are presented as texts. Thus, reading comprehension and the construction of an adequate mental representation of the real-world situation are essential for solving modelling problems. Based on this mental representation, the solution plan can be developed and carried out. As Leiss et al. (2019) showed, text comprehension was a significant predictor of students' performance in solving modelling problems. Thus, there is a need for research on text comprehension and task design in modelling. Initial attempts to conduct research on the design of modelling problems were carried out in the past by posing open modelling problems with missing information (e.g., Fermi problems) or by asking students to develop multiple solutions to the problem or to draw a picture that represents the situation (for an overview, see Schukajlow, Kaiser, and Stillman, 2021). However, to date, not much is known about how task design affects the reading comprehension and solving of modelling problems. One variation in task design involves so-called *question-placement effects* (Thevenot et al., 2007). The theory of question-placement effects suggests that comprehension should benefit from reading the question before reading the text of the mathematical problem. The aims of this theoretical

paper are (1) to summarize and link theoretical and empirical findings from research in the areas of reading comprehension and modelling, (2) to ground the influence of questions and their placement on comprehension in mathematical modelling problems, and (3) to present consequences for research and task design.

Theoretical background

Text comprehension

Van Dijk and Kintsch (1983) distinguished between three levels of text comprehension that are built on each other but differ primarily in their complexity: the surface code, the text base, and the situation model. The surface code of a text consists of the exact wording and syntax of what is read, and thus, it is a pure representation of what has been read. In most cases, readers retain only the text surface of the last sentence or part of a sentence they read. Comprehension based on the surface code alone would be indicated through verbatim reproduction of the given text.

The text base reduces the text surface to the semantic content of the text – the wording and syntax are therefore no longer exactly the same as the surface code. However, the meaning of the text remains, represented in so-called *propositions* (van Dijk & Kintsch, 1983). This level illustrates a mental representation of the semantic content of the text via the process of building coherence locally, that is, the relationships between the subjects, objects, and so forth, and the predicates (e.g., traits or actions) given in the surface code in a sentence or between adjacent sentences. Successful comprehension on the level of the text base means the text base leads to an understanding of the basic features of the text, but deeper comprehension of the content of the text is not sufficient.

The highest level of comprehension is the situation model (van Dijk & Kintsch, 1983). It contains the content or context of the text and therefore no longer represents the explicit text but rather what the text is about. This mental model is constructed from the text base by drawing inferences and is augmented by prior knowledge and even external sources, such as the content of other texts or pictures. Building coherence across the entire text (i.e., global coherence) is crucial for constructing this mental model. The situation model may then be used to detect inconsistencies presented in external information and to evaluate the memory contents (Radvansky & Copeland, 2004). In the past, research on reading comprehension has been carried out by using different research approaches, including analyses of eye movements.

Eye movements during reading

The comprehension process is based on eye movements. Eye movements during reading can be classified into two different types: fixations and saccades. The former is “the state when the eye remains still over a period of time” (Holmqvist et al., 2011, p. 21). During reading, fixations occur when the eye temporarily stops moving on a specific word in order to process its meaning. Between two fixations (e.g., while switching from one word two another during reading), rapid eye movements occur, the so-called *saccades*. In reading research, a saccade that opposes the typical reading direction (e.g., *left to right* in Latin typeface) is called *regression*. Clifton et al. (2016) observed that the processes of word recognition and text comprehension (i.e., the construction of a situation model) are especially likely to have a strong influence on eye movements during reading.

Text comprehension in mathematical modelling

According to research by Reusser (1990) and Leiss et al. (2010), a situation model must be constructed to solve complex word problems and modelling problems. The real-world situation is represented in the so-called *situation model*, and it serves as the foundation for further steps in the solution processes. Blum and Leiß (2007) acknowledged this necessity and included the situation model in their modelling cycle, which has often been used to describe the process of solving modelling problems in the past. By comprehending the text, students construct an idiosyncratic mental representation of the real-world situation. This process can be described as a task-oriented structuring of individual knowledge. Importantly, the situation model contains more than just extramathematical knowledge (i.e., prior knowledge and prior experience). Rather, over the course of constructing a viable situation model of the problem, the student must identify a mathematically significant gap in the real-world situation in order to be able to complete the remaining steps in the modelling cycle on the basis of this situation model (Reusser, 1990). While constructing a situation model, the mathematically relevant information related to the question needs to be represented, and relevant information needs to be distinguished from irrelevant information regarding the modelling problem's question. On average, students spend around 40% of the time necessary to solve modelling problems on the comprehension process (Leiss et al., 2019). Therefore, there is a need to investigate how different task designs affect students' efficiency during this process. The process of comprehending the modelling problem is then followed by the process of structuring and simplifying the given information and thus constructing the so-called *real model*. Based on this deliberately structured mental model, mathematization is subsequently used to construct a mathematical model, which will allow the student to work mathematically within the mathematical model in order to generate mathematical results. These results then have to be interpreted on the basis of the real-world situation, followed by the deliberate validation of these results.

Hegarty et al. (1995) proposed two approaches used by successful and less successful word problem solvers: the direct-translation strategy and the problem model strategy. Problem solvers who utilize this first approach tend to base their solution plan on the selection of keywords and numbers given in the text of a problem. This rather superficial approach (for an overview, see Verschaffel et al., 2020) then leads to an incorrect solution more often when compared with the second approach the authors identified: the problem model strategy. Problem solvers who utilize this second approach deliberately construct a situation model of the situation described in the text and are therefore able to detect inconsistencies between their situation model and the given information by applying their solution process, a practice that is necessary for correctly validating the solution. In line with these results, Strohmaier et al. (2020) identified eye movements corresponding with these two approaches, plus a third approach that characterized readers who struggled while solving word problems. According to Strohmaier et al., utilizing the direct-translation strategy manifested in "a very linear and intense reading pattern with long mean fixation durations, short saccades and few regressions"(p. 10). On the other hand, the problem model strategy manifested in a shorter mean fixation duration and a high frequency of regressions. The third pattern Strohmaier et al. (2020) identified, which was linked to struggling readers, consisted of shorter saccades, higher fixation counts, as well as a higher regression count, resulting in overall longer reading times.

Effects of the placement of the question on comprehension in mathematical modelling problems

The reading goal is important for comprehension processes. In complex word problem solving, the reading goal is set by the question that accompanies the text of the task. While solving mathematical problems with relationships with the real world, the reading goal is to generate an accurate and deep understanding of the situation in order to answer the question. Usually, the question points to a mathematically relevant gap in the situation, which can be closed by using mathematics as a tool (Reusser, 1990), that is, by constructing a mathematical model and applying mathematical procedures. Because of the importance of questions for reading comprehension processes and for solving problems, the placement of the question needs to be evaluated critically with respect to task design. Depending on whether the question is placed before or after the text describing the real-world situation, comprehension processes can differ significantly. Placing the question before the text (i.e., reading the question first) should enhance the comprehension process because the reading goal is made more specific. Additionally, the question may serve as a tool for organizing and structuring the reading material, thus helping a student focus their attention on the relevant information. Therefore, decisions about the relevance of information given in the text should be made more precisely and quickly and, in conclusion, more efficiently. Furthermore, constructing coherence between the title and the question may enable initial inferences to be drawn and may therefore generate initial comprehension. The question and the title should facilitate the comprehension process, which should help a student solve a modelling problem similar to the ways a title facilitates a reader's understanding of a narrative text. This hypothesis was derived from numerous studies that have shown the effect of placing questions before the text on the retrieval of goal-relevant information (cf. Carpenter et al., 2018). As a result, the situation model should be more adequate (i.e., goal-related comprehension should be improved). The question could also be placed after the descriptive text. By placing the question after the descriptive text, the automated comprehension process would usually result in reading the text first and the question afterwards. Therefore, readers would have to construct hypotheses about the relevance of information for the superordinate reading goal, that is, solving the (at this point still unknown) problem. Placing the question after the text would result in the problem solver having to revisit the text after reading the question or adapting their situation model accordingly by shifting the relevance attributed to the information, ultimately leading the problem solver to learn from the text.

When discussing question-placement effects, some considerations have to be kept in mind. First, a question-placement effect on comprehension might only occur if the question cannot be inferred immediately and unambiguously from reading the text at hand. This is because, if the question is already clear to the reader, although they have not read it yet, the reading goal can be adjusted accordingly, thus organizing the process of distinguishing between goal-relevant and goal-irrelevant information. Second, the benefits of placing the question before the text are assumed to be specific to goal-relevant information (Carpenter et al., 2018). Therefore, the use of questions placed before the text might impair a student's ability to learn from the text. Third, the beneficial effects of the placement of the question in mathematical modelling problems should depend on the problem solver's mathematical competence. This is the case because the mathematically significant gap in the

real-world situation needs to be identified, that is, the problem solver needs to construct a basic mathematical model that is based on the question in order to distinguish relevant from irrelevant information. The construction of the mathematical model requires adequate mathematical competence, and it was found to be affected by mathematical performance in the past (Schukajlow, Blomberg, et al., 2021).

Research on question-placement effects in mathematics education is scarce. Thevenot et al. (2007) showed that performance in solving arithmetic word problems was significantly better if the question was placed before the text. Their results opposed the results found by Arter and Clinton (1974), who did not find significant effects of placing the question before the text on error counts in a study on irrelevant information in arithmetic word problems. However, effects on the time needed to solve the problems were found, thus influencing the efficiency (the ratio of accuracy and time).

We would like to propose two prototypical comprehension processes regarding the two possible question placements in modelling problems by illustrating these induced effects with the sample problem called “Buddenturm” (Figure 1). We suggest that problems used for research in the field of question-placement effects should contain a title, a question, and a brief text that describes the situation. According to Thevenot et al. (2007), the descriptive text needs to be constructed in such a way that the problem’s actual question cannot be inferred immediately and unambiguously. Otherwise, effects of question placement might be impaired as stated above. To do so, from our point of view, each problem should contain additional numerical information that is not relevant to answering the question. In the “Buddenturm” problem, the annual data, the length of the city wall, and the tower’s former height are examples of irrelevant information.

Buddenturm

Question: How much area was refurbished in 2002?

The so-called "Buddenturm" is the only remaining fortified tower from the former city wall of Münster and was built in 1150. A few remaining rudiments of the 8-10-meter-high and 4-kilometer-long city wall can still be seen in two parts of the round tower. The Buddenturm is 30 meters high in total and has a 5-meter-high conical roof. In 2002, the outer wall of the 12.5-meter-wide tower was renovated along with the remains of the city wall by applying a new layer of limestone. This repair is only one of the few that the Buddenturm has experienced in its almost 900-year history; for example, it was still 40 meters high until 1945.

Figure 1: “Buddenturm” modelling problem with the question placed before the descriptive text

Assuming a linear reading pattern, the title of the problem is read first and does not depend on the placement of the question. By recognizing the name of the tower or by decoding the word *Turm* (in English: *tower*), readers can activate prior knowledge and integrate it into their initial situation model. After that, if the question is placed *before* the text (as is the case in Figure 1), readers will then read the question. By integrating the question and its content into the situation model, the reader may be able to draw inferences from it, such as that the tower is antique or at least in need of refurbishment. In conjunction with the information that the size of an area should be calculated to solve the problem, readers can enrich their situation model even further. After that, while reading the descriptive text,

relevant and irrelevant information can be identified as such, simplifying the process of structuring the situation model for mathematization later on. In summary, the comprehension process initiated by placing the question before the text can be processed on a deeper level from the very beginning, rather than the problem solver focusing on generating a rough understanding of the situation. In turn, if the question is placed *after* the text, all information from the descriptive text needs to be integrated into the situation model initially, resulting in an understanding of the overall situation. To some extent, this is the result of the descriptive text being structured in such a way that the question (and thus, the reading goal) cannot be inferred unambiguously while solving modelling problems. Therefore, the ability to distinguish between relevant and irrelevant information during reading might be impaired. Although readers might anticipate a particular question, they cannot evaluate the relevance of information with certainty. This is because the actual question has not yet been integrated into the situation model. After reading the question and integrating it into the situation model, the situation model can be (re)structured and simplified for mathematization later on. Rereading the descriptive text might be necessary during this phase so that the problem solver can adequately represent all the relevant information.

These two prototypical comprehension processes might differ across the range of novices to experts as well as interindividually, as the processes depend strongly on prior knowledge. Experts in understanding modelling problems might structure their comprehension process in a specific way so that they read the title of the problem first but then immediately read the question, effectively simulating the question being placed before the text and thus possibly benefitting from the early integration of the question in their situation model.

Consequences for task design and future research

There are two possible conditions for the placement of the question, both already being used by teachers and textbooks: *before the text* and *after the text*. As described above, both conditions may have a positive influence on parts of students' cognition while also impairing other parts. The latter may challenge the problem solver by making it harder to initially comprehend the modelling problem, especially if prior knowledge is poor. But the reader will ultimately learn from the text, that is, the reader's ability to learn about the content of the text by solving the problem might be improved (e.g., learning that the tower's height in Figure 1 has changed across the centuries). The former may impair long-term learning about the content of the text but may in turn improve text comprehension, ultimately leading to an improved solution process, which in turn may result in improved modelling competence. However, both possible conditions and the extent of their influence have yet to be studied thoroughly. Thevenot et al. (2007) examined question-placement effects for arithmetic word problems, which were designed for fourth-graders. For modelling problems, research needs to be done on the influence of the placement of the question on text comprehension and the efficiency of text comprehension. Eye-tracking technology can be utilized productively to identify how the placement of the question influences students' comprehension of mathematical modelling problems and how students attempt to comprehend them. If a question-placement effect can be demonstrated through carefully designed studies, research on possible effects on modelling performance through effects on text comprehension may be conducted. However, the question-placement effect and its extent should furthermore depend on students' cognitive characteristics, such as working memory

capacity, mathematical competence, and reading competence and on the problems' characteristics (e.g., the difficulty) (Thevenot et al., 2007) or the type of problem (e.g., "dressed up" word problems or intramathematical problems). As prior research was conducted on word problems, we can mostly speculate about how question placement might affect how students solve modelling problems.

Conclusion

In this theoretical paper, we discussed the theoretical background for question-placement effects in the reading comprehension of mathematical modelling problems and derived why placing the question before the text that describes the real-world situation should be beneficial for learners' comprehension processes, ultimately benefitting the solution processes. In short, placing the question before the text should help the reader better distinguish between relevant and irrelevant information by having a specified reading goal and constructing an adequate initial situation model just from the title and the question about the modelling problem.

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