Observerless Output-feedback Consensus-based Formation Control of 2nd-order Nonholonomic Systems

Antonio Loria, Emmanuel Nuno, Elena Panteley

Observerless Output-feedback Consensus-based Formation Control of 2nd-order Nonholonomic Systems

Antonio Loría Emmanuel Nuño Elena Panteley

***Abstract*—We present a solution to the problem of formation consensus control of second-order nonholonomic systems via output feedback. We contribute with a distributed consensus controller for force-controlled nonholonomic systems under the assumption that the forward and angular velocities are not measurable. Our main statement establishes uniform global asymptotic stability for the closed-loop system; this guarantees robustness with respect to bounded disturbances, in the sense of Malkin's total stability, also known as local input-to-state stability.**

***Index Terms*—Formation control, persistency of excitation, nonholonomic systems, consensus, output feedback**

I. Introduction

Formation control consists in making a group of multiple mobile robots acquire positions (and possibly orientations) in a desired geometric pattern, in order to maneuver as a whole [1]. As such, it consists in two distinct problems: that of consensus-based formation, (also known as rendezvous [2], [3]) and formation-tracking, in which case a trajectory imposed by a (possibly fictitious) leader is to be followed [4]–[6]. The distinction is important because, in contrast to the case of holonomic systems, for systems with nonholonomic constraints stabilization is not a particular case of trajectory tracking. Indeed controllers that solve one problem *generally* cannot solve the other [7]. Let alone for multi-agent systems.

The consensus-based formation problem fundamentally differs from that of formation-tracking in that in the former there is no pre-specified reference. That is, consensus-based formation is a pure consensus-seeking problem in which the stabilized manifold of *equilibria* depends on the systems' initial conditions, the graph topology, and the systems' dynamics [8]. Now, although the understanding of the consensus paradigm is well established for robots modeled as simple first- and second-order integrators, even under consideration of network aspects as proximity constraints [9] or communication delays [10], none of these approaches apply to consensus of nonholonomic systems. Indeed, this problem inherits the difficulties found both in cooperative control of networked systems and those related to set-point stabilization under non-integrable velocity constraints [11], so the literature on consensus of nonholonomic systems is more scarce.

For instance, in [12] and [13] the problem for systems in so-called chain form is addressed and in [2] a discontinuous controller that applies even under proximity constraints is proposed; see also [14] in which, in addition, the collision avoidance problem is addressed. In [15] the authors use the elegant reduction theorems to address a problem of consensus stabilization on circular formations. Nevertheless, with the exception of [5] and [15], in the latter papers, as well as in several works on formation-tracking control, including [4] and [6], nonholonomic systems are modeled as first-order systems involving only the kinematics equations that encode the non-integrable velocity constraints. Nonholonomic systems, however, are best modeled as second-order systems in which a force-balance equation, most typically expressed in Lagrangian form, is also considered —see, e.g., [16]–[19].

In this Technical Note we address the consensus-based formation control problem for 2nd-order nonholonomic systems —cf. [5], [15]. The originality of the controller proposed here resides in that it relies only on position and orientation feedback, but the velocities are not measured. Indeed, output feedback control of nonholonomic systems has been addressed in the context of trajectory-tracking for a single nonholonomic robot, e.g., in [16], [18], [20], and [21], but it has been scarcely studied in a multi-agent-systems setting. For instance, in [5] a discontinuous leader-follower formation-tracking controller is designed to follow a predetermined reference trajectory. The controller that we propose here is smooth and time-varying. It contains correcting terms proportional to consensus errors and their dirty derivatives, as well as a persistently-exciting term that excites all modes in the system to overcome the stabilization obstacles imposed by the nonholonomic constraints —cf. [22]. A downside of such controllers, however, is the difficulty of tuning the control gains to avoid excessive oscillatory behavior [23].

In contrast to some of the above-cited references, important aspects related to the network, such as preservation of connectivity, time-delays or, in robotics applications, obstacle-collision-avoidance, are not broached. Yet, a formal analysis is made to establish uniform global asymptotic stability for the closed-loop system. This property goes well beyond the mere (non)-uniform convergence property more often encountered in the literature. Indeed, for time-varying systems, only uniform global asymptotic stability (UGAS) guarantees robustness with respect to bounded disturbances, in the sense of Malkin's total stability [24].

The rest of this Technical Note is organized as follows. The problem formulation is presented in Section II, followed by the rationale of our control approach, in Section III. The main result is stated in Section IV and numerical simulations are provided in Section V. Concluding remarks are given in Section VI.

II. Model and problem formulation

Consider a group of N differential-wheel mobile robots moving on a plane. Denoting by $z_i := [x_i\ y_i]^\top \in \mathbb{R}^2$, with $i \in \bar{N} := \{1, ..., N\}$, the Cartesian position of one robot's center of mass and by $\theta_i \in \mathbb{R}$ the vehicle's orientation, the velocity kinematics for the i-th robot is given by the equations

$$\dot{x}_i = v_i \cos(\theta_i) \tag{1a}$$

$$\dot{y}_i = v_i \sin(\theta_i) \tag{1b}$$

$$\dot{\theta}_i = \omega_i, \qquad i \in \bar{N} \tag{1c}$$

where $v_i, \omega_i \in \mathbb{R}$ are the linear and angular velocities of the center of mass. Such model is often considered in the literature on cooperative

Corresponding author: Emmanuel Nuño; Department of Computer Science at the University of Guadalajara. Guadalajara, Mexico (e-mail: emmanuel.nuno@cucei.udg.mx). A. Loría and E. Panteley are with the Laboratoire des signaux et systèmes, Univ Paris-Saclay, CNRS, Gif-sur-Yvette, France; (e-mail: {antonio.loria}{elena.panteley}@cnrs.fr). E. Panteley is also with ITMO University, St. Petersburg Russia.

E. Nuño's work is partially supported by the Government of Mexico via the Basic Scientific Research grant CB-282807, sponsored by CONACyT. The work of A. Loría and E. Panteley is supported by the French National Research Agency (ANR) via the project "HANDY" — contract number: ANR-18-CE40-0010, and by CEFIPRA under the grant number 6001-A. The work of E. Panteley is also the supported Government of the Russian Federation under grant 074-U01.

control of nonholonomic systems —see, e.g., [2], [4], [6] and [11]. In this Technical Note, however, we adopt a more appropriate 2nd-order model which includes the velocity dynamics equation

$$\begin{bmatrix} m_i & 0 \\ 0 & I_i \end{bmatrix} \begin{bmatrix} \dot{v}_i \\ \dot{\omega}_i \end{bmatrix} = \frac{1}{r_i} \begin{bmatrix} 1 & 1 \\ 2R_i & -2R_i \end{bmatrix} \tau_i \tag{2}$$

where m_i is the robot's mass; I_i is the moment of inertia; R_i is the distance between the point z_i and the wheels; r_i is the radius of the wheels; and τ_i is the control input torque of the left and right wheels, i.e., $\tau_i = [\tau_{il}, \tau_{ir}]^\top$. For analysis purposes, it is assumed that the center of mass is placed on the axis joining the two wheels' centers. Otherwise, Coriolis and centrifugal forces terms should be included in (2) —cf. [16]–[19]. These are considered in Section V.

Let $z_c := [x_c \; y_c]^\top$ denote a point on the Cartesian plane that is not specified a priori and is unknown to any robot. Let this point be the center of a formation pattern constructed by defining constant vectors originating in z_c, denoted $\delta_i = [\delta_{ix} \; \delta_{iy}]^\top$. For each $i \in \{1 \dots N\}$, the vector δ_i is defined so as to determine a desired position for the i-th agent with respect to z_c, thereby defining a formation centered at z_c. Then, defining $\bar{z}_i := z_i - \delta_i$ the consensus-based formation control goal consists in ensuring that

$$\lim_{t\to\infty} (\bar{z}_i(t), \theta_i(t), v_i(t), \omega_i(t)) = (z_c, \theta_c, 0, 0), \quad \forall i \in \bar{N}. \tag{3}$$

It is assumed that each robot possesses sensors to measure its Cartesian position, z_i, and its orientation, θ_i, but the forward and angular velocities, v_i and ω_i, are not measurable.

Regarding the network topology we assume that the robots communicate their own measurements with a set of neighbor robots located in sufficient proximity for the network interconnection to be established, but not necessarily close enough for relative-distance sensors to be effective [11]. Note that it is natural to assume that if a pair of robots establish an interconnection, the flow of information is bidirectional hence, the robots' network topology may be described using an undirected weighted graph. This is commonly defined via a Laplacian matrix, $L \in \mathbb{R}^{N\times N}$, whose entries are defined as

$$\ell_{ij} = \begin{cases} \sum\limits_{k\in\mathcal{N}_i} w_{ik} & \text{if } i = j, \quad \forall\, i,\, j \in \bar{N} \\ -w_{ij} & \text{otherwise,} \end{cases} \tag{4}$$

where $\mathcal{N}_i$ is the set of agents communicating with the i-th robot. The interconnection weight, w_{ij}, is positive if the pair of nodes (i, j) is connected; otherwise, $w_{ij} = 0$. It is also assumed that the graph is connected, so L is symmetric positive semi-definite and possesses a unique null eigenvalue.

Remark 1: Consensus of nonholonomic systems under less stringent graph conditions are considered, e.g., in [2], but in a state-feedback control context. •

III. Consensus-based formation Control

For the purpose of control design and stability analysis, we derive now a dynamical model for which the control problem may be formulated as one of stabilization of the origin. To that end, we define $\varphi_i : \mathbb{R} \to \mathbb{R}^2$ and $\Phi : \mathbb{R}^N \to \mathbb{R}^{2N\times N}$, as

$$\varphi_i(\theta_i) := \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix} \quad \Phi(\theta) := \text{blockdiag}\big[\varphi_i(\theta_i)\big] \tag{5}$$

where $\theta := [\theta_1 \; \cdots \; \theta_N]^\top$. Then, we use (1) and $\bar{z}_i = z_i - \delta_i$ to derive the equations

$$\dot{\bar{z}}_i = \varphi_i(\theta_i) v_i, \tag{6a}$$
$$\dot{\theta}_i = \omega_i. \tag{6b}$$

Next, we perform a change of variable to "normalize" the control inputs. That is, let

$$\tau_i = \frac{r_i}{2} \begin{bmatrix} m_i & I_i/2R_i \\ m_i & -I_i/2R_i \end{bmatrix} \begin{bmatrix} u_{vi} \\ u_{\omega i} \end{bmatrix}, \tag{7}$$

so the velocity-dynamics equations become

$$\dot{v}_i = u_{vi} \tag{8a}$$
$$\dot{\omega}_i = u_{\omega i}. \tag{8b}$$

From a control viewpoint, beyond the weak convergence property expressed by (3), it is desired to design decentralized dynamic output-feedback controllers with state ζ_c and control laws $(t, \bar{z}_i, \theta_i, \zeta_c) \mapsto u_{vi}(t, \bar{z}_i, \theta_i, \zeta_c)$ and $(t, \bar{z}_i, \theta_i, \zeta_c) \mapsto u_{\omega i}(t, \bar{z}_i, \theta_i, \zeta_c)$ which depend on the measurable positions and orientations, z_i and θ_i, as well as on the controller state variable, ζ_c, such that the origin for the closed-loop system is uniformly globally asymptotically stable. To that end, we define the consensus errors

$$e_i = \sum_{j\in\mathcal{N}_i} w_{ij} \left(\bar{z}_i - \bar{z}_j\right), \tag{9}$$

$$e_{\theta i} = \sum_{j\in\mathcal{N}_i} w_{ij} \left(\theta_i - \theta_j\right) \tag{10}$$

and we introduce the smooth control laws

$$u_{vi} := -k_{dvi}\varphi_i(\theta_i)^\top \vartheta_{vi} - k_{pvi}\varphi_i(\theta_i)^\top e_i, \tag{11}$$
$$u_{\omega i} := -k_{d\omega i}\vartheta_{\omega i} - k_{p\omega i} e_{\theta i} + \alpha_i\left(t, \theta_i, e_i\right), \tag{12a}$$
$$\alpha_i := k_{\alpha i}\psi_i(t)\varphi_i(\theta_i)^\perp e_i, \tag{12b}$$

where k_{dvi}, k_{pvi}, $k_{d\omega i}$, $k_{p\omega i}$, $k_{\alpha i} > 0$,

$$\varphi_i(\theta_i)^\perp = [-\sin(\theta_i) \;\; \cos(\theta_i)]$$

is the left annihilator of $\varphi_i(\theta_i)$, i.e., $\varphi_i(\theta_i)^\perp\varphi_i(\theta_i) = \varphi_i(\theta_i)^\top[\varphi_i(\theta_i)^\perp]^\top = 0$, and the function $\psi_i : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is $\mathcal{C}^1$, bounded, has bounded derivative, and is persistently exciting —see Proposition 1 farther below. The variables ϑ_{vi} and $\vartheta_{\omega i}$ are defined dynamically as

$$\dot{q}_{vi} = -a_{vi}\left(q_{vi} + b_{vi}\bar{z}_i\right), \tag{13a}$$
$$\vartheta_{vi} = q_{vi} + b_{vi}\bar{z}_i, \quad a_{vi},\; b_{vi} > 0 \tag{13b}$$

and

$$\dot{q}_{\omega i} = -a_{\omega i}\left(q_{\omega i} + b_{\omega i}\theta_i\right), \tag{14a}$$
$$\vartheta_{\omega i} = q_{\omega i} + b_{\omega i}\theta_i, \qquad a_{\omega i},\; b_{\omega i} > 0. \tag{14b}$$

The dynamic controller defined by Eqs. (11)–(14) is, essentially, of the proportional-derivative type, with the added time-varying term α_i —cf. [22]. In particular, the terms $-k_{dvi}\varphi_i(\theta_i)^\top\vartheta_{vi}$ and $-k_{d\omega i}\vartheta_{\omega i}$ are responsible for injecting appropriate damping to achieve asymptotic stabilization. However, in place of velocity measurements $\dot{\bar{z}}_i$ and $\dot{\omega}_i$ the variables ϑ_{vi} and $\vartheta_{\omega i}$ are used. These may be regarded as the outputs of *approximate-differentiation* (low-pass) filters which, in the frequency domain, correspond to

$$\vartheta_{vi} = \frac{b_{vi}}{s + a_{vi}}\dot{\bar{z}}_i, \quad \vartheta_{\omega i} = \frac{b_{\omega i}}{s + a_{\omega i}}\omega_i. \tag{15}$$

Such *ad hoc* replacement for the velocities is often used, e.g., in control of robot manipulators, since the seminal paper [25]. Now, the system (6) in closed-loop with the controller defined by Eqs. (11)–(14) is given by

$$\Sigma_{\omega i} \quad \begin{cases} \dot{\theta}_i = & \omega_i \\ \dot{\omega}_i = & -k_{d\omega i}\vartheta_{\omega i} - k_{p\omega i}e_{\theta i} + \alpha_i\left(t, \theta_i, e_i\right) \\ \dot{\vartheta}_{\omega i} = & -a_{\omega i}\vartheta_{\omega i} + b_{\omega i}\omega_i. \end{cases} \tag{16}$$

$$\Sigma_{vi}\quad\begin{cases}\dot{\bar z}_i = & \varphi_i(\theta_i)v_i\\ \dot v_i = & -k_{dvi}\varphi_i(\theta_i)^\top\vartheta_{vi}-k_{pvi}\varphi_i(\theta_i)^\top e_i\\ \dot\vartheta_{vi} = & -a_{vi}\vartheta_{vi}+b_{vi}\varphi_i(\theta_i)v_i.\end{cases}\tag{17}$$

Several structural properties of the closed-loop system above are worth remarking. On one hand, note that both Σ_{v_i} and $\Sigma_{\omega i}$ are reminiscent of second-order systems coupled with first-order low pass filters. To better see this, consider $\Sigma_{\omega i}$ with $\alpha_i\equiv 0$ and $\vartheta_{\omega_i}=\omega_i$. Then, we recover a linear second-order system for which the consensus manifold $\{\omega_i=0,\, e_{\theta_i}=0\}$ is asymptotically stable [8]. What is more, replacing $e_{\theta i}$ with θ_i in the first two equations in (16), we observe that these equations correspond to an ordinary second-order linear system and the control gains $k_{p\omega i}$ and $k_{d\omega i}$ determine the fundamental frequency and damping coefficient respectively. Thus, with $\alpha_i\equiv 0$, the equations (16) correspond to those of two stable feedback-interconnected systems. Furthermore, letting aside the nonholonomy expressed via φ_i in (17), Σ_{vi} has similar structural properties. On the other hand, note that if in (17) we replace the state variable θ_i with the corresponding trajectories $\theta_i(t)$, which are (part of the) solutions of (16), then Σ_{vi} may be regarded as a decoupled linear time-varying system —*cf.* [26, p. 657]. Correspondingly, $\Sigma_{\omega i}$ may be considered as a system that is "perturbed" by α_i, which is a function of e_i so it is both an output of Σ_{vi} and an "input" to $\Sigma_{\omega i}$. Thus, the closed-loop system $\Sigma_{\omega i}$–Σ_{vi} may be considered as if interconnected in cascade[1].

These observations are useful in understanding the stabilization mechanism in play. For cascaded systems uniform global asymptotic stability of the origin follows if three basic conditions are met: 1) the origin for the perturbed system without input (i.e., with $\alpha_i\equiv 0$), $\Sigma^\circ_{\omega i}$, is UGAS; 2) the origin for the perturbing system, Σ_{vi}, is UGAS, uniformly in the trajectories $\theta_i(t)$; and 3) the solutions of $\Sigma_{\omega i}$ are uniformly globally bounded. Now, UGAS of the origin for $\Sigma^\circ_{\omega i}$ may be established as per the rationale above. UGAS of the origin for Σ_{vi} comes after similar considerations as for $\Sigma_{\omega i}$, in addition to the fact that ψ_i is persistently exciting. Indeed, this property guarantees that $\alpha_i\not\equiv 0$ unless $e_i\equiv 0$ —see (12b). Therefore, the "perturbation" α_i persistently prevents Σ_{ω_i} to stabilize at unwanted equilibria while the consensus errors e_i perdure. Uniform global boundedness of the solutions of $\Sigma_{\omega i}$ with $\alpha_i\not\equiv 0$ follows by proving, in addition, that the trajectories $e_i(t)$, and hence the "perturbation" $\alpha_i(t,e_i(t),\theta_i(t))$, are bounded. Our main result and its proof constitute a formal statement of these intuitive arguments.

IV. Main result

We start by rewriting the closed-loop equations for the N robots in compact form. To that end, let $e:=[e_1^\top\ \cdots e_N^\top]^\top$, $\bar z:=[\bar z_1^\top\ \cdots \bar z_N^\top]^\top$, $v:=[v_1\ \cdots\ v_N]^\top$, $e_\theta:=[e_{\theta 1}^\top\ \cdots e_{\theta N}^\top]^\top$, $\theta:=[\theta_1^\top\ \cdots \theta_N^\top]^\top$, $\omega:=[\omega_1\ \cdots\ \omega_N]^\top$, $\vartheta_v:=[\vartheta_{v1}^\top\ \cdots\ \vartheta_{vN}^\top]^\top$, and $\vartheta_\omega:=[\vartheta_{\omega 1}\ \cdots\ \vartheta_{\omega N}]^\top$, as well as $\alpha:=[\alpha_1\cdots\alpha_N]^\top$ and the control gains $K_{dv}:=\mathrm{diag}[k_{dvi}]$, $K_{d\omega}:=\mathrm{diag}[k_{d\omega i}]$, $K_{pv}:=\mathrm{diag}[k_{pvi}]$, $K_{p\omega}:=\mathrm{diag}[k_{p\omega i}]$, $A_v:=\mathrm{diag}[a_{vi}]\otimes I_2$, $A_\omega:=\mathrm{diag}[a_{\omega i}]$, $B_v:=\mathrm{diag}[b_{vi}]\otimes I_2$, $B_\omega:=\mathrm{diag}[b_{\omega i}]$, where the matrix I_2 corresponds to the 2×2-identity-matrix and $\otimes$ denotes the Kronecker product. Then, the closed-loop may be rewritten as

$$\Sigma_\omega\quad\begin{cases}\dot\theta &=& \omega\\ \dot\omega &=& -K_{d\omega}\vartheta_\omega-K_{p\omega}e_\theta+\alpha(t,\theta,e)\\ \dot\vartheta_\omega &=& -A_\omega\vartheta_\omega+B_\omega\omega\end{cases}\tag{18}$$

[1] See [27] for a more detailed explanation of how feedback-interconnected systems may be considered as if interconnected in cascade.

$$\Sigma_v\quad\begin{cases}\dot{\bar z} &=& \Phi(\theta)v\\ \dot v &=& -K_{dv}\Phi(\theta)^\top\vartheta_v-K_{pv}\Phi(\theta)^\top e\\ \dot\vartheta_v &=& -A_v\vartheta_v+B_v\Phi(\theta)v\end{cases}\tag{19}$$

where, in addition, we stress that $e=\mathcal{L}\bar z$, with $\mathcal{L}:=L\otimes I_2$.

Proposition 1: (Main result) Consider the system (18)–(19). Assume that for each $i\in\bar N$, there exist $c_{\psi i}$, μ_i and $T_i>0$, such that

$$\max\{\,|\psi_i|_\infty,\ |\dot\psi_i|_\infty\,\}\quad\le\quad c_{\psi i}\tag{20}$$

$$\int_t^{t+T_i}\psi_i(s)^2ds\quad\ge\quad\mu_i\qquad\forall\, t\ge 0\tag{21}$$

where $|\psi_i|_\infty:=\sup_{t\ge 0}|\psi_i(t)|$. In addition, assume that all the control gains A_v, A_ω, B_v, B_ω, K_{dv}, $K_{d\omega}$, K_{pv}, $K_{p\omega}$, and K_α are positive definite and B_ω is sufficiently large, such that

$$B_\omega\ge 2L,\tag{22}$$

i.e., $(B_\omega-2L)$ is positive semi-definite. Then, the origin, $(e,e_\theta,v,\omega,\vartheta_v,\vartheta_\omega)=(0,0,0,0,0,0)$ is uniformly globally asymptotically stable. *In particular*, under the action of the controller defined by Eqs. (11)–(14), the system (6) achieves full consensus, i.e., (3) holds.

Remark 2: The condition (22) imposes a lower bound on each b_{wi} depending only on a bound on the interconnection strength of the links involving the i-th robot and its neighbors. Also, the factor '2' is used to simplify some computations in the proof, but (22) may be relaxed to imposing that $(B_\omega-L)$ be positive definite. •

Proof of Proposition 1. According with the rationale given previously, we follow three logical steps and we invoke a cascades argument, specifically, [28, Lemma 2].

Step 1: UGAS of Σ°_w.– Consider the system (18) with $e=0$, hence, with $\alpha\equiv 0$. The proof follows invoking the Generalized Matrosov's Theorem [29]. Let $K_\omega:=K_{p\omega}^{-1}K_{d\omega}B_\omega^{-1}$ and consider the functions

$$W_1(\theta,\omega,\vartheta_\omega):=\frac{1}{2}\Big[\omega^\top K_{p\omega}^{-1}\omega+\vartheta_\omega^\top K_\omega\vartheta_\omega+\theta^\top L\theta\Big],\tag{23a}$$

$$W_2(\omega,\vartheta_\omega):=-\omega^\top\vartheta_\omega,\tag{23b}$$

$$W_3(\theta,\omega):=e_\theta^\top\omega,\tag{23c}$$

where $e_\theta=L\theta$. The function W_1 is positive definite in the space of $(e_\theta,\omega,\vartheta_\omega)$ since there exist c_1 and $c_2>0$ such that

$$c_1|e_\theta|^2\le\theta^\top L\theta\le c_2|e_\theta|^2.\tag{24}$$

—see [30, Lemma 1]. Next, we evaluate the derivatives of W_1, W_2, and W_3, along the trajectories of (18) with $\alpha\equiv 0$. For W_1 we obtain

$$\dot W_1=-\vartheta_\omega^\top K_\omega A_\omega\vartheta_\omega\ =:Y_{w1}(e_\theta,\omega,\vartheta_\omega)\le 0.\tag{25}$$

It follows from the latter that the origin, $\{x_\omega=0\}$, where $x_\omega:=[e^\top,v^\top,\vartheta_\omega^\top]^\top$, is uniformly globally stable for Σ°_ω. Now, the total derivatives of W_2 and W_3 yield

$$\begin{aligned}\dot W_2&=-\omega^\top B_\omega\omega+\vartheta_\omega^\top\big[A_\omega\omega+K_{p\omega}e_\theta+K_{d\omega}\vartheta_\omega\big]\\&=:Y_{w2}(e_\theta,\omega,\vartheta_\omega)\end{aligned}\tag{26}$$

$$\begin{aligned}\dot W_3&=-e_\theta^\top K_{p\omega}e_\theta-\vartheta_\omega^\top K_{d\omega}e_\theta+\omega^\top L\omega\\&=:Y_{w3}(e_\theta,\omega,\vartheta_\omega)\end{aligned}\tag{27}$$

We see that $\dot W_2$ is negative semi-definite on the set $\{Y_{w1}=0\}$, $\dot W_3$ is negative definite on the set $\{Y_{w1}=0\}\cap\{Y_{w2}=0\}$ and all Y_{wi}, with $i\in\{1,2,3\}$, are zero simultaneously if and only if $(e_\theta,\omega,\vartheta_\omega)=(0,0,0)$. UGAS of the origin for Σ°_ω follows from [29, Theorem 1].

Step 2: Uniform boundedness of the solutions of Σ_ω.– Let $\varepsilon_1, \varepsilon_2 > 0$ and consider the function $W : \mathbb{R}_{\geq 0} \times \mathbb{R}^{3N} \to \mathbb{R}_{\geq 0}$,

$$W(t,\theta,\omega,\vartheta_\omega) := W_1(\theta,\omega,\vartheta_\omega) + \varepsilon_1 W_2(\omega,\vartheta_\omega) + \varepsilon_2 W_3(\theta,\omega).$$

This function is positive definite in $(e_\theta, \omega, \vartheta_\omega)$ for sufficiently small ε_1 and $\varepsilon_2 > 0$. Its total derivative, using (18), yields

$$\begin{aligned}\dot{W} \;\; = \;\; & Y_{w1} + \varepsilon_1 Y_{w2} + \varepsilon_2 Y_{w3} \\ & + \alpha(t,\theta,e)^\top [K_{p\omega}^{-1}\omega - \varepsilon_1 \vartheta_\omega + \varepsilon_2 e_\theta].\end{aligned}$$

Now, define k_*^m and k_*^M as the smallest and the largest elements of a diagonal matrix $K_* > 0$. After some direct computations, using (26) and (27), we see that

$$\begin{aligned}\dot{W} \leq & -\frac{1}{2}\Big[k_\omega^m a_\omega^m |\vartheta_\omega|^2 + \frac{\varepsilon_1}{2} b_\omega^m |\omega|^2 + \varepsilon_2 k_{p\omega}^m |e_\theta|^2\Big] \\ & + |\alpha| \Big[\frac{1}{k_{p\omega}^m}|\omega| + \varepsilon_1 |\vartheta_\omega| + \varepsilon_2 |e_\theta|\Big] \\ & -\omega^\top \Big[\frac{\varepsilon_1}{2} B_\omega - \varepsilon_2 L\Big]\omega - \frac{\varepsilon_1}{4}\Big[b_\omega^m - 2\lambda a_\omega^M\Big]|\omega|^2 \\ & -\frac{1}{2}\Big[k_\omega^m a_\omega^m - \frac{\varepsilon_1}{\lambda}\big[a_\omega^M + k_{p\omega}^M + 2\lambda k_{d\omega}^M\big] - \frac{\varepsilon_2}{\lambda} k_{d\omega}^M\Big]|\vartheta_\omega|^2 \\ & -\frac{1}{2}\Big[\varepsilon_2 k_{p\omega}^m - \lambda\big[\varepsilon_1 k_{p\omega}^M + \varepsilon_2 k_{d\omega}^M\big]\Big]|e_\theta|^2 \end{aligned} \tag{28}$$

where ε_1, ε_2, and $\lambda \in (0,1)$. On the right-hand side of the previous expression, the third term is non-positive, e.g., if $\varepsilon_1 = \varepsilon_2$ and in view of (22), the fourth and last terms are non-positive for sufficiently small values of λ, while the fifth term is non-positive for sufficiently small values of ε_1 and ε_2. Hence, let $\varepsilon_1 = \varepsilon_2 =: \varepsilon$ and $b_\omega^m := 2c\lambda a_\omega^M$, with $c \geq 1$. Then, the last term is non-positive if

$$\lambda \leq \frac{k_{p\omega}^m}{k_{p\omega}^M + k_{d\omega}^M} \tag{29}$$

because

$$k_\omega^m = \frac{k_{d\omega}^m}{b_\omega^M k_{p\omega}^M} = \frac{b_\omega^m k_{d\omega}^m}{2c\lambda a_\omega^M b_\omega^M k_{p\omega}^M},$$

while the before-last term is non-positive if

$$\varepsilon \leq \frac{b_\omega^m k_{d\omega}^m a_\omega^m}{2c\, a_\omega^M b_\omega^M k_{p\omega}^M \big[a_\omega^M + k_{p\omega}^M + (2\lambda+1)k_{d\omega}^M\big]}. \tag{30}$$

In turn, (29) and $b_\omega^m = 2c\lambda a_\omega^M$ impose that

$$c \geq \frac{k_{p\omega}^M + k_{d\omega}^M}{k_{p\omega}^m}\frac{b_\omega^m}{a_\omega^M}.$$

Remark 3: If necessary, the value of ε computed to satisfy (30) for an arbitrary $c \geq 1$ may be redefined. •

Thus, $\dot{W}$ is bounded from above by the first two terms on the right-hand side of (28) so, for all $|x_\omega| \gg 1$, any $c_\alpha > 0$ and all $\alpha \leq c_\alpha$, we have $\dot{W} \leq 0$. That is, the solutions $t \mapsto x_\omega(t)$ are uniformly globally bounded provided that so is $\alpha(t,\theta,e(t))$ which, in view of (20), holds if $e(t)$ is uniformly bounded. This is proved next.

Consider, the system Σ_v with $\theta = \theta(t)$, the latter corresponding to (part of) the solution to Eqs. (18), and the Lyapunov function candidate

$$V_1(\bar{z},v,\vartheta_v) := \frac{1}{2}\Big[v^\top K_{pv}^{-1} v + \vartheta_v^\top K_{pv}^{-1} K_{dv} B_v^{-1}\vartheta_v + \bar{z}^\top \mathcal{L}\bar{z}\Big]. \tag{31}$$

This function is positive definite in (e, v, ϑ), where $e = \mathcal{L}\bar{z}$. Indeed, there exist c_1 and $c_2 > 0$ such that

$$c_1|e|^2 \leq \bar{z}^\top \mathcal{L}\bar{z} \leq c_2|e|^2. \tag{32}$$

—see [30, Lemma 1]. Furthermore, using the first equation in (19) we find that the total derivative of V_1 along the trajectories of Σ_v yields

$$\dot{V}_1 = -\vartheta_v^\top K_v \vartheta_v, \quad K_v := K_{pv}^{-1} K_{dv} B_v^{-1} A_v \tag{33}$$

for all $(\bar{z}, v, \vartheta_v) \in \mathbb{R}^{5N}$. Since $\dot{V}_1 \leq 0$, it also follows that $V_1(\bar{z}(t), v(t), \vartheta_v(t)) \leq V_1(\bar{z}(t_\circ), v(t_\circ), \vartheta_v(t_\circ))$ for all $t \geq t_\circ \geq 0$. In view of (23), it also follows that, defining,

$$x_v := [e^\top, v^\top, \vartheta_v^\top]^\top, \tag{34}$$

there exists $c_v > 0$ such that

$$|x_v|_\infty \leq c_v |x_v(t_\circ)|, \tag{35}$$

which implies uniform global stability for the origin of Σ_v, hence uniform global boundedness of $x_v(t)$.

Remark 4: Technically, the equations (19) are defined only on the maximal interval of solutions of $t \mapsto \theta(t)$, $[t_\circ, t_{\max})$ with $t_{\max} \leq +\infty$, so the bound (35) holds only for all $t \in [t_\circ, t_{\max})$. Forward completeness of (18)-(19), however, follows from the fact that (33) and (28) hold along the system's solutions on *any* maximal interval of existence, $[t_\circ, t_{\max})$ for any $t_{\max} \geq t_\circ$. By continuity of the solutions in the initial conditions this interval may be extended up to $t_{\max} = +\infty$. •

Step 3: UGAS of the origin for Σ_v with $\theta = \theta(t)$.– We proceed as in Step 1. First, in view of (33) and the uniform global boundedness of the solutions, the origin for Σ_v with $\theta = \theta(t)$ is globally stable, uniformly in the initial conditions and in $\theta(t)$. Next, consider the functions $\tilde{\Phi}(\cdot) := \Phi(\theta(\cdot))$,

$$V_2(t,v,\vartheta_v) := -v^\top \tilde{\Phi}(t)^\top \vartheta_v, \tag{36}$$

$$V_3(t,e,v,) := v^\top \tilde{\Phi}(t)^\top e, \tag{37}$$

and

$$V_4(t,e) := -e^\top \tilde{\Phi}(t)^{\perp\top} \int_t^{t+T} \Psi(\tau)^\top \Psi(\tau) d\tau \, \tilde{\Phi}(t)^\perp e,$$

where $\Psi(t) := \mathrm{diag}[\psi_i(t)]$, $i \in \bar{N}$. These functions are defined on $[t_\circ, \infty)$ for any $t_\circ \geq 0$, are smooth, and uniformly bounded in t. Now, the total derivative of $V_2(t,v,\vartheta_v)$ along the trajectories of Σ_v with $\theta = \theta(t)$ yields

$$\begin{aligned}\dot{V}_2(t,v,\vartheta_v) = & -v^\top B_v v + \Big[[\Omega(t)\otimes I_2]\tilde{\Phi}(t)^{\perp\top} + A_v\tilde{\Phi}(t)\big]v\Big], \\ & + \vartheta_v^\top \Big[\tilde{\Phi}(t)\big[K_{dv}\tilde{\Phi}(t)^\top \vartheta_v + K_{pv}\tilde{\Phi}(t)^\top e\Big]\end{aligned}$$

which is negative semi-definite on the set $\{\dot{V}_1 = 0\}$. In the computation of $\dot{V}_2$ we used $\dot{\Phi}(\theta) = [\Omega \otimes I_2]\Phi(\theta)^{\perp\top}$ where $\Omega := \mathrm{diag}[\omega_i]$. Next, define $\xi_1(t,e) := \tilde{\Phi}(t)^\top e$; then, the total derivative of $V_3(t,e,v)$ yields

$$\begin{aligned}\dot{V}_3(t,e,v) = & -\xi_1(t,e)^\top K_{pv}\xi_1(t,e) - \vartheta_v^\top \tilde{\Phi}(t) K_{dv}\xi_1(t,e) \\ & + v^\top \Big[\tilde{\Phi}(t)^\perp [\Omega(t)\otimes I_2]e + \tilde{\Phi}(t)^\top \mathcal{L}\tilde{\Phi}(t)v\Big],\end{aligned}$$

which is negative semi-definite on the set $\{\dot{V}_1 = 0\} \cap \{\dot{V}_2 = 0\}$. Finally, we evaluate the total derivative of V_4. To that end, define $\xi_2(t,e) := \tilde{\Phi}(t)^\perp e$ then,

$$\begin{aligned}\dot{V}_4(t,e) = & V_4(t,e) - 2\xi_2(t,e)^\top \left[\int_t^{t+T} \Psi(\tau)^\top \Psi(\tau) d\tau\right] \times \\ & \Big[-\tilde{\Phi}(t)^\top [\Omega \otimes I_2]e + \tilde{\Phi}(t)^\perp \mathcal{L}\tilde{\Phi}(t)v\Big],\end{aligned}$$

where we used $\dot{\Phi}(\theta)^\perp = -\Phi(\theta)^\top [\Omega \otimes I_2]$. Now, let $\mu := \min\{\mu_i\}$ and $T := \max\{T_i\}$ where μ_i and T_i are as in (21) and observe that $V_4 \leq -\mu e^{-T}|\xi_2(t,e)|^2$. We see that $\dot{V}_4$ is negative semi-definite on the set $\{\dot{V}_1 = 0\} \cap \{\dot{V}_2 = 0\} \cap \{\dot{V}_3 = 0\}$. Moreover, $\dot{V}_i$, with $i \leq 4$

are equivalently equal to zero if and only if $v = \vartheta_v = \xi_1 = \xi_2 = 0$, while $\xi_1 = \xi_2 = 0$ if and only if $e = 0$. Thus, $\dot{V}_i = 0$ for all $i \leq 4$ only at the origin, $\{x_v = 0\}$. UGAS follows invoking the Generalized Matrosov's Theorem [29, Theorem 1]. ■

Remark 5: The proposed controller may be modified to address the problem in which the robots are required to converge to a given desired constant orientation θ_d, as opposed to a common, not pre-imposed one, θ_c. This may be achieved by replacing the orientation error (10) with

$$e_{\theta i} = \sum_{j \in \mathcal{N}_i} w_{ij} \left(\theta_i - \theta_j\right) + \sum_{j \in \mathcal{N}_i} b_i \tilde{\theta}_i,$$

where $\tilde{\theta}_i := \theta_i - \theta_d$, $b_i > 0$ if the ith-robot has access to the desired orientation θ_d and $b_i = 0$ otherwise. The result follows provided that *at least* one robot has access to θ_d. •

V. Simulations

Numerical simulations in Matlab™ Simulink™ under two different scenarii were performed to illustrate the controller's performance. The simulation setup consists in six robots required to meet at an unknown rendezvous point while forming a hexagonal pattern. The latter is defined by setting the offsets $(\delta_{xi}, \delta_{yi})$ to the values showed in Table I below. The robots' initial conditions are also given in Table I and the robots' parameters were taken as in [31]: $m_i = 10.4$kg, $I_i = 3$kgm^2, $R_i = 0.3$m, and $r_i = 0.05$m.

TABLE I
Initial conditions, relative desired positions, and desired orientations.

	R1	R2	R3	R4	R5	R6
$x_i(0)$	5	7	7	3	1	1
$y_i(0)$	2	5.5	3.5	2	3.5	5.5
$\theta_i(0)$	0	$-\pi/4$	$-\pi/2$	$\pi/4$	$\pi/2$	$\pi/4$
δ_{x_i}	2	1	-1	-2	-1	1
δ_{y_i}	0	2	2	0	-2	-2

It is assumed that the vehicles are interconnected according to a cyclic graph topology as illustrated in Fig. 1 below, with unitary interconnecting weights.

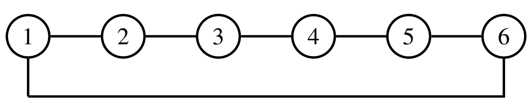

Fig. 1. Graph representation of the interconnection topology

The linear- and angular-velocity dynamics, Σ_{v_i} and Σ_{ω_i} corresponding, essentially, to second-order systems coupled with low-pass filters, the controller gains may be selected following a rule of thumb based on the tuning of an ordinary second-order system. Hence, they were set to $k_{pi} = 300$, $k_{di} = 600$, $k_{p\omega i} = 30$, $k_{d\omega i} = 60$, $k_{\alpha i} = 15$ in order to damp transient oscillations. Regarding the gains of the filters, as expressed in (15), they were set so as to have a unitary DC gain and filter out "high" frequencies. Hence, they were set to $a_{vi} = b_{vi} = a_{\omega i} = b_{\omega i} = 10$. Finally, for ψ_i to satisfy (21) we used the periodic (hence persistently exciting) function $\psi_i(t) := 2.5 + (4/\pi)\sin(0.5t)$ for all agents, but it may be chosen differently for each of them.

The paths followed by the robots on the plane are illustrated in Figure 2. We observe that all the robots converge to the desired hexagonal formation. The center of the latter is at the point $(x_c, y_c) =$

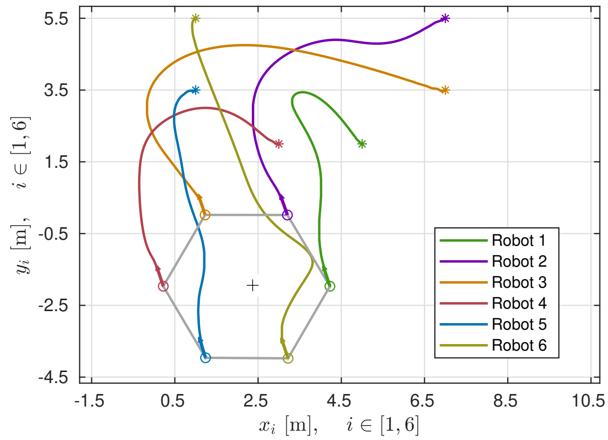

Fig. 2. Path followed by the robots on the plane

$(2.22, -1.95)$. The final coinciding orientations of the robots, which settle at approximately $\theta_c = 1.85$rad, are depicted using vectors.

For the purpose of illustrating the robustness of the controller relative to parametric uncertainty, neglected dynamics, and measurement noise, a second simulation was performed in which there is a discrepancy between the actual values of the mass, the inertia, and the location of the center of mass and those considered for the control design. Also, white noise of zero mean and standard deviation equal to 2% was added to the position measurements, to account for sensor defects [32]. It is assumed that the value of the mass and moment of inertia are inaccurate by a 10% of their value and the center of mass is located off the axis connecting the two wheels. The misplacement of the center of mass entails neglected quadratic Coriolis terms, $\frac{r_i}{3}\omega_i^2$ on the left-hand side of equation (8a) and $-\frac{r_i m_i}{3I_i}\omega_i v_i$ on the left-hand side of equation (8b) —*cf.* [17].

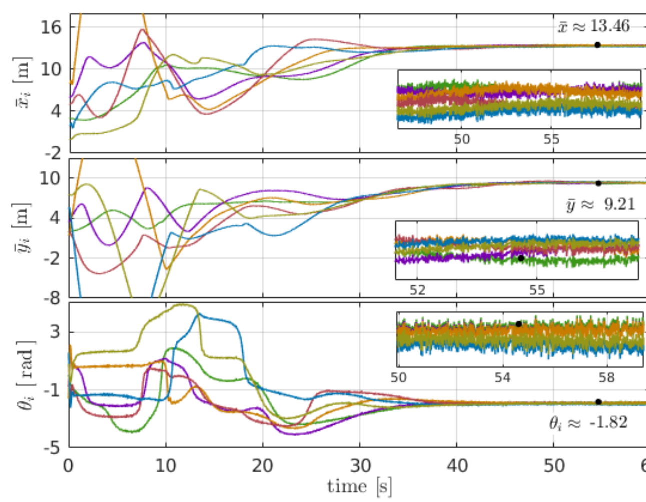

Fig. 3. Cartesian positions and orientations converge to consensual values despite parametric uncertainty and unmodeled dynamics

In Figure 3 are showed the Cartesian positions and orientations converging to, and remaining within a neighborhood of, consensual values. The relatively small steady-state error is also appreciated in Figure 4 —notice the deformed gray hexagon.

The transient behavior, which in this simulation is clearly unfitting from a robotics viewpoint, may be improved by re-tuning the control gains. For the sake of fair comparison, however, the control gains and the initial conditions are deliberately set as in the first scenario (in which the Coriolis terms, the measurement noise, and the parametric uncertainty are neglected). Indeed, the purpose of the second simulation is to illustrate the robustness provided by uniform global *asymptotic* stability, that is, in the sense of total stability. In that regard, it is also worth recalling that only uniform asymptotic stability, and not mere non-uniform convergence (as it is more often encountered in the literature), guarantees total stability.

The paths on the plane are illustrated in Figure 4. Even though the intersections do not necessarily happen simultaneously, it shows the importance of improving our controller with collision-avoidance strategies.

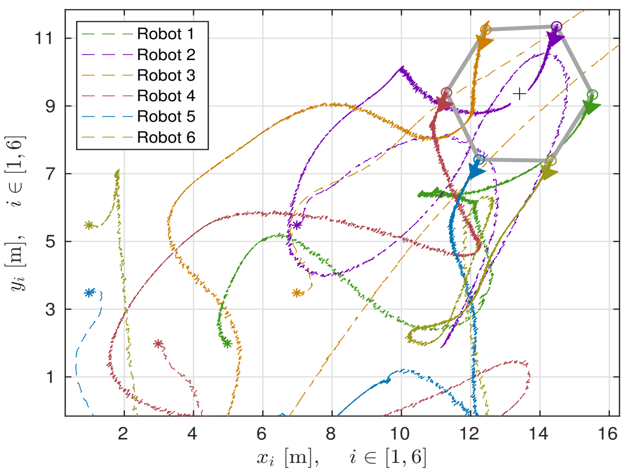

Fig. 4. The robots on the plane converge to a hexagonal formation and consensual orientation despite parametric uncertainty and unmodeled dynamics, albeit an oscillatory transient

VI. Conclusions

The rendezvous control problem for 2nd-order nonholonomic systems, without velocity measurements, may be addressed via smooth control laws of proportional-derivative type, by replacing the unavailable measurements with an *ad hoc* dirty-derivative filter. This result may serve as basis for further work on output-feedback formation control problems involving the relaxation of the assumptions made here on the network's topology and the nature of the interconnections. Also, it appears important to extend this work to models of nonholonomic systems involving Lagrangian dynamics.

References

[1] S. Zhao, "Affine formation maneuver control of multiagent systems," *IEEE Transactions on Automatic Control*, vol. 63, no. 12, pp. 4140–4155, 2018.

[2] D. Dimarogonas and K. Kyriakopoulos, "On the rendezvous problem for multiple nonholonomic agents," *IEEE Transactions Automatic Control*, vol. 52, pp. 916–922, May 2007.

[3] A. Roza, M. Maggiore, and L. Scardovi, "A Smooth Distributed Feedback for Global Rendezvous of Unicycles," *IEEE Transactions Control Netw. Syst.*, vol. 5, pp. 640–652, Mar. 2018.

[4] L. Consolini, F. Morbidi, D. Prattichizzo, and M. Tosques, "On a class of hierarchical formations of unicycles and their internal dynamics," *IEEE Trans. on Automatic Control*, vol. 57, no. 4, pp. 845–859, 2012.

[5] Y. Cheng, R. Jia, H. Du, G. Wen, and W. Zhu, "Robust finite-time consensus formation control for multiple nonholonomic wheeled mobile robots via output feedback," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 6, pp. 2082–2096, 2018.

[6] J. A. Marshall, M. E. Broucke, and B. A. Francis, "Formations of vehicles in cyclic pursuit," *IEEE Transactions on Automatic Control*, vol. 49, no. 11, pp. 1963–1974, 2004.

[7] D. A. Lizárraga., "Obstructions to the existence of universal stabilizers for smooth control systems," *Mathematics of Control, Signals and Systems*, vol. 16, pp. 255–277, 2004.

[8] W. Ren and R. W. Beard, *Distributed consensus in multivehicle cooperative control*. Springer verlag, 2005.

[9] H. A. Poonawala and M. W. Spong, "Preserving strong connectivity in directed proximity graphs," *IEEE Trans. Automatic Control*, vol. 62, no. 9, pp. 4392–4404, 2017.

[10] A. Abdessameud, I. Polushin, and A. Tayebi, "Synchronization of Lagrangian systems with irregular communication delays," *IEEE Trans. on Automatic Control*, vol. 59, no. 1, pp. 187–193, 2014.

[11] Z. Lin, B. Francis, and M. Maggiore, "Necessary and sufficient graphical conditions for formation control of unicycles," *IEEE Trans. on Automatic Control*, vol. 50, no. 1, pp. 121–127, 2005.

[12] A. Dong and J. A. Farrell, "Cooperative control of multiple nonholonomic mobile agents," *IEEE Trans. on Automatic Control*, vol. 53, no. 6, pp. 1434–1447, 2008.

[13] M. Defoort, G. Demesure, Z. Zuo, A. Polyakov, and M. Djemai, "Fixed-time stabilization and consensus of nonholonomic systems," *IET Control Theory & Applications*, vol. 10, no. 8, pp. 2497–2505, 2016.

[14] H. Poonawala, A. C. Satici, H. Eckert, and M. W. Spong, "Collision-free formation control with decentralized connectivity preservation for nonholonomic-wheeled mobile robots," *IEEE Trans. on Control of Network Systems*, vol. 2, pp. 122–130, 2015.

[15] M. I. El-Hawwary and M. Maggiore, "Distributed circular formation stabilization for dynamic unicycles," *IEEE Trans. on Automatic Control*, vol. 58, no. 1, pp. 149–162, 2013.

[16] K. D. Do, Z.-P. Jiang, and J. Pan, "A global output-feedback controller for simultaneous tracking and stabilization of unicycle-type mobile robots," *IEEE Trans. on Robotics and Automation*, vol. 20, no. 3, pp. 589–594, 2004.

[17] S. G. Tzafestas, *Introduction to mobile robot control*. Elsevier, 2013.

[18] J. Huang, C. Wen, W. Wang, and Z.-P. Jiang, "Adaptive output feedback tracking control of a nonholonomic mobile robot," *Automatica*, vol. 50, pp. 821–831, 2014.

[19] R. Dhaouadi and A. A. Hatab, "Dynamic modelling of differential-drive mobile robots using Lagrange and Newton-Euler methodologies: A unified framework," *Advances in Robotics and Automation*, vol. 2, no. 2, 2013.

[20] B. Yousuf, "Robust output-feedback formation control design for nonholonomic mobile robot," *Robotica*, vol. 37, no. 6, pp. 1033–1056, 2019.

[21] K. Wu and C.-Y. Sun, "Output feedback control for nonholonomic systems with non-vanishing disturbances," *International Journal of Control*, vol. 93, no. 10, pp. 2480–2489, 2019.

[22] E. Nuño, A. Loría, A. T. Hernández, M. Maghenem, and E. Panteley, "Distributed consensus-formation of force-controlled nonholonomic robots with time-varying delays," *Automatica*, no. 120, p. 109114, 2020.

[23] B.-M. Kim and P. Tsiotras, "Controllers for unicycle-type wheeled robots: Theoretical results and experimental validation," *IEEE Trans. on Robotics and Automation*, vol. 18, no. 3, pp. 294–307, 2002.

[24] I. G. Malkin, *Theory of Stability of Motion*, vol. AEC-tr-3352 of *Translation Series, Physics and Mathematics*. United States Atomic Energy Commission, Technical Information Service Extension, 1952.

[25] R. Kelly, "A simple set-point robot controller by using only position measurements," in *12th. IFAC World Congress*, vol. 6, (Sydney, Australia), pp. 173–176, 1993.

[26] H. Khalil, *Nonlinear systems*. New York: Macmillan Publishing Co., 2nd ed., 1996.

[27] A. Loría, "From feedback to cascade-interconnected systems: Breaking the loop," in *Proc. 47th. IEEE Conf. Decision Contr.*, (Cancun, Mex.), pp. 4109–4114, 2008.

[28] E. Panteley and A. Loría, "Growth rate conditions for stability of cascaded time-varying systems," *Automatica*, vol. 37, no. 3, pp. 453–460, 2001.

[29] A. Loría, E. Panteley, D. Popović, and A. Teel, "A nested Matrosov theorem and persistency of excitation for uniform convergence in stable non-autonomous systems," *IEEE Trans. on Automat. Contr.*, vol. 50, no. 2, pp. 183–198, 2005.

[30] M. Maghenem, A. Bautista-Castillo, E. Nuño, A. Loría, and E. Panteley, "Consensus of multi-agent systems with nonholonomic restrictions via Lyapunov's direct method," *IEEE Control Systems Letters*, vol. 3, no. 2, pp. 344–349, 2019.

[31] K. Shojaei, A. M. Shahri, A. Tarakameh, and B. Tabibian, "Adaptive trajectory tracking control of a differential drive wheeled mobile robot," *Robotica*, vol. 29, no. 3, pp. 391–402, 2011.

[32] R. C. Kavanagh, "Shaft encoder characterization via theoretical model of differentiator with both differential and integral nonlinearities," *IEEE Trans. on Instrumentation and Measurement*, vol. 49, pp. 795–801, Aug 2000.