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Ahmad Amin, Laurence Girolami, Frédéric Risso. Fall of a large sphere in a suspension of small fluidized particles. *Physical Review Fluids*, 2022, 7 (8), pp.L082301. 10.1103/PhysRevFluids.7.L082301 . hal-03750976

**HAL Id: hal-03750976**

**<https://hal.science/hal-03750976v1>**

Submitted on 13 Aug 2022

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# Fall of a large sphere in a suspension of small fluidized particles

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(Dated: August 13, 2022)

## Abstract

The investigation of the fall of a sphere at finite Reynolds number in a concentrated suspension of small fluidized particles leads to unexpected results. By analyzing the drag force, it is shown that the average surface stress on the sphere is independent of the size of the sphere. It is proportional to an effective viscosity determined from the sedimentation velocity of the particles multiplied by the velocity of the sphere and divided by the size of the particles. These results question the role of concentration inhomogeneities that occur on a large scale in the overall flow around a moving obstacle and on a small scale near its surface.

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21 Suspensions, consisting of small particles dispersed in a fluid, are very common in na-  
 22 ture (turbidity currents, pyroclastic flows, blood, etc.) as well as in industry (food and  
 23 cosmetic, fluidized beds, etc.). A suspension is a complex two-phase mixture that is desir-  
 24 able to model as an equivalent fluid of effective density  $\rho_m$  and viscosity  $\mu_m$ . The mixture  
 25 density  $\rho_m$  is simply the average density of both phases weighted by their respective volume  
 26 fraction. However, defining an effective viscosity  $\mu_m$  for the mixture, always larger than the  
 27 suspending-fluid viscosity  $\mu_f$ , remains a challenge. Since the first attempt of Einstein [1, 2],  
 28 numerous works have been devoted to this issue, mainly focused on sheared suspensions of  
 29 neutrally buoyant solid particles with negligible inertia. This case has been thoroughly re-  
 30 viewed in Ref. [3] for non-Brownian suspensions. Under these conditions, the stress  $\tau$  within  
 31 the mixture is linear with the shear rate  $\dot{\gamma}$  and, for a given fluid-particle system,  $\mu_m/\mu_f$  is  
 32 only a function of the particle volume fraction  $\Phi$ . This result may not hold with deformable  
 33 particles, such as droplets in emulsions [4] or red cells in blood [5], since their deformation  
 34 is affected by the shear rate  $\dot{\gamma}$  and thus  $\mu_m/\mu_f$  may depend on it. As well, when inertia is  
 35 no longer negligible,  $\mu_m/\mu_f$  may depend on the local Reynolds number and vary with  $\dot{\gamma}$ .

36 The flow around an obstacle is known as a reference case from which the rheology of  
 37 a fluid can be analyzed. However, it has rarely been applied to the investigation of the  
 38 effective behavior of suspensions, with the notable exception of [6], where the rise of a  
 39 bubble through a dispersion of neutrally buoyant particles was studied. The present work  
 40 investigates the fall of a large solid sphere through a suspension of small beads in a liquid.  
 41 The beads, heavier than the liquid, are maintained in suspension by imposing a weak upward  
 42 flow. Using a fluidized bed makes it possible to deal with buoyant particles and to easily  
 43 control the volume fraction  $\Phi$  by changing the fluidization velocity  $U_f$ . Here,  $U_f$  is taken in  
 44 the range of the stable homogeneous fluidization regime, in which the particle distribution  
 45 remains steady and uniform. The terminal velocity  $V_t$  of three large spheres of different  
 46 diameters  $D$  is measured within four suspensions of different beads of diameters  $d \ll D$ , at  
 47 concentrations  $\Phi$  from 0.3 to 0.85. As shown later,  $V_t$  is much larger than  $U_f$  and the inertia  
 48 of the suspension is not negligible as its flows around the large sphere. On the other hand,  
 49 the inertia of the small beads is low compared to viscous effect.

50 The experimental setup is depicted in Fig. 1. The fluidization column has a rectangular  
 51 cross-section of sides  $w_1=0.2$  m and  $w_2=0.3$  m. It is filled with a mixture of water and  
 52 particles. In the absence of flow, the particles form a loose packed bed of height  $h_0$  at

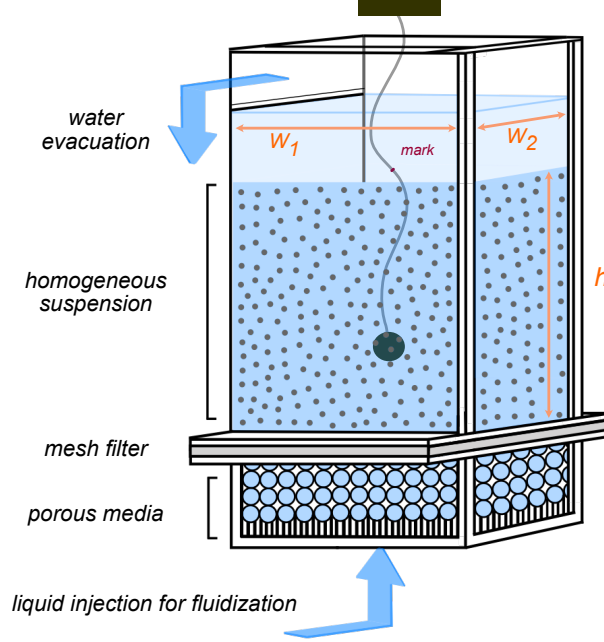


FIG. 1. Scheme of the experimental setup.

Suspension properties	$GB^1$	$GB^2$	$GB^3$	<i>Sand</i>
Particle diameter $d$ [ $\mu\text{m}$ ]	160	240	335	310
Particle density $\rho_d$ [ $\text{kg m}^{-3}$ ]	$2.50 \times 10^3$	$2.50 \times 10^3$	$2.50 \times 10^3$	$2.66 \times 10^3$
Fluid density $\rho_f$ [ $\text{kg m}^{-3}$ ]	$1.0 \times 10^3$	$1.0 \times 10^3$	$1.0 \times 10^3$	$1.0 \times 10^3$
Fluid viscosity $\mu_f$ [ $\text{Pa s}$ ]	$1.15 \times 10^{-3}$	$1.11 \times 10^{-3}$	$1.11 \times 10^{-3}$	$1.10 \times 10^{-3}$
$St_0 = \frac{(\rho_d - \rho_f)(\rho_d + \frac{1}{2}\rho_f)gd^3}{18 \mu_f^2}$	7.6	28	76	80

TABLE I. Physical properties of the suspensions

53 a concentration  $\Phi_{\text{pack}}$  between 0.58 and 0.60. Then, water is injected from the bottom  
54 at a flow rate  $Q$  through a porous media, which ensures a uniform flow, and a mesh filter,  
55 which prevents the passage of particles. For a given fluidization velocity,  $U_f = Q/(w_1 w_2)$ , the  
56 suspension expands up to reach a height  $h$ , corresponding to a concentration  $\Phi/\Phi_{\text{pack}} = h_0/h$ .  
57 The properties of the suspensions are given in Table I. We used three sets of spherical glass  
58 beads of different sizes ( $GB^1$ ,  $GB^2$ ,  $GB^3$ ) and one set of natural sand grains. Following  
59 [10, 11], we introduce an effective viscosity of the suspension  $\mu_{\text{m}_d}$  determined from the  
60 fluidization velocity. Let's consider a spherical bead of diameter  $d$  and density  $\rho_d$  falling  
61 at velocity  $U_f$  into a fluid of viscosity  $\mu_{\text{m}_d}$  and density  $\rho_m$ . Balancing the Stokes' drag,

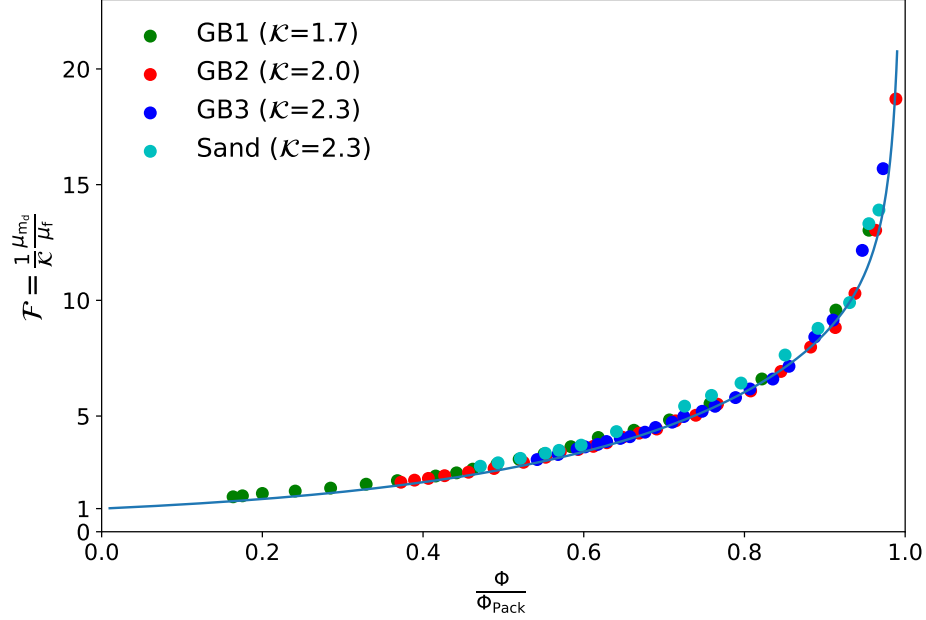


FIG. 2. Mixture effective viscosity defined from the fluidization velocity of the suspension. Symbols: measurements. Line: model from [10], taking  $\mathcal{F}\left(\frac{\Phi_s}{\Phi_{\text{pack}}}\right) = \frac{1}{(e^{-3}+0.08)} \left[ e^{-3\left(1-\frac{\Phi_s}{\Phi_{\text{pack}}}\right)} + 0.08 \left(1 - \frac{\Phi_s}{\Phi_{\text{pack}}}\right)^{-2/3} \right]$ .

62  $3\pi\mu_{\text{m}_d}dU_f$ , by the reduced weight of the bead,  $\pi d^3/6(\rho_d - \rho_m)g$ , where  $g$  is the gravity  
 63 acceleration and  $(\rho_d - \rho_m) = (1 - \Phi)(\rho_d - \rho_f)$ , yields

$$\frac{\mu_{\text{m}_d}}{\mu_f} = \frac{g(\rho_d - \rho_f)(1 - \Phi)d^2}{18\mu_f U_f}. \quad (1)$$

64 From the analysis of many fluid-particle systems, it has been shown in [10] that, provided  
 65 that the fluid inertia is negligible, the fluidization velocity of a suspension can be modeled  
 66 as

$$\frac{\mu_{\text{m}_d}}{\mu_f} = \mathcal{F}\left(\frac{\Phi}{\Phi_{\text{pack}}}\right) \mathcal{K}(St_0). \quad (2)$$

67  $\mathcal{F}$  is only a function of  $\Phi/\Phi_{\text{pack}}$ , which tends towards unity as  $\Phi/\Phi_{\text{pack}}$  tends to zero, and  
 68 towards infinity when  $\Phi/\Phi_{\text{pack}}$  tends to unity.  $\mathcal{K}$  only depends on the Stokes number defined  
 69 as  $St_0 = \frac{(\rho_d - \rho_f)(\rho_d + \frac{1}{2}\rho_f)gd^3}{18\mu_f^2}$  and accounts for the role played by the inertia of the dispersed  
 70 particles through their fluctuating motion. It is constant for a given fluid-particle system  
 71 and increases from 1 to 3 as  $St_0$  increases from zero to infinity. Figure 2 shows that the  
 72 experimental results obtained with the present suspensions collapse on the master curve  
 73 proposed by [10], which validates the relevance of the viscosity  $\mu_{\text{m}_d}$  determined from Eq. 1.

Sphere properties	$S1$	$S2$	$S3$
Diameter $D$ [mm]	12.2	15.7	22.4
Density $\rho_D$ [ $\text{kg m}^{-3}$ ]	$2.64 \times 10^3$	$2.60 \times 10^3$	$2.50 \times 10^3$

TABLE II. Physical properties of the falling spheres

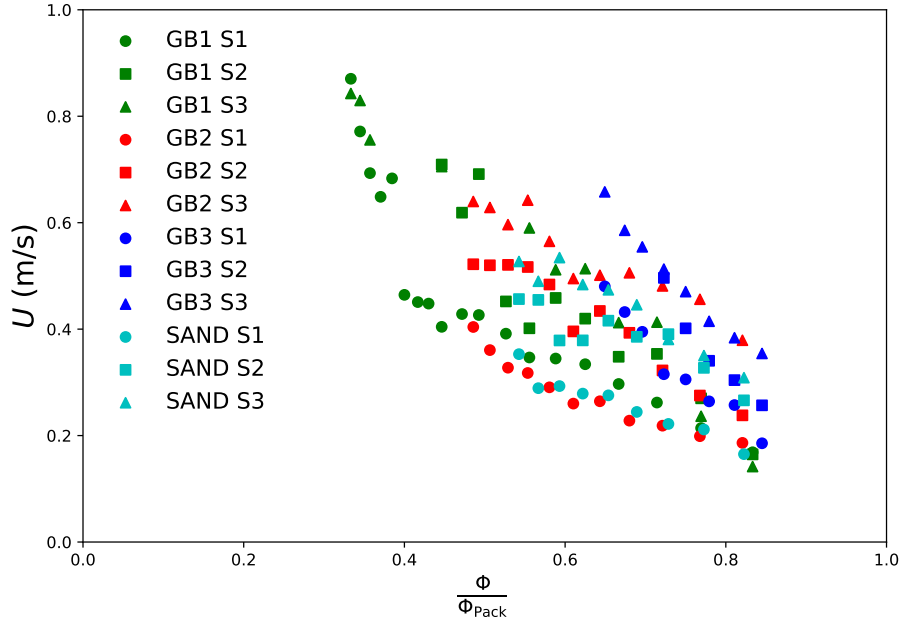


FIG. 3. Relative sphere velocity versus particle concentration.

74 However,  $\mu_{\text{md}}$  characterizes the viscous stresses at the scale of the dispersed beads. It is  
75 therefore not expected to be relevant to describe the macroscopic behavior of the mixture  
76 when the suspension is subjected to a shear at a scale that is large compared to  $d$  [12], as  
77 it was confirmed in [11] from comparisons with classic correlations for the effective viscosity  
78 of a sheared suspension. This motivated us to study the fall of a large sphere of diameter  
79  $D \gg d$  through such fluidized suspensions.

80 The characteristics of the falling spheres are given in Table II. They are made of glass  
81 and have a density close to that of the dispersed particles ( $\pm 6\%$ ) and approximately 2.5  
82 times that of the liquid. Their diameter ranges between 12.2 and 22.4 mm, corresponding  
83 to diameter ratios  $D/d$  from 36 to 140. The sphere falling experiments are conducted as  
84 follows. Since the suspension is opaque, we needed to find an alternative to optical methods.  
85 A thread of nylon with a diameter of 0.4 mm is attached to a support above the column,  
86 at one extremity, and glued to the sphere, at the other one. The thread length is adjusted

87 so that the sphere can be suspended within the column without touching the bottom. A  
 88 mark is made on the thread at a location that coincides with the top of the suspension  
 89 while the sphere is hanging from the support. At the beginning of a test, the sphere is fully  
 90 immersed in the suspension and positioned just below the top of the fluidized bed. Then,  
 91 the sphere is released and falls through the suspension until the thread is taut. A high-speed  
 92 Phantom VEO 340L camera with a LED lighting is used to record the process at a rate of  
 93 1000 frames per second. The release of the sphere is visible on the movie and the end of  
 94 the fall corresponds to the instant when the mark on the thread reaches the top of the bed.  
 95 The uncertainties on the detection of the times of release and fall end are of  $\pm 3$  images.  
 96 Depending on the system under consideration, the fall time  $T$  lies between 500 and 1300 ms  
 97 and is measured with an accuracy of  $\pm 6$  ms. The fall length  $L$  is known from the thread  
 98 length and varies from 20 to 60 cm, depending on the suspension height.

99 Because the sphere velocity  $V(t)$  takes a certain time to reach its terminal value  $V_t$ , the  
 100 average velocity  $\langle V \rangle = L/T$  is not equal to  $V_t$ . A better approximation of  $V_t$  is obtained by  
 101 assuming that the sphere motion includes a stage of constant acceleration  $\dot{V}_0$  followed by a  
 102 stage of constant velocity  $\tilde{V}_t$ . Considering that the fall length is given by  $L = \int_0^T V(t)dt$ ,  
 103 one gets that  $\tilde{V}_t$  is a solution of the following second-degree equation,

$$\tilde{V}_t^2 - (2T\dot{V}_0)\tilde{V}_t + (2L\dot{V}_0) = 0, \quad (3)$$

104 the initial acceleration being obtained from the balance between the inertial forces and the  
 105 reduced weight acting on the sphere,

$$\dot{V}_0 = \frac{(\rho_d - \rho_m)g}{\rho_d + \frac{1}{2}\rho_m}, \quad (4)$$

106 where  $\frac{1}{2}\rho_m$  accounts for the added mass. With this model, the terminal velocity is reached  
 107 at time  $t_t = \tilde{V}_t/\dot{V}_0$ . Thus  $\tilde{V}_t$  tends towards  $V_t$  when  $t_t/T$  becomes small, i.e. when the  
 108 acceleration stage is short compared to the whole fall duration. We have determined  $\langle V \rangle$ ,  $\tilde{V}_t$   
 109 and  $t_t/T$  for all the tests made. In the following, only the tests with  $t_t/T \leq 0.3$  have been  
 110 retained. In this case, the difference between  $\langle V \rangle$  and  $\tilde{V}_t$  is less than 15% and we estimate  
 111 that the discrepancy between  $\tilde{V}_t$  and  $V_t$  is less than 5%. All the subsequent analysis is thus  
 112 done by using  $\tilde{V}_t$  as the terminal velocity of the spheres. Note that the experimental data  
 113 have also been processed by considering a less demanding criterion  $t_t/T \leq 0.5$ , which does  
 114 not change the present conclusions and proves the robustness of the results regarding the  
 115 determination of  $\tilde{V}_t$ .

116 The terminal velocity  $U$  of the sphere relative to the fluid-particle mixture is obtained  
 117 by adding the fluidization velocity  $U_f$ , so that  $U = V_t + U_f$ . Figure 3 shows  $U$  as a function  
 118 of  $\Phi/\Phi_{\text{pack}}$  for the three spheres and the four types of suspensions. The values of  $U$  ranges  
 119 between 0.1 and 0.9 m/s and are much larger than the fluidization velocities, which remain  
 120 less than 0.01 m/s. In any case,  $U$  is thus almost equal to  $V_t$ . It is a decreasing function of  
 121  $\Phi/\Phi_{\text{pack}}$ , since both the density and the effective viscosity of the suspension increase with  
 122 the solid volume fraction. For a given type of bead,  $U$  is also observed to decrease with  $D$ .  
 123 However, it is difficult to draw physical conclusions from these dimensional plots.

124 As shown by [6], an important dimensionless group is the other Stokes number defined  
 125 by  $St = \tau_d/\tau_D$ , which compares the response time of the dispersed particles,  $\tau_d = (\rho_d +$   
 126  $\frac{1}{2}\rho_m)d^2/18\mu_{m_d}$ , to the time scale of the flow generated by the motion of the large body,  
 127  $\tau_D = D/U$ . For  $St < 1$ , the particles are expected to follow the stream lines of the suspending  
 128 fluid, whereas, for  $St > 1$ , they may collide with the large body. In the present case, this  
 129 Stokes number is much less than unity ( $2 \times 10^{-3} < St < 9 \times 10^{-2}$ ). However, this analysis  
 130 is not sufficient to conclude that the suspension remains homogeneous. First, shear-rate  
 131 gradients in the flow around the sphere may induce particle migration [7], leading to non-  
 132 uniform concentration. In addition, a depletion of particles in the wake behind the obstacle  
 133 have been reported in previous studies [8, 9].

134 Now, the results are analyzed in terms of the relationship between the drag coefficient and  
 135 the Reynolds number of the falling sphere. If the drag coefficient is obtained directly from  
 136 the balance between the drag force and the reduced weight of the sphere,  $C_d = \frac{4}{3} \frac{(\rho_D - \rho_m)gD}{\rho_m U^2}$ ,  
 137 the Reynolds number requires the knowledge of the effective viscosity of the suspension.  
 138 Let us consider the viscosity  $\mu_{m_d}$  defined by Eq. 1 and introduce  $Re_m = \frac{\rho_m U D}{\mu_{m_d}}$ . Figure. 4  
 139 shows log-log plots of the experimental values of  $C_d$  versus  $Re_m$ , for all investigated cases.  
 140 For any given pair of sphere and suspension, the values of  $C_d$  collapse on a  $Re_m^{-1}$  straight  
 141 line. On these plots, the Reynolds number has been divided by a constant  $k$ , which has  
 142 been arbitrarily adjusted to make the data of the various systems to coincide with the drag  
 143 Stokes law,  $C_d = 24/Re$ . The values of  $k$  varies from one system to another, but remain  
 144 constant for a given system, which means that they are independent of  $Re_m$ . In Fig. 5,  $k$   
 145 is plotted against  $D/d$  and turns out to be a linear function of the sphere-to-bead diameter  
 146 ratio:  $k = \alpha D/d$ , with  $\alpha \approx 0.58$ . Therefore, the experimental results lead to the following



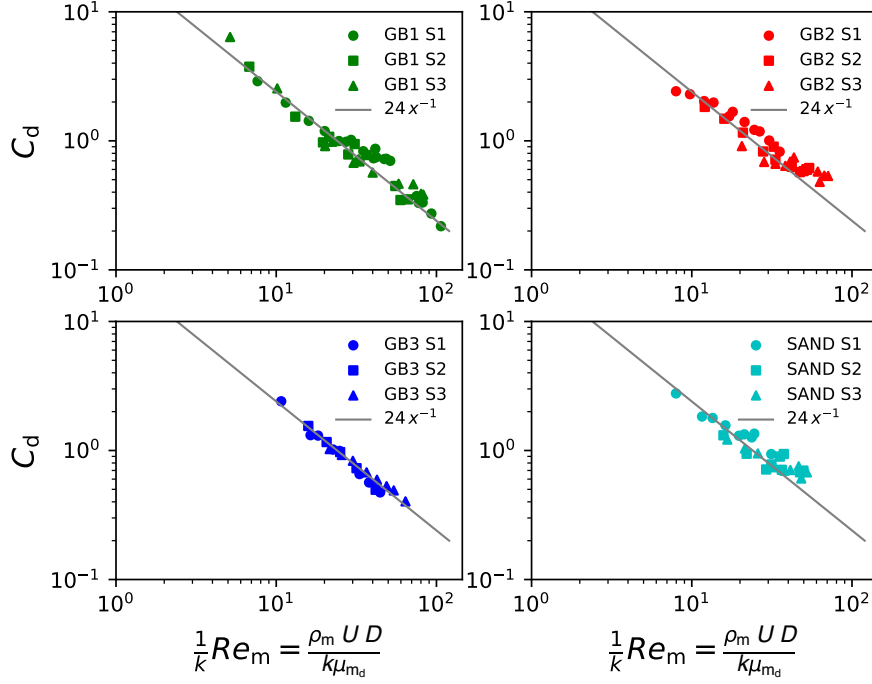


FIG. 4. Drag coefficient of the sphere versus the Reynolds number (values of  $k$  in Fig 5).

147 quite unexpected expressions of the drag coefficient,

$$C_d = 24 \frac{\mu_{\text{md}}}{\rho_{\text{m}} U D} \alpha \frac{D}{d} = 24 \alpha \frac{\mu_{\text{md}}}{\rho_{\text{m}} U d}, \quad (5)$$

148 and the drag force

$$F_D = (\pi D^2) \mu_{\text{md}} \frac{3\alpha U}{d}. \quad (6)$$

149 Thus it turns out that the drag coefficient  $C_d$ , as well as the average stress on the sphere  
 150 surface  $\tau_p = F_D/\pi D^2 = (3\alpha) \mu_{\text{md}} \frac{U}{d}$ , are independent of the size  $D$  of the falling sphere and  
 151 proportional to  $\mu_{\text{md}}$ .

152 We now discuss possible interpretations of this surprising result. A first naive approach  
 153 is to assume that the suspension remains homogeneous and that its behavior is controlled  
 154 by an effective viscosity  $\mu_{\text{m}}$  that is constant throughout the flow. By equating Eq. 6 to the  
 155 Stokes' drag,

$$F_{D\text{St}} = (\pi D^2) \mu_{\text{m}} \frac{3U}{D}, \quad (7)$$

156 one finds

$$\mu_{\text{m}} = \alpha \mu_{\text{md}} \frac{D}{d}. \quad (8)$$

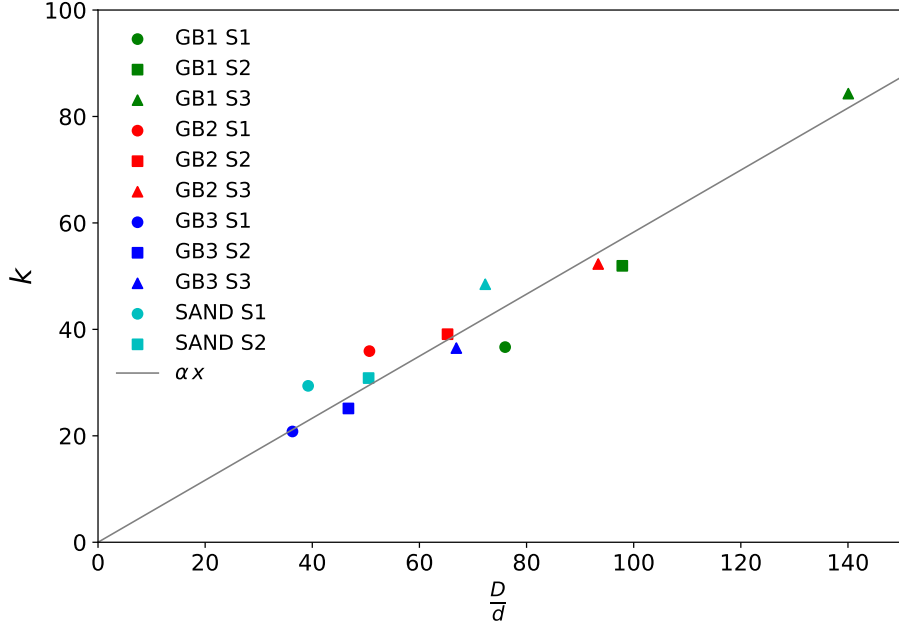


FIG. 5. Coefficient  $k$  against  $\frac{D}{d}$ .

157 However, this expression is inconsistent. As  $\mu_{m_d}$  represents the effective viscosity at the  
 158 scale  $d$  of the beads, one could expect the effective viscosity at a much larger scale  $D$  to  
 159 be different, but converge towards a constant values at large  $D/d$ , which is not the case  
 160 here. Otherwise, one can assume that  $\mu_m$  actually varies with  $D$  because it would depend  
 161 on the shear rate  $\dot{\gamma} \propto U/D$ , but that leads to contradictory behaviors according to that  $\dot{\gamma}$   
 162 varies by changing either  $U$  or  $D$ . Moreover, the sphere Reynolds number based on  $\mu_m$  is  
 163 too large ( $5 < \rho_m U D / \mu_m < 100$ , see Fig. 4) for the Stokes' drag law to be valid. Therefore,  
 164 the effective viscosity of the suspension cannot be determined from Eq. 8.

165 As noted by [13], determining the effective viscosity of a suspension from the measurement  
 166 of the force exerted on a wall requires the homogeneity of the suspension everywhere, and  
 167 in particular near the wall. The present result is probably associated with the fact that  
 168 the particle concentration is not uniform. We think that it is relevant to discuss separately  
 169 the effect of inhomogeneity at the scale  $D$  of the flow around the large sphere and that of  
 170 inhomogeneity at the scale  $d$  of the dispersed particles. Regarding large scales, the expected  
 171 increase of the particle concentration at the sphere front and the decrease at its rear can  
 172 significantly influence the drag coefficient [8, 9] and eventually lead to unexpected behaviors.  
 173 In our opinion, such a mechanism can hardly result in a drag coefficient that both decreases

174 as the reciprocal of the velocity and does not depend on  $D$ . However, due to the complexity  
175 of such flows, the question of its relevance remains open.

176 At the scale of the particles, the homogeneity of the suspension is never rigorously fulfilled  
177 in the vicinity of a solid surface. Since a particle cannot approach an obstacle at a distance  
178 that is closer than its radius, the volume fraction of the dispersed phase tends to zero at a  
179 solid surface [14, 15]. In addition, the interactions between a solid surface and the dispersed  
180 particles differ from the interactions between a solid surface and the suspending fluid. A fluid  
181 adheres to a solid because of molecular interactions such as van der Waals forces, whereas  
182 dispersed particles can move relative to a solid. Considering the blood flow for example, the  
183 red blood cells may experience a slip velocity of 40% of the maximum flow velocity relative  
184 to the vessel wall [16]. In the framework of two-fluid approaches, this can be modeled  
185 by increasing the viscosity of the plasma near the vessel wall in order to account for the  
186 additional dissipation induced by the slip motion of the cells [17]. However, it is not relevant  
187 to model the whole mixture as a homogeneous fluid satisfying a non-slip condition at a solid  
188 boundary. This suggests another possible interpretation of the experimental result. We can  
189 assume that there is a thin layer of liquid at the surface of the sphere of thickness  $\delta$  which is  
190 devoid of particles, and that the particles just outside this layer move at a speed of the order  
191 of  $U$  with respect to the surface of the sphere. Within this layer, the liquid is submitted to a  
192 stress  $\tau_p \approx \mu_f \frac{U}{\delta}$ . As  $\mu_{md} \frac{U_f}{d}$  is the average stress submitted by the liquid passing through the  
193 fluidized particles, it seems relevant to assume that  $\tau_p \approx \mu_{md} \frac{U}{d}$ , which corresponds to the  
194 experimental result. By considering that the flow in the near vicinity of the sphere surface  
195 is independent of the large-scale flow around the sphere, this interpretation is naturally  
196 consistent with the fact the drag coefficient is proportional to  $U^{-1}$  and independent of  $D$ .

197 Apart from the blood circulation, a few other studies of dispersed two-phase flows have  
198 reported evidences of such a slip of the dispersed phase near a solid surface. A foam in a  
199 pipe was shown to behave as a rigid body slipping on a lubricated layer at the wall and  
200 the authors concluded that “the flow of such foams is not controlled by foam rheology”  
201 [18]. The flow of a concentrated gas-solid suspension released after a dam break was also  
202 observed to flow as an inviscid fluid which slips on the wall [19]. Regarding an imposed wall  
203 shear rate, it is worth mentioning an investigation of the flow of a homogeneous oil-in-water  
204 droplet emulsion in a pipe [4]. While the effective viscosity of the emulsion  $\mu_m$  was found to  
205 vary over the pipe cross-section and to depend on the bulk velocity  $U$ , the viscosity at the

206 wall  $\mu_{m_w}$  was observed to be independent of  $U$  and the pressure drop along the pipe to be  
 207 proportional to  $\mu_{m_w}U$ . This surprising outcome is fully compatible with the present result,  
 208  $\tau_p = \mu_{m_d} \frac{3\alpha U}{d}$ , where  $\mu_{m_d}$  is independent of  $U$  and implies a pressure drop that is proportional  
 209 to  $\mu_{m_d}U$ , whatever the nature of the flow or the mixture rheology away from the wall.

210 To conclude, the fall of a large sphere in a fluidized suspension of small particles has been  
 211 investigated in the regime where the flow inertia is negligible at the scale of the particle but  
 212 not at that of the sphere. The drag force (Eq. 6) is found to be the product of the sphere area  
 213  $\pi D^2$ , the viscosity  $\mu_{m_d}$  determined from the fluidization velocity of the dispersed particles  
 214 (Eq. 1), and the ratio  $U/d$  of the sphere velocity and the particle diameter. The fact that  
 215 the drag coefficient depends on the Reynolds number based on the diameter of the dispersed  
 216 particles rather than that of the falling sphere is quite unexpected. It is likely associated with  
 217 the fact that the suspension does not remain homogeneous. Two interpretations, based on  
 218 either a large-scale inhomogeneity or a particle slip velocity at the sphere surface, have been  
 219 discussed. They are not mutually exclusive. Indeed, the stress along the sphere surface is  
 220 probably not constant. The most likely situation is that the stress near the front stagnation  
 221 point is mainly normal and rather scales as  $U/D$ , and that the pressure difference between  
 222 the front and the rear of the sphere are mainly controlled by the large-scale inhomogeneity.  
 223 On the other hand, the shear rate near the equator probably scales as  $\tau_p \approx \mu_f \frac{U}{\delta}$  with  
 224  $\delta \ll d$ . We are inclined to think that the magnitude of the latter may be much larger  
 225 than the former, so that its contribution dominates the overall friction. However, since the  
 226 local particle concentration and velocity have not been measured, no definitive conclusions  
 227 regarding the physical mechanism can be reached. Future numerical works should determine  
 228 whether a large-scale inhomogeneity may be consistent with the present experimental scaling  
 229 and future experimental works should assess the existence of a strong slip velocity at the  
 230 sphere surface.

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- 231 [1] A. Einstein, Eine neue Bestimmung der Moleküldimensionen, *Ann. Phys.* **19**, 289 (1906).  
 232 [2] A. Einstein, Berichtigung zu meiner Arbeit: Eine neue Bestimmung der Moleküldimensionen,  
 233 *Ann. Phys.* **34**, 591 (1911).  
 234 [3] É Guazzelli and O. Pouliquen, Rheology of dense granular suspensions, *J. Fluid Mech.* **852**,

- 235 P1 (2018).
- 236 [4] M. Abbas, M., A. Pouplin, O. Masbernat, A. Liné and S. Décarre, Pipe flow of a dense emul-  
 237 sion: Homogeneous shear-thinning or shear-induced migration?, *AIChE J.* **63**, 5182 (2017).
- 238 [5] A. S. Popel and P. C. Johnson, Microcirculation and hemology, *Annu. Rev. Fluid Mech.* **37**,  
 239 43 (2005).
- 240 [6] N. Hooshyar, van J. R. Ommen, P. J. Hamersma, S. Sundaresan and R. F. Mudde, Dynamics  
 241 of single rising bubbles in neutrally buoyant liquid-solid suspensions, *Phys. Rev. Lett.* **110**,  
 242 244501 (2013).
- 243 [7] D. Leighton and A. Acrivos, The shear-induced migration of particles in concentrated suspen-  
 244 sions, *J. Fluid Mech.* **181**, 415 (1987).
- 245 [8] H. Haddadi, S. Shojaei-Zadeh and J. F. Morris, Lattice-Boltzmann simulation of inertial  
 246 particle-laden flow around an obstacle, *Phys. Rev. Fluids* **1**, 024201 (2016).
- 247 [9] T. Dbouk, A suspension balance direct-forcing immersed boundary model for wet granular  
 248 flows over obstacles, *J. Non-Newtonian Fluid Mech.* **230**, 68 (2016).
- 249 [10] A. Amin, L. Girolami and F. Risso, On the fluidization/sedimentation velocity of a homoge-  
 250 neous suspension in a low-inertia fluid, *Powder Tech.* **391**, 1 (2021).
- 251 [11] L. Girolami and F. Risso, Sedimentation of gas-fluidized particles with random shape and  
 252 size, *Phys. Rev. Fluids* **4**, 074301 (2019).
- 253 [12] E. J. Hinch, An averaged-equation approach to particle interactions in a fluid suspension, *J.*  
 254 *Fluid Mech.* **83**, 695 (1977).
- 255 [13] G. Ovarlez, F. Bertrand and S. Rodts, Local determination of the constitutive law of a dense  
 256 suspension of noncolloidal particles through magnetic resonance imaging, *J. Rheol.* **50**, 259  
 257 (2006).
- 258 [14] N. Lubchenko, B. Magolan, R. Sugrue, E. Baglietto, A more fundamental wall lubrication force  
 259 from turbulent dispersion regularization for multiphase CFD applications, *Int. J. Multiphase*  
 260 *Flow.* **98**, 36 (2018).
- 261 [15] A. du Cluzeau, G. Bois and A. Toutant, Analysis and modelling of Reynolds stresses in  
 262 turbulent bubbly up-flows from direct numerical simulations, *J. Fluid Mech.* **866**, 132 (2019).
- 263 [16] S. Roman, S. Lorthois, P. Duru and F. Risso, Velocimetry of red blood cells in microvessels  
 264 by the dual-slit method: Effect of velocity gradients, *Microvasc. Res.* **84**, 249 (2012).
- 265 [17] M. Sharan and A. S. Popel, A two-phase model for flow of blood in narrow tubes with increased

- 266 effective viscosity near the wall, *Biorheology* **38**, 415 (2001).
- 267 [18] M. I. Briceño and D. D. Joseph, Self-lubricated transport of aqueous foams in horizontal  
268 conduits, *Int. J. Multiphase Flow* **29**, 1817 (2003).
- 269 [19] L. Girolami and F. Risso, Physical modeling of the dam-break flow of sedimenting suspensions,  
270 *Phys. Rev. Fluids* **5**, 084306 (2020).