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A Third order Semi-Implicit Homogeneous differentiator: Experimental Results

Loïc MICHEL, Malek GHANES, Yannick Aoustin and Jean-Pierre BARBOT

Abstract—It is well-known that implicit-based and projector-based differentiator schemes offer better performances, like reducing high frequency oscillations, compared to the corresponding explicit schemes. To keep the advantages of such implicit Euler approximations, when this approximation is applied in case of homogeneous differentiators, a semi-implicit Euler approximation has been recently proposed for second-order systems. In this paper, a generalization to third order semi-implicit differentiation is proposed taking into account the attenuation of the noise considering advanced iterative projectors definition. Validation on experimental data is conducted to highlight the well-founded of the proposed differentiation strategy.

Index Terms—Discretization, Homogeneous differentiator, Noise attenuation, Projectors, Experimental Data.

I. INTRODUCTION

Real-time discrete signal differentiation can be performed using sliding-modes techniques [1] and the recent implicit discretization technique, introduced in [2] would overcome some limitations of the classical sliding-mode such as a cancellation of the chattering effect as well as robustness of the estimation under lower sampling frequencies. The principle of the implicit-based differentiation is to replace the classical sign function by an *implicit* projector and some recent successful experiments through implicit based sliding mode control algorithms have been obtained (see e.g. [3], [4], [5], [6], [7], [8], [9], [10]). In the framework of the discrete homogeneous differentiation, the proposed differentiator combines explicit terms with implicit one including two *projectors* in order to reduce the effects of chattering [11]. Recent contributions include a derivation to second order system [12], and an adaptive version regarding the rejection of the measurement noise, that has been proposed in [13]. A comparison of performances between classical and advanced differentiation structures has been performed in [14]–[16] and highlights the fact that implicit based methods are more efficient and robust to the noise and chattering than the corresponding explicit one.

Recently, the authors presented the derivation of a cascaded version of second order semi-implicit homogeneous differentiator [12], [16], considering thus two stages of

differentiators for which the homogeneous exponents can be adjusted separately, but it includes delays (toward the propagation from the first stage to the second stage) and requires more states than the system to differentiate (for example, four states are required to estimate the acceleration from the measured position of a mechanical system). To better exploit the advantages of the implicit projectors, in this work, we propose an extension to the third order semi-implicit differentiator, keeping the homogeneous properties of the differentiation and for which, the tuning of the parameters is simpler than the cascade since a single homogeneous exponent is considered. Moreover, it has the same order than the dimension of the system to differentiate. Note that additional Taylor expansion corrective terms, inspired from [17], [18], can be included in the differentiator to improve the precision, as well as an adaptive tuning procedure for the homogeneous exponent in order to reject the noise [13].

To highlight the merit of the proposed strategy, experiments are conducted within an electrical RLC circuit where the output voltage across the capacitor can be differentiated twice and compared easily with the corresponding measurements but other applications can be considered, for example in mechanics.

This paper is outlined as follows. Section II describes the background of the homogeneous differentiation approach. Section III presents the main contribution and describes the third order semi-implicit homogeneous approach including iterative projectors. The differentiation approach is applied to experimental data in Section IV in which the benefits are discussed. Section V concludes the paper and suggests some future works.

II. BACKGROUND ON HOMOGENEOUS DIFFERENTIATION

Considering a signal to differentiate y , at least of class \mathcal{C}^2 with bounded third derivative. From [1], [19], [20], a continuous-time homogeneous differentiator of order three can be designed as follows:

$$\begin{cases} \dot{z}_1 = z_2 + \lambda_1 \mu [e_1]^\alpha \\ \dot{z}_2 = z_3 + \lambda_2 \mu^2 [e_1]^{2\alpha-1} \\ \dot{z}_3 = \lambda_3 \mu^3 [e_1]^{3\alpha-2} \\ \hat{y} = z_1 \end{cases} \quad (1)$$

where $\alpha \in [\frac{2}{3}, 1]$ has to be fixed [21]; $e_1 = y - z_1$ (i.e. $e_1 = 0$ defines the sliding surface), including the notation $[\bullet]^\alpha = |\bullet|^\alpha \text{sgn}(\bullet)$.

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The gain $\mu > 0$ is chosen greater than the perturbation on the 3^{rd} output derivative and the gains $(\lambda_1, \lambda_2, \lambda_3) > 0$ are tuned such that e_1 , $e_2 = \dot{y} - z_2$, and $e_3 = \ddot{y} - z_3$ converge towards zero. Then, the system (1) allows to get an estimation of y , \dot{y} and \ddot{y} .

III. MAIN RESULTS

Let us consider now the following restriction that is to define $y(t) \in \mathcal{C}^\infty$ i.e. an analytic signal; the exact discretization of y is:

$$y(t+h) = y(t) + \sum_{j=1}^{\infty} y^{(j)}(t) \frac{h^j}{j!} \quad (2)$$

where $y^{(j)}(t)$ denotes the j^{th} time derivative of $y(t)$.

Assumption 1: $y^{(1)}(t)$, $y^{(2)}(t)$ and $y^{(3)}(t)$ are bounded for all $t > 0$ and there exists a known $P > 0$ such that $|y^{(j)}(t)| < P^{\frac{j}{3}} \quad \forall t > 0$ and $j \in \{1, 2, 3\}$.

Note that the continuous time homogeneous differentiator (1) is not analytic, that is why only Euler discretization of (1) is used, the explicit one is (see [17], [18] for an improved and efficient version of (3)):

$$\begin{cases} z_1^+ = z_1 + h(z_2 + \lambda_1 [e_1]^\alpha) \\ z_2^+ = z_2 + h(z_3 + \lambda_2 [e_1]^{2\alpha-1}) \\ z_3^+ = z_3 + h(\lambda_3 [e_1]^{3\alpha-2}) \end{cases} \quad (3)$$

where $e_1 = y - z_1$ and for ease of reading, the notations " $\bullet = \bullet(kh)$ " and " $\bullet^+ = \bullet((k+1)h)$ " are kept in the discrete-time framework, where $h > 0$ is the sampling period. This solution is not attractive since it suffers from chattering phenomena.

The implicit Euler discretization [2] of (1) is not directly applicable, this is due to the homogeneous term $|e_1|^\alpha$ and that is why semi-implicit Euler discretization methods are used (see [12] for some efficient methods). The first proposed semi-implicit Euler discretization of (1) given hereafter is mainly based on the projector [2] and this is an extension to order three of the second order differentiator proposed in [12]:

$$\begin{cases} z_1^+ = z_1 + h(z_2^+ + \lambda_1 \mu |e_1|^\alpha \mathcal{N}_1) \\ z_2^+ = z_2 + E_1^+ h(z_3^+ + \lambda_2 \mu^2 |e_1|^{2\alpha-1} \mathcal{N}_2) \\ z_3^+ = z_3 + E_1^+ E_2^+ h(\lambda_3 \mu^3 |e_1|^{3\alpha-2} \mathcal{N}_3) \end{cases} \quad (4)$$

where the associated projectors \mathcal{N}_q with $q \in \{1, 2, 3\}$ and $\alpha \in [\frac{2}{3}, 1[$ are defined by (see [22] for $q \in \{1, 2\}$):

$$\mathcal{N}_q := \begin{cases} \text{if } |e_1|^{q(1-\alpha)} < \lambda_q (\mu h)^q \rightarrow \mathcal{N}_q = \frac{[e_1]^{q(1-\alpha)}}{\lambda_q (\mu h)^q} \\ \text{if } |e_1|^{q(1-\alpha)} \geq \lambda_q (\mu h)^q \rightarrow \mathcal{N}_q = \text{sgn}(e_1) \end{cases} \quad (5)$$

Moreover, E_i^+ is defined for $i = \{1, 2\}$ such as:

$$E_i^+ := \begin{cases} \text{if } |e_1|^{i(1-\alpha)} < \lambda_i (\mu h)^i \rightarrow E_i^+ = 1 \\ \text{if } |e_1|^{i(1-\alpha)} \geq \lambda_i (\mu h)^i \rightarrow E_i^+ = 0 \end{cases} \quad (6)$$

Each line of the differentiator must be activated once the convergence of the corresponding previous line has converged and therefore each variable E_i^+ is iteratively activated depending on the $(i)^{th}$ projector convergence. In particular, the activation of the third line in (4) depends simultaneously on the two previous projectors E_1^+ and E_2^+ in order to avoid unexpected perturbations that may destabilize the convergence. This follows the scheme of a recursive algorithm.

Comparing the differentiator (4), (including (5) and (6)) to the following implicit Euler discretization of order three:

$$\begin{aligned} x_1^+ &= x_1 + h x_2^+ \\ x_2^+ &= x_2 + h x_3^+ \\ x_3^+ &= x_3 + h y^{(3)} \end{aligned} \quad (7)$$

where $x_1 = y$, $x_2 = \dot{y}$, and $x_3 = \ddot{y}$, it is possible to set the following theorem:

Theorem 1: Considering the implicit Euler discretization of third order integrator (7) and the differentiator (4), under the assumption 1, there exist μ and λ_q ($q \in \{1, 2, 3\}$) such as the differentiator errors $e_1 = x_1 - z_1$, $e_2 = x_2 - z_2$ and $e_3 = x_3 - z_3$ are respectively smaller or equal to $h^3 P$, $h^2 P$ and hP .

A sketch of proof is given hereafter.

Considering the error dynamics:

$$\begin{aligned} e_1^+ &= e_1 + h(e_2^+ - \lambda_1 \mu |e_1|^\alpha \mathcal{N}_1) \\ e_2^+ &= e_2 + E_1^+ h(e_3^+ - \lambda_2 \mu^2 |e_1|^{2\alpha-1} \mathcal{N}_2) \\ e_3^+ &= e_3 + E_1^+ E_2^+ h(y^{(3)+} - \lambda_3 \mu^3 |e_1|^{3\alpha-2} \mathcal{N}_3) \end{aligned} \quad (8)$$

Step 1 Under the assumption 1, it is possible to choose $\mu \gg P^{\frac{1}{3}}$, and after some iterations, the condition $|e_1|^{1-\alpha} \leq h \lambda_1 \mu$ implies $E_1^+ = 1$ and $e_1^+ = h e_2^+$, because $h \lambda_1 \mu \mathcal{N}_1$ is exactly equal to e_1 , see [12] for more details.

Step 2 When $E_1 = 1$ then $e_1^+ = h e_2^+$, and as $\mu^2 \gg P^{\frac{2}{3}}$ again under assumption 1, after some iterations, the condition $|e_1|^{2(1-\alpha)} \leq \lambda_2 (\mu h)^2$ is verified, which gives $E_2 = 1$ and $|e_2|^{2(1-\alpha)} \leq \lambda_2 \mu^2 h^{2\alpha}$, this implies that e_2 is equal to $\lambda_2 \mu^2 |e_1|^{2(\alpha-1)} \mathcal{N}_2$ and so $e_2^+ = h e_3^+$.

Step 3 Finally, for $E_1 = 1$ and $E_2 = 1$, as $\mu^3 \gg P$ and under assumption 1, after some iterations $|e_1|^{3(1-\alpha)} \leq \lambda_3 \mu^3 h^3$ which gives $|e_3|^{3(1-\alpha)} \leq \lambda_3 \mu^3 h^{6\alpha-3}$ and thus $\lambda_3 \mu^3 |e_1|^{3\alpha-2} \mathcal{N}_3$ is exactly equal to e_3 then $e_3^+ = h y^{(3)+}$ and as $|y^{(3)}| < P$. It is possible to conclude that $|e_1| < h^3 P$, $|e_2| < h^2 P$ and $|e_3| < hP$. ■

Remark 1: The previous theorem gives the error between the state of the integrator (7) and the state of the differentiator (4). Nevertheless, the integrator (4) is only an approximation in $\mathcal{O}(h^2)$. It is why hereafter and based on [17], [18], some corrective terms are added to take into account that the signal to differentiate is analytical and consequently, an exact explicit solution¹ of 3rd order integrator can be used.

$$\begin{aligned} x_1^+ &= x_1 + h x_2 + \frac{h^2}{2!} x_3 + \frac{h^3}{3!} \ddot{y} \\ x_2^+ &= x_2 + h x_3 + \frac{h^2}{2!} \ddot{y} \\ x_3^+ &= x_3 + h \ddot{y} \end{aligned} \quad (9)$$

Then, the exact implicit solution of (9) of 3rd order integrator, assuming that $\ddot{y} = \ddot{y}^+$, is obtained as follow considering e.g. in

$$x_2^+ = x_2 + h x_3 + \frac{h^2}{2!} \ddot{y}$$

substituting x_3 by x_3^+ , and $\ddot{y}^+ = \ddot{y}$ this gives:

$$x_2^+ = x_2 + h (x_3 + h \ddot{y}^+) - \frac{h^2}{2!} \ddot{y}^+$$

Finally², recursively, we obtain:

$$\begin{aligned} x_1^+ &= x_1 + h x_2^+ - \frac{h^2}{2!} x_3^+ + 7 \frac{h^3}{3!} \ddot{y}^+ \\ x_2^+ &= x_2 + h x_3^+ - \frac{h^2}{2!} \ddot{y}^+ \\ x_3^+ &= x_3 + h \ddot{y}^+ \end{aligned} \quad (10)$$

From the exact implicit solution (10), the corrective term $-E_2^+ h \frac{1}{2} z_3^+$ has been added in (4); and the semi-implicit differentiator of order three including the corrective term (in bold) reads:

$$\begin{cases} z_1^+ = z_1 + h \left(z_2^+ - \mathbf{E}_2^+ h \frac{1}{2} z_3^+ + \lambda_1 \mu |e_1|^\alpha \mathcal{N}_1 \right) \\ z_2^+ = z_2 + E_1^+ h \left(z_3^+ + \lambda_2 \mu^2 |e_1|^{2\alpha-1} \mathcal{N}_2 \right) \\ z_3^+ = z_3 + E_1^+ E_2^+ h \left(\lambda_3 \mu^3 |e_1|^{3\alpha-2} \mathcal{N}_3 \right) \end{cases} \quad (11)$$

Remark that the correcting term $-E_2^+ h \frac{1}{2} z_3^+$ is different from the one computed in [18]. First, this is due to the fact that in the studied case, the implicit discretization is considered instead of the explicit one.

Secondly, the term E_2^+ has been added in order to inject the corrective term only when z_3 is closed to its switching surface. As usual, all terms in \ddot{y} are not taken into account in the differentiator (11) because they represent unknown perturbations.

¹under the assumption that \ddot{y} is constant during the sampling period.

²The term in h^3 in x_1^+ is obtained as follow. In (9), all the coefficients of corrective terms in h^2 are equal to $-\frac{h^2}{2!}$ and the coefficient X of the corrective term in h^3 is obtained from $-\frac{h^2}{2!} - \frac{h^3}{2!} + X = \frac{h^3}{3!}$.

IV. EXPERIMENTAL APPLICATION

A. Experimental setup

Previous work of the authors [12], [16] considered some homogeneous differentiation algorithms that have been compared in the framework of velocity/acceleration estimation regarding the position of a pneumatic actuator, and the estimation of the current in the case of an electronics RC low pass circuit.

In this study, the objective is to differentiate the output of a RLC series circuit (see Fig. 1 with $R = 890\Omega$, $L = 1\text{H}$ and $C = 1\mu\text{F}$); the input signal being a sine function of frequency $f = 160\text{ Hz}$, set at the resonance of the circuit $\omega_0 = 1000\text{ rad/s}$.

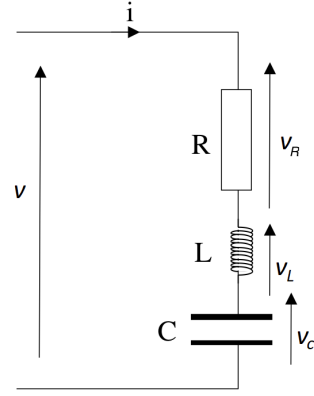


Fig. 1. RLC circuit.

The measured signal to differentiate (Fig. 2 without normalization), has been obtained by scoping the voltage v_C across the capacitor C .

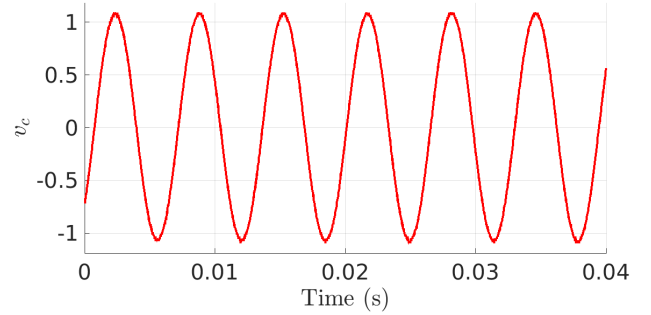


Fig. 2. Measured v_C versus time (s).

Firstly, the voltage $v_R = Ri$ is measured across the terminals of the resistor R , and constitutes the reference differentiated signal \dot{v}_C since

$$\dot{v}_C = \frac{1}{RC} v_R, \quad (12)$$

then, the voltage $v_L = L di/dt$ is measured across the terminals of the inductance L and constitutes the reference twice differentiated signal \ddot{v}_C since

$$\ddot{v}_C = \frac{1}{LC} v_L. \quad (13)$$

The proposed differentiator allows avoiding the use of a current sensor given that $i = C\dot{v}_C$ and an extra voltage sensor given that $v_L = L di/dt$.

B. Attenuation noise projectors

In this experiment, the measured signal y_m is noisy i.e. $y_m = y + \eta$ where η is a measurement noise and then, the output variable e_1 becomes $e_{1m} = y_m - z_1$. Consequently, a modified projector including a new parameter θ is introduced in order to mitigate the influence of noise on the sliding surface. Roughly speaking, on the sliding surface, for θ equals to zero, finite-time convergence is ensured and when θ is different from zero, the convergence is only asymptotic but the differentiator is less sensitive to noise. This is due to the fact that the influence of noise on the sliding surface is multiplied by $(1 - \theta)$; thus, this improves the behavior of the differentiator in regard to the measurement noise. To take into account the measurement noise, instead of being on the "sliding" domain (i.e. $|e_{1m}|^{(1-\alpha)} < \lambda_1 \mu h$), $e_{1m}^+ = y_m^+ - z_1^+$ converges in one step to $h e_2^+$ for which the convergence is now only exponential and realized as follows:

$$\begin{cases} z_1^+ = z_1 + h(z_2^+ + \lambda_1 \mu |e_{1m}|^\alpha \mathcal{N}_{\theta_1}) \\ z_2^+ = z_2 + E_{\theta_1}^+ h(z_3^+ + \lambda_2 \mu^2 |e_{1m}|^{2\alpha-1} \mathcal{N}_{\theta_2}) \\ z_3^+ = z_3 + E_{\theta_1}^+ E_{\theta_2}^+ h(\lambda_3 \mu^3 |e_{1m}|^{3\alpha-2} \mathcal{N}_{\theta_3}) \end{cases} \quad (14)$$

with

$$\mathcal{N}_{\theta_q} := \begin{cases} (1 - \theta_q) |e_{1m}|^{q(1-\alpha)} < \lambda_q (\mu h)^q & \rightarrow \mathcal{N}_{\theta_q} = \frac{(1-\theta_q) |e_{1m}|^{q(1-\alpha)}}{\lambda_q (\mu h)^q} \\ (1 - \theta_q) |e_{1m}|^{q(1-\alpha)} \geq \lambda_q (\mu h)^q & \rightarrow \mathcal{N}_{\theta_q} = \text{sgn}(e_{1m}) \end{cases} \quad (15)$$

where $q = \{1, 2, 3\}$ and E_{θ_i} is defined only for $i = \{1, 2\}$

$$E_{\theta_i} := \begin{cases} (1 - \theta_i) |e_{1m}|^{i(1-\alpha)} < \lambda_i (\mu h)^i & \rightarrow E_{\theta_i} = 1 \\ (1 - \theta_i) |e_{1m}|^{i(1-\alpha)} \geq \lambda_i (\mu h)^i & \rightarrow E_{\theta_i} = 0 \end{cases} \quad (16)$$

C. Results

To operate the differentiator, the measured sampling period $h_m = 4 \mu s$ has been increased in the sequel to the sampling period $h_d = 10^3 h_m = 4 ms$ to avoid dealing with huge numbers due to the second derivative of the sine function, that is of the order $(2\pi f)^2 \approx 10^6$. This "normalization" allows an easier tuning of the parameters. For the seek of simplicity, after the normalization, the time derivative of v_C becomes v'_C and is called "velocity". In the same way, the second time derivative of v_C becomes v''_C and is noted "acceleration".

The μ parameter is chosen according to the magnitude of the third derivative of the sine function such as $\mu^3 \gg (2\pi f)^3$. The λ_i parameters are chosen such as the linear part is stable and typically $\lambda_1 = \lambda_2 = 3$ and $\lambda_3 = 1$. The

homogeneous exponent α is chosen between the coefficient of Levant's differentiator and the linear solution i.e. $\alpha = 0.81$. The θ_i parameters are chosen to allow a good filtering of the noise i.e. $0.5 < \theta_i < 1$ and can be refined thanks to an optimization procedure to improve the SSE performance index³ [23]: the parameters θ_i are set such as: $\theta_1 = 0.8980$, $\theta_2 = 0.9979$, and $\theta_3 = 0.9999$.

Without the normalization, the μ parameter is of order 10^4 ; the normalization allows manipulating much smaller numbers, thus avoiding truncation and numerical errors. The normalization would allow to systematize the tuning of the differentiator depending on the considered application.

Figures (3) and (4) show respectively the estimated first derivative and the second derivative of the capacitor voltage according to their measured reference regarding the explicit differentiator and Figures (5) and (6) show respectively the estimated first derivative and the second derivative of the capacitor voltage according to their measured reference regarding the semi-implicit differentiator.

The Sum of Square Error (SSE) index of "velocity" and "acceleration" are displayed in Fig. (7) for both explicit and semi-implicit methods; the smallest the SSE index, the better the result.

Even if the "velocity" estimation seems to be similar for explicit (Fig. (3)) and semi-implicit (Fig. (5)) solutions, the SSE index with respect to "velocity" (Fig. (7)) highlights the fact that the semi-implicit solution has better performances. In the same way, with respect to the "acceleration", the semi-implicit solution (Fig. (6)) has better rejection to the noise than the explicit one (Fig. (4)). This is confirmed by the SSE index in Fig. (7). Moreover, the semi-implicit solution seems to have less phase delay.

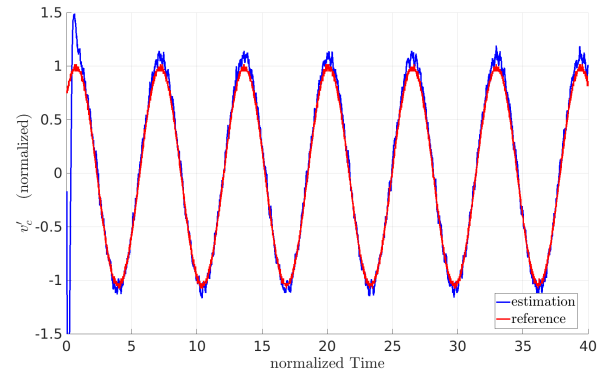


Fig. 3. Measured and estimated (normalized) v'_C for the explicit differentiator.

V. CONCLUSIONS AND FUTURE WORKS

This paper deals with an homogeneous differentiator of high order in the implicit framework proposed in [2]. The proposed scheme considers semi-implicit homogeneous differentiators instead of classical sliding mode differentiators

³The SSE is given by $SSE(\bullet) = \frac{1}{n} \sum_{i=1}^n \bullet_i^2$ with n the data size.

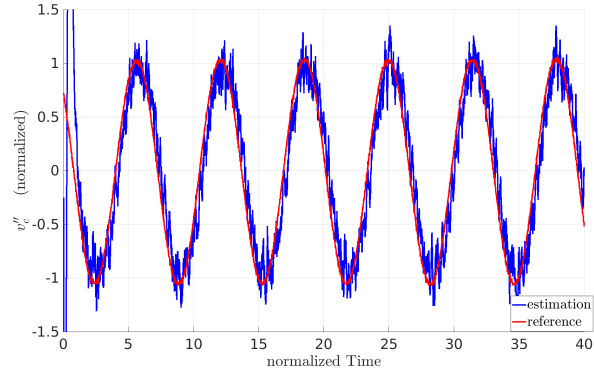


Fig. 4. Measured and estimated (normalized) v'_C for the explicit differentiator.

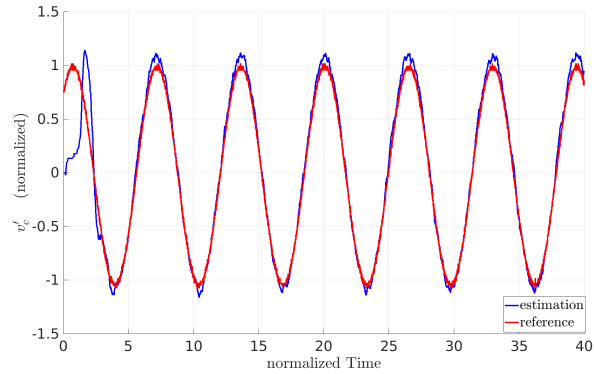


Fig. 5. Measured and estimated (normalized) v'_C for the semi-implicit differentiator.

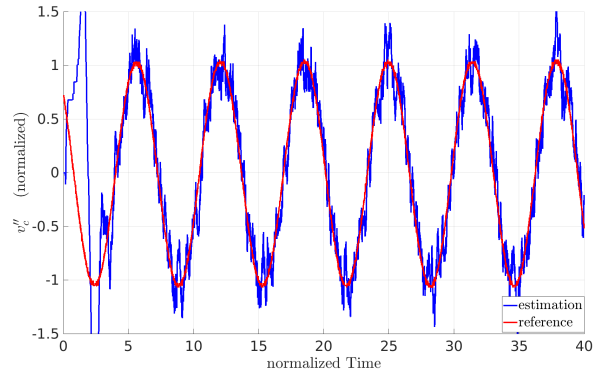


Fig. 6. Measured and estimated (normalized) v''_C for the semi-implicit differentiator.

including three projectors. These theoretical projectors, defined in the main part, have been modified in order to reduce chattering and measurement noise effects. An experimental validation has been conducted on an electronics *RLC* filter for which promising performances of the differentiators have been obtained in term of estimation error and noise rejection. Future investigations include the generalization of the proposed approach to variable exponent differentiators [1]. In addition, more investigations will be conducted with respect

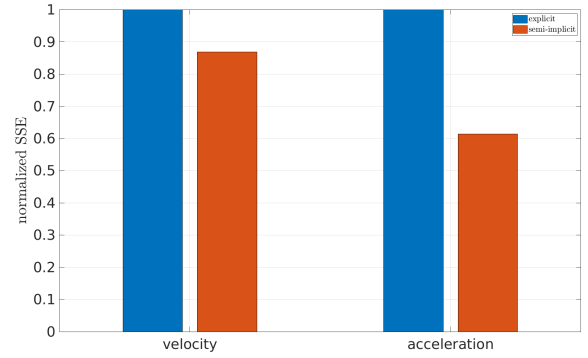


Fig. 7. Normalized SSE.

to recent implicit differentiation techniques. Moreover, it will be very interesting to adapt the Taylor expansion corrective term proposed in [17] for the explicit Euler differentiator. In addition, one can also consider not only a regular sampling but also the self and event triggering case [18], [24], [25].

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