



# **Preservice primary teachers' curricular reasoning when anticipating primary students' answers to geometrical figure classification tasks**

Melania Bernabeu, Mar Moreno, Salvador Llinares

## **► To cite this version:**

Melania Bernabeu, Mar Moreno, Salvador Llinares. Preservice primary teachers' curricular reasoning when anticipating primary students' answers to geometrical figure classification tasks. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. <hal-03751484>

**HAL Id: hal-03751484**

**<https://hal.science/hal-03751484v1>**

Submitted on 14 Aug 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

# Preservice primary teachers' curricular reasoning when anticipating primary students' answers to geometrical figure classification tasks

Melania Bernabeu<sup>1</sup>, Mar Moreno<sup>2</sup> and Salvador Llinares<sup>3</sup>

<sup>1</sup>University of Alicante, Spain; [melania.bernabeu@ua.es](mailto:melania.bernabeu@ua.es)

<sup>2</sup>University of Alicante, Spain; [mmoreno@ua.es](mailto:mmoreno@ua.es)

<sup>3</sup>University of Alicante, Spain; [sllinares@ua.es](mailto:sllinares@ua.es)

*This study aims to characterize the preservice primary teachers' curriculum reasoning when anticipating primary students' answers to figure classification tasks. Anticipating answers to figure classification tasks involves considering the relationship between the definition and classification and the characteristics of geometrical thinking progression. We analyzed the definitions of geometric figures posed by twenty-eight preservice primary teachers to quadrilateral classification tasks. The results show three preservice primary teachers' profiles taking into account the specialization/generalization of the quadrilateral definitions provided and the transitivity of inclusion relations. These results indicate features of curricular reasoning in anticipating tasks as part of preservice primary teachers' curricular noticing.*

*Keywords: curricular noticing, anticipating students' answers, geometric reasoning, hierarchical classification, definition of geometric figures.*

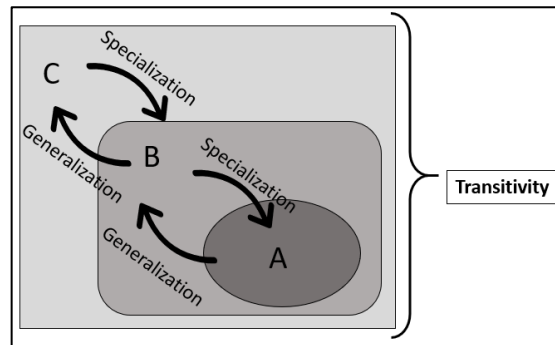
## Introduction

Previous researches (Jones & Tzekaki, 2016; Sinclair et al., 2016) argue that it is important for teachers to understand the elements that condition understanding of inclusive relationships to plan geometry instruction. This generates the need to know how preservice primary teachers (PPT) reason when anticipating student answers as an aspect of *curricular noticing* (Dietiker et al., 2018). This construct describes a set of professional practices that allow teachers or preservice teachers to recognize, interpret, and generate learning opportunities from teaching-learning situations when they interact with curricular materials. In these situations, they had to carry out the *curricular reasoning* (Breyfogle et al., 2010), which has been defined as the thought processes in which teachers or preservice teachers engage when working with curricular documents or materials to plan, implement and reflect on instruction.

## Theoretical Background and Framework

A key aspect in the progression of geometric thinking, whatever the educational level, is understanding a geometric figure as an example of a class of figures (hierarchical classification). The understanding of hierarchical classifications is linked to the definition of the figure. These hierarchical classifications imply that, for example, a figure (square) in a class A (rectangle) has all the properties of the class B (parallelograms) that includes it. There are two processes for defining these figures. On the one hand, the *specialization* process, in which to get an example of A, attributes

have been added to the definition of B (more general class); and, on the other hand, the *generalization* process, by suppressing attributes from the definition of an example of A to create a more general class, like B. The process of including an example within a more general class, and so on (A is an example of B, B is an example of C, so A is an example of C) is known as *transitivity* (de Villiers, 1994) (Figure 1).



**Figure 1: The specialization and generalization of the definitions and the transitivity of the inclusion relations**

However, understanding the *specialization* and the *transitivity* in the hierarchical relationships between figures is difficult (Fujita, 2012; Fujita & Jones, 2007). Previous research (Brunheira & da Ponte, 2019) showed that prospective teachers, after a teacher education experiment that includes this kind of classifications, had a limited figural concept, and therefore, difficulties in the hierarchical classifications using the transitivity. These difficulties may be a problem for prospective teacher when they will have to plan the teaching/instruction.

For planning the teaching of hierarchical relationships of geometric figures in primary education, PPT should anticipate the thinking processes that the tasks may promote in students considering the cognitive abilities of the students (both mathematical and didactic contents). For example, PPT should consider, as key aspects in learning, the process of *specialization* of the definition (adding properties to a general class) as well as the *transitivity* of the inclusion relations ( $A \rightarrow B$  and  $B \rightarrow C$ , then  $A \rightarrow C$ ) (de Villiers, 1994). The way in which PPT use these two features of inclusion relations between geometric figures could provide information on how they reason about the hierarchical classification tasks. In this sense, anticipating students' answers to hierarchical classification tasks in a lesson is a skill linked to *curricular reasoning* (Breyfogle et al., 2010) as an aspect of *curricular noticing* (Dietiker et al., 2018). Thus, this study aims to find out:

- How do PPT reason when anticipating correct answers in hierarchical classification tasks as an aspect of curricular noticing?

## Method

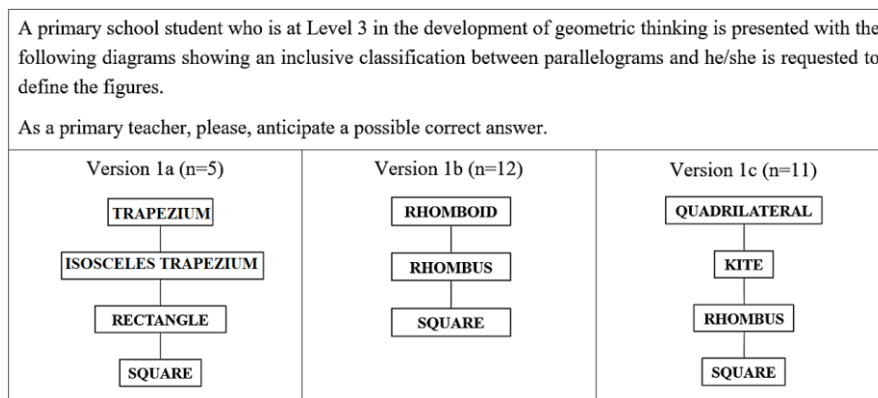
This study involved 28 PPT who participated in a learning environment (4 sessions of 2 hours) aimed at developing *curricular noticing*. The content of the learning environment includes the characteristics of geometric thinking development in primary education and the types of tasks for enhancing the students' geometric thinking along the lesson plan. The PPT, during the learning environment, analysed primary school students' answers, anticipated answers from primary school

students to the tasks of classifying geometric figures and shapes, and analysed teaching activities to support understanding of inclusion relationships (Lehrer et al., 2014). These actions related with the *curricular noticing* are what the PPT would put into practice during their teaching practices at school and in their future as teachers. At the end of the learning environment, PPT solved a task on anticipating primary school students' answers to an activity in inclusive relations between geometric figures (Usiskin & Griffin, 2008). In this study, we analysed the PPT's answers given to this last task through a qualitative analysis (Figure 1).

## The task

The task is contextualised in the planning of a lesson with the learning objective: *inclusive relations between geometric objects*, for pupils aged 10-12 years. The focus was the relation between definitions of geometric objects and classification. Three versions of the same task were proposed in the domain of quadrilaterals (Figure 2). The task highlights the *curricular reasoning* processes (Breyfogle et al., 2010) of PPT by anticipating possible student answers considering mathematical knowledge and the development of geometric thinking in primary education (Sánchez-Matamoros et al., 2019). PPT should take into account the process of specialization (add conditions to generate subclasses) and the transitivity of the inclusion relationships of quadrilaterals. For example, in version 1b task, the rhombus must have *four equal sides* to be a particular example of the rhomboid class (*quadrilateral with two pairs of opposite sides parallel*) (Table 1).

Some of these quadrilaterals often appear in the primary school curriculum with partitive (non-inclusive) definitions, such as defining trapezium as a quadrilateral with only two parallel sides. Thus, this task has a high cognitive demand for PPT, as it requires modifying and overcoming the definitions associated with prototypical images and partitive definitions.



**Figure 2: Task of anticipating student answer as part of teaching planning**

Version 1a	Version 1b	Version 1c
Trapezium: quadrilateral with two parallel sides.	Rhomboid: quadrilateral with two pairs of opposite sides parallel	Quadrilateral: polygon with four sides

Isosceles trapezium: trapezium <u>with congruent diagonals.</u>	Rhombus: rhomboid <u>with congruent sides.</u>	Kite: quadrilateral <u>with perpendicular diagonals and two pairs of adjacent congruent sides.</u>
Rectangle: isosceles trapezium <u>with congruent angles.</u>	Square: rhombus <u>with congruent angles.</u>	Rhombus: kite <u>with congruent sides.</u>
Square: rectangle <u>with congruent sides.</u>		Square: rhombus <u>with equal angles.</u>

**Table 1: Possible definitions and inclusive relationships between quadrilaterals**

## Analysis

For the analysis of the task answers, we considered three criteria:

- The economy of the definition, i.e., if the definition included the minimum and sufficient conditions or redundant conditions. In addition, we considered if the definitions derived from the perceptual references of prototypical examples (*prototypical judgement, type 1*), from the attributes of prototypical examples (*prototypical judgement, type 2*) or using attributes relevant to the concept (*analytical features, type 3 judgement*) (Hershkowitz, 1990).
- The *specialization* of the definitions, i.e., to what extent it was considered that adding attributes to the general class generate a subclass. For example, in task 1c, to define a rhombus, it is sufficient to add having equal sides to the definition of a kite (see Table 1).
- The *transitivity* of the definitions ( $A \rightarrow B$  and  $B \rightarrow C$  then  $A \rightarrow C$ ). For example, that the definitions given allow us to see that, if a square is a rhombus, and a rhombus is a rhomboid, then the square is a rhomboid, but not inversely (there are rhomboids that are not squares) (asymmetric relations between the inclusion relations between geometric figures).

Regarding of procedure followed, we analysed the answers following the above criteria (Figure 3) and then compared the answers with each other, identifying similarities and differences, generating patterns of answers. These patterns of answers were considered as profiles PPT's anticipatory competence of anticipating primary school students' answers to a task on figure classification as a part of the acquisition of the curricular noticing. One student out of 28 provided an answer that was inconsistent with the required task, so it was not considered in characterising the profiles.

Preservice teacher 17 (PPT17)'s answer	Pre-analytical comments
<i>Rhomboid: Parallelogram with interior angles less than 180°, and with 4 sides parallel two by two.</i>	<ul style="list-style-type: none"> <li>- <i>Uneconomical</i> definitions (non-minimal) (e.g., if you say the rhomboid is a parallelogram, you do not need to say 4 parallel sides two by two)</li> <li>- There is <i>transitivity</i> (the square is a rhombus; the rhombus is a rhomboid).</li> <li>- Use <i>specialization</i> by adding attributes to the more general class it belongs to.</li> </ul>
<i>Rhombus: Rhomboid with 4 equal sides and perpendicular diagonals</i>	
<i>Square: Rhombus with four right angles</i>	

**Figure 3: Example of analysis of the PPT17's answer**

## Results

The analysis has generated three PPT's profiles.

### Profile 1. No use of hierarchical definitions nor of the transitivity process (n=7)

Preservice primary teachers in this profile do not recognise the process of generating subclasses as they do not take into account any case the specialization of definitions, and they fail to consider the transitivity between inclusion relations. In this profile, PPT seem to base their definitions on prototypical images, generating *prototypical judgements of type 1*. For example, the preservice teacher PPT10 in task 1b, Rhomboid→Rhombus→Square, indicated,

- *Rhomboid: A geometric figure with four sides that do not form right angles, of which the opposite sides are equal, and the adjacent sides are unequal.*
- *Rhombus: Geometric figure with four equal sides that do not form right angles.*
- *Square: Figure having four equal sides that form four right angles.*

This PPT use partitive definitions and removes the possibility of generating subclasses from the more general classes by indicating that: the rhomboid has the *adjacent sides unequal*, excluding the rhombus as a subclass of the rhomboid; the rhombus *has no right angles*, eliminating the possibility of squares as a subclass of the rhombus; and the rhomboid *has no right angles*, excluding squares as a subclass and thus preventing *transitivity* between inclusion relations.

### Profile 2. Partial use of hierarchical definitions, but not transitivity process (n=12)

PPT in this profile show a partial understanding of the specialization process. They define the figures considering only some of the inclusion conditions indicated in the task. Furthermore, they show a lack of understanding of the transitivity of inclusion relations by defining some figures using attributes of the prototypical figures, but not all the necessary attributes or using some definitions incorrectly. In this profile, PPT seem to use attributes of the prototypical examples as reference, generating *prototypical judgements of type 2*, as they use attributes of the identified figures. For example, the preservice teacher PPT2 in task 1a, Trapezium→Isosceles Trapezium→Rectangle→Square, indicated

*As the student is at level 3, he can analyse the properties of geometric figures giving the sufficient and necessary indications, relating the properties to each other:*

- *The trapezium is a quadrilateral with only two parallel sides.*
- *The Isosceles trapezium is a quadrilateral with two equal angles two by two.*
- *The rectangle is a quadrilateral with all right angles.*
- *The square is a quadrilateral with all sides equal and all right angles.*

By defining trapezium as a quadrilateral with *only two parallel sides*, it excludes the possibility of considering the rest of the parallelograms (rectangle and square) as subclasses, and therefore does not take transitivity into account. By defining isosceles trapezium as a *quadrilateral with [two] equal angles two by two* it seems to be using some properties of the prototypical figure, but not all of them.

Finally, the definitions and the relation of inclusion between squares and rectangles are adequately indicated, reflecting judgements supported by the attributes of the prototypical figures and evidencing the characteristic of *specialization* between the rectangle and the square.

### **Profile 3. Use of hierarchical definitions involving the transitivity of inclusion relations (although with some non-economical definitions) (n=8)**

In this profile, PPT use the specialization of definitions and the transitivity of inclusion relations. They define geometric figures considering, in all cases, the process of adding attributes to a class to generate a subclass (*specialization*), which entails the *transitivity* of inclusion relations. Furthermore, in most cases, they define geometric figures using the relevant attributes of the concepts (*analytical features, type 3 judgement*), although in some cases they provide redundant data. For example, the participant PPT17 in task 1b, Rhomboid→Rhombus→Square, explains,

- *Rhomboid: Parallelogram with interior angles less than 180°, and with 4 sides parallel two by two.*
- *Rhombus: rhomboid with four equal sides and perpendicular diagonals.*
- *Square: rhombus with four right angles.*

This PPT adds redundant information in the definition of rhomboid: *parallelogram with angles less than 180° and parallel sides two by two*. However, he explicitly uses the process of *specialization* of definitions (*a rhombus is a rhomboid that ...; a square is a rhombus that ...*), showing the *transitivity* of inclusion relations.

## **Discussion**

The aim of this study was to characterise the PPT's curricular reasoning when anticipating correct answers in tasks involving inclusive relations between geometric figures as an aspect of *curricular noticing*. The results show three profiles of the PPT's curricular reasoning in the classification tasks, considering how they considered the *specialization* of definitions and the *transitivity* of inclusion relations. The three profiles described in the result section show two ideas. Firstly, about the preservice primary teachers' understanding of the relationship between defining and classifying, and secondly, about the characteristics of the PPT's curricular reasoning regarding the student's geometrical thinking in anticipating tasks.

### **Preservice primary teachers' understanding of the relationship between defining and classifying**

The *specialization* of definitions and *transitivity* of inclusion relations are key to understanding the relationship between the processes of defining and classifying. On the one hand, the process of *specialization* allows us to understand how adding properties to figures generates a subclass of a class. Furthermore, considering the definitions of geometric figures to reflect two (or more) inclusion relations is a key aspect that must be understood by PPT when they think about the cognitive demand of tasks for the primary students. On the other hand, the *transitivity* of inclusion relations is linked to the process of *generalization* of definitions, which allows to include a figure (A) within a class (B) and, to include this second class, within another more general class (C) (Figure 1). Understanding

these relationships between figures, dismissing misconceptions generated by prototypical examples (profile 3) and using *Analytical Features* (type 3) (Hershkowitz, 1990) seems to be a necessary condition in PPT's understanding of the potential of teaching activities in primary education. Thus, PPT using in their definitions some references to prototypical examples or attributes of figures to express the inclusive relationship between geometric figures (*prototypical judgement 1 and 2*) (Hershkowitz, 1990), seem have limited ability to anticipate primary school students' answers to the classification tasks of geometric figures.

### **Characteristics of preservice primary teachers' curricular reasoning in anticipating tasks**

The PPT's curricular reasoning in anticipatory tasks indicates ways of thinking about teaching activities considering the students' geometrical thinking. In this study, the ideas of *specialization/generalization* of definitions and *transitivity* of inclusion relations have been considered as key aspects when PPT anticipate student answers in the teaching activities focused on the relationships between defining and classifying. In this way, anticipating student answers has shown features (profiles) of the PPT's curricular reasoning as part of *curricular noticing* (Amador et al., 2017). Furthermore, these results show that if a PPT uses correctly all the process to anticipate a hierarchical definition from a classification (specialization and transitivity), he/she has acquired one of the aspects of the curricular noticing.

For example, the inclusion relationship between the Rhomboid  $\rightarrow$  Rhombus  $\rightarrow$  Square has been the one in which the least students have applied the process of *specialization*, generating non-inclusive definitions and using the attributes of the prototypical figures, which tend to be exclusive. Possibly, using prototypical examples of the concept of quadrilateral makes it difficult for PPT to use the relevant attributes of different quadrilaterals to establish inclusive relationships between them. Moreover, in the definitions in which excluding attributes have been used the most, so that neither *specialization* of the definition nor *transitivity* of the inclusive relations can occur, it has been in the quadrilaterals in the first inclusion relation of the three graphs (e.g., in task 1b Rhomboid  $\rightarrow$  Rhombus). We think that this may be due to the difficulty involved in the process of *generalization* (eliminating certain properties or substituting some for more general ones) when the concepts have already been acquired (*a posteriori classification*) (de Villiers, 1994) (Figure 3), to include the rest of the quadrilaterals that make up the inclusive relation within this more general class.

### **Acknowledgment**

Study carried out with the support of the project: PID2020-116514GB-I00, Agencia Estatal de Investigación, Ministerio de Ciencia e Innovación, Spain and coReflect@maths 2019-1-DE01-KA203-004947. The European Commission's support to produce this publication does not constitute an endorsement of the contents, which reflect the views only of the authors, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

### **References**

Amador, J., Males, L., Earnest, D., & Dietiker, L. (2017). Curricular Noticing: Theory on and Practice of Teachers' Curricular Use. In E. Schak, M. Fisher, & J. Wilhelm, (Eds.), *Teacher Noticing: Bridging and Broadening Perspectives, Contexts and Frameworks* (pp. 427-443). Springer.



- Breyfogle, M. L., Roth McDuffie, A., & Wohlhuter, K. A. (2010). Developing curricular reasoning for grades pre-K-12 mathematics instruction. In B. Reys, R. E. Reys, & R. Rubenstein (Eds.), *Mathematics curriculum: Issues, trends, and future directions* (pp. 307–320). National Council of Teachers of Mathematics.
- Brunheira, L., & da Ponte, J. P. (2019). From the classification of quadrilaterals to the classification of prisms: An experiment with prospective teachers. *The Journal of Mathematical Behavior*, 53, 65–80.
- De Villiers, M. (1994). The role and function of a hierarchical classification of quadrilaterals. *For the Learning of Mathematics*, 14(1), 11–18. <https://www.jstor.org/stable/40248098>
- Dietiker, L., Males, J., Amador, J., & Earnest, D. (2018). Research Commentary: Curricular Noticing: A Framework to Describe Teachers' Interactions with Curriculum Materials. *Journal for Research in Mathematics Education*, 49(5), 521–532. <https://doi.org/10.5951/jresmetheduc.49.5.0521>
- Fujita, T. (2012). Learners' level of understanding of the inclusion relations of quadrilaterals and prototype phenomenon. *The Journal of Mathematical Behavior*, 31(1), 60–72. <https://doi:10.1016/j.jmathb.2011.08.003>
- Fujita, T., & Jones, K. (2007). Learners' understanding of the definitions and hierarchical classification of quadrilaterals: Towards a theoretical framing. *Research in Mathematics Education*, 9(1), 3–20. <https://doi.org/10.1080/14794800008520167>
- Hershkowitz, R. (1990). Psychological aspects of learning geometry. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and cognition* (pp. 70–95). Cambridge University Press.
- Jones, K., & Tzekaki, M. (2016). Research on teaching and learning geometry. In A. Gutierrez, G. Leader, & P. Boero (Eds.), *The second Handbook of research on the psychology of mathematics education* (pp. 109–149). Sense.
- Lehrer, R., Slovin, H., & Dougherty, B. (2014). *Developing Essential Understanding of Geometry and Measurement for Teaching Mathematics in Grades 3–5*. NCTM.
- Sánchez-Matamoros, G., Fernández, C., & Llinares, S. (2019) Relationships among prospective secondary mathematics teachers' skills of attending, interpreting and responding to students' understanding. *Educational Studies in Mathematics*, 100(1), 83–99. <https://doi.org/10.1007/s10649-018-9855-y>
- Sinclair, N., Bussi, M. G. B., de Villiers, M., Jones, K., Kortenkamp, U., Leung, A., & Owens, K. (2016). Recent research on geometry education: An ICME-13 survey team report. *ZDM*, 48(5), 691–719. <https://doi.org/10.1007/s11858-016-0796-6>
- Usiskin, Z., & Griffin, G. (2008). *The classification of Quadrilaterals. A Study of definition*. IAP.