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Ice floe fracture in a granular model

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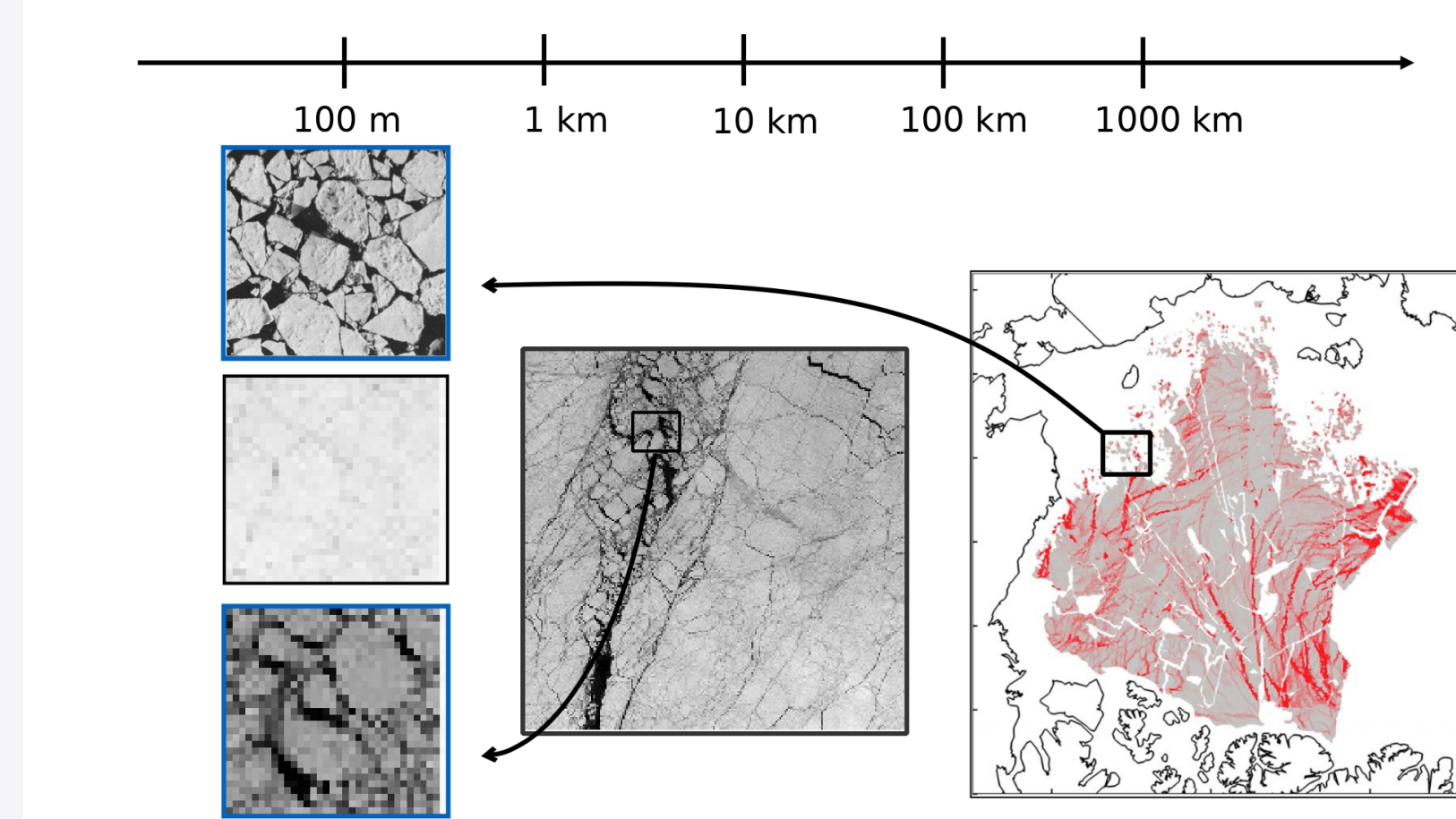
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Motivations

The Scale-Aware Sea Ice Project

The sea ice cover in the Marginal Ice Zone (MIZ) is thin and fragmented into individual plates called floes. It is vulnerable and sensitive to melting and to external forcings. Our is to improve the representation of the MIZ in large-scale, continuum sea ice models through a better understanding of its mechanical and dynamical behavior. Using FloeDyn, a realistic discrete model of the MIZ, and available observations, we study the behavior of an aggregate of ice floes at small scales to **develop physically-sound sub-gridscale parameterizations** destined to these continuum models.

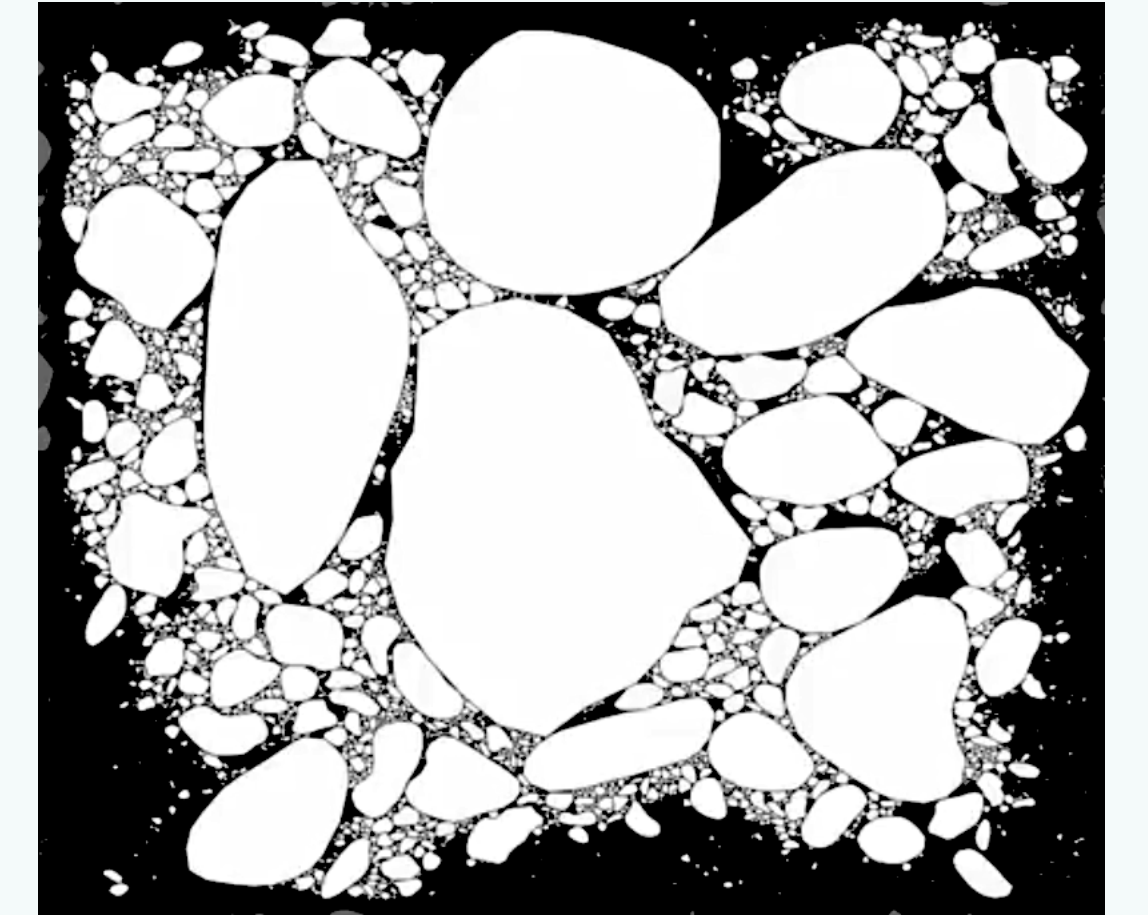


Reological transition in sea ice

FloeDyn, a discrete C++ model of ice floe dynamics

Developed by Rabatel (2015) and Rabatel et al. (2015):

- manages collisions between floes without interpenetration
- generates large assemblies of floes with realistic shapes and floe size distributions
- takes into account the atmosphere and ocean drags



An ice floe assembly in FloeDyn

Mathematical theory of fracture

Fracture propagation : state of the art

In the classical brittle fracture model of Griffith and Taylor (1921), the fracture propagation problem is solved by assigning an energy to the fracture. The fracture extends if it allows reaching a lower energy state. Francfort and Marigo (1998) have reformulated this approach as a variational problem which allows considering fractures with a low regularity.

The surface energy:

- The elastic energy of the fractured material, for a given fracture, σ , and a given displacement, u , is given by

$$E_{el}(u, \sigma) = \int_{\Omega \setminus \sigma} W(x, e(u)) \quad \text{with} \quad e(u) = \frac{1}{2} (\nabla u + \nabla u^T)$$

with W , the energy density function. In practical case studies, displacement is known only in the subdomain of $\partial\Omega \subset \partial\Omega$ as a Dirichlet condition $u|_{\partial\Omega_D} = u_D$. "Admissible" displacements are given by

$$A_\sigma = \left\{ u \in H^1(\Omega \setminus \sigma, \mathbb{R}^2) \mid u = u_D \text{ sur } \partial\Omega_D \setminus \sigma \right\}$$

and the associated elastic energy for a given fracture σ is therefore given by

$$E_{el}(\sigma) = \inf_{u \in A_\sigma} E_{el}(u, \sigma)$$

B. Bourdin and Marigo (2008) have shown that the infimum is reached if σ is smooth enough.

- The energy associated to a fracture σ is given by

$$E_{frac}(\sigma) = \int_\sigma k(x) d\mathcal{H}^{N-1}$$

with $d\mathcal{H}^{N-1}$, the Hausdorff dimension and N , the dimension.

- The total energy of the material for a given fracture σ is then :

$$E_{tot}(\sigma) = E_{el}(\sigma) + E_{frac}(\sigma) = \min_{u \in A_\sigma} E_{el}(u, \sigma) + \int_\sigma k(x) d\mathcal{H}^{N-1}$$

Fracture propagation criteria: After some simplification to this framework, the fracture propagates to $\sigma^*(t)$ such that

$$E_{tot}(t, \sigma^*(t)) = \min_{\sigma(t) \in \Gamma} E_{tot}(\sigma(t))$$

With Γ the admissible fracture path.

Fracturation of ice floes : our mathematical problem

Our questions in the context of ice floes are :

- "where and how is the fracture initiated ?" \Rightarrow consider an infinitesimal crack that can extend from the border to the interior.
- "does the floe break?" \Rightarrow consider a simple fracture path that separates the floe in two parts.

Following the Francfort and Marigo model, a floe is broken by fracture σ^* if

$$E_{tot}(\sigma^*) < E_{tot}^{initial}$$

Fracture due to a collision with another floe or with an obstacle.

Using the Francfort and Marigo theory, we need to determine the Dirichlet condition to calculate the fracture initiation.

- Model ice floe as a mass-spring network** : Balasoiu (2020) have shown that a mass-spring network can approximate an elastic material.
- Determination of the Dirichlet condition during a collision** using the mass-spring network : current Phd thesis of Dang Toai Phan.

Fracture due to waves : preview of a 1D test-case

- For a given wave field, η , the displacement, w , at each position, x , of the floe, Ω_f , is given by:

$$\begin{cases} YI \partial_x^4 w = -\rho g(w + \eta) & \text{on } \Omega_f \\ \partial_x^3 w = \partial_x^2 w = 0 & \text{on } \partial\Omega_f \end{cases}$$

with, ρ , the water density, g , the gravity and Y, I parameters depending on the ice mechanical strength and thickness.

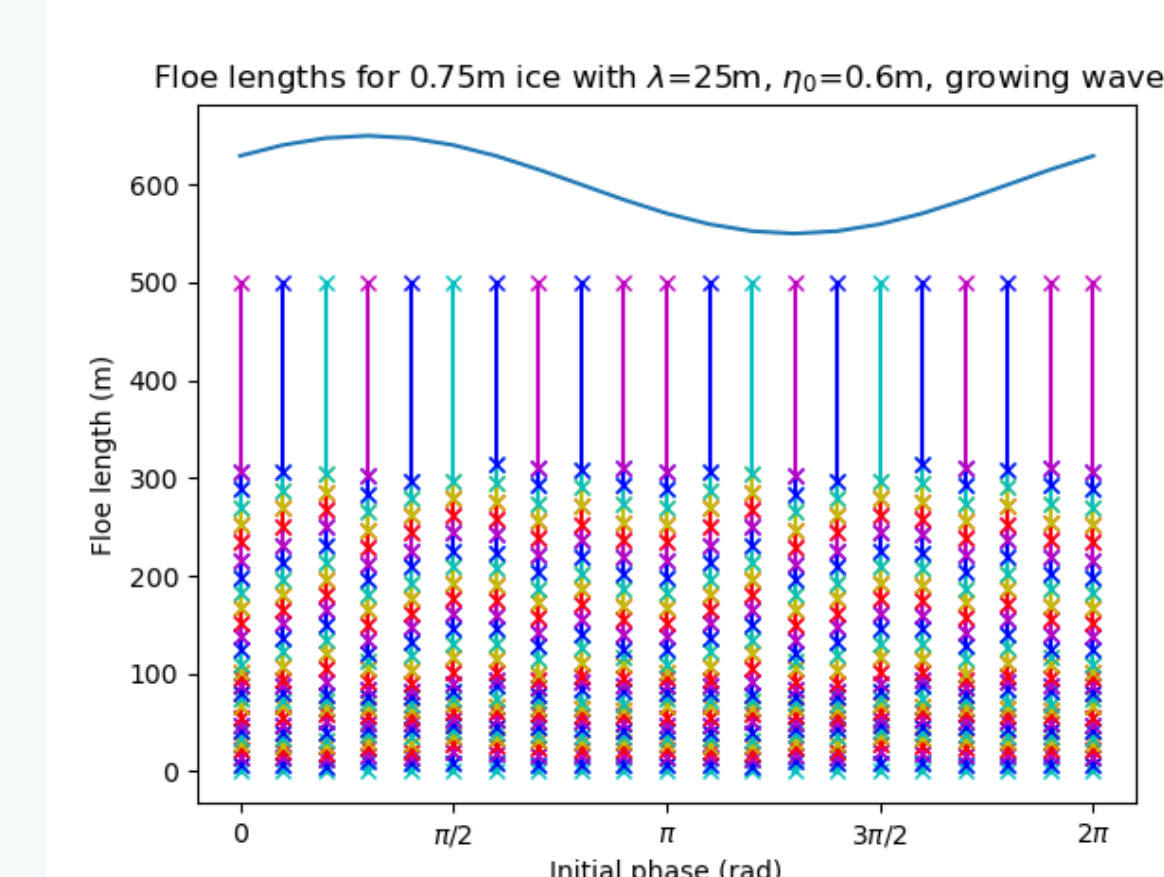
- The elastic energy of a floe is given by: $E_{el} = \int_{\Omega_f} (\lambda + 2\mu) |\partial_x^2 w|^2 dx$. with λ and μ the Lamé coefficients.

- Following the Francfort & Marigo formulation : a floe is fractured by the wave if

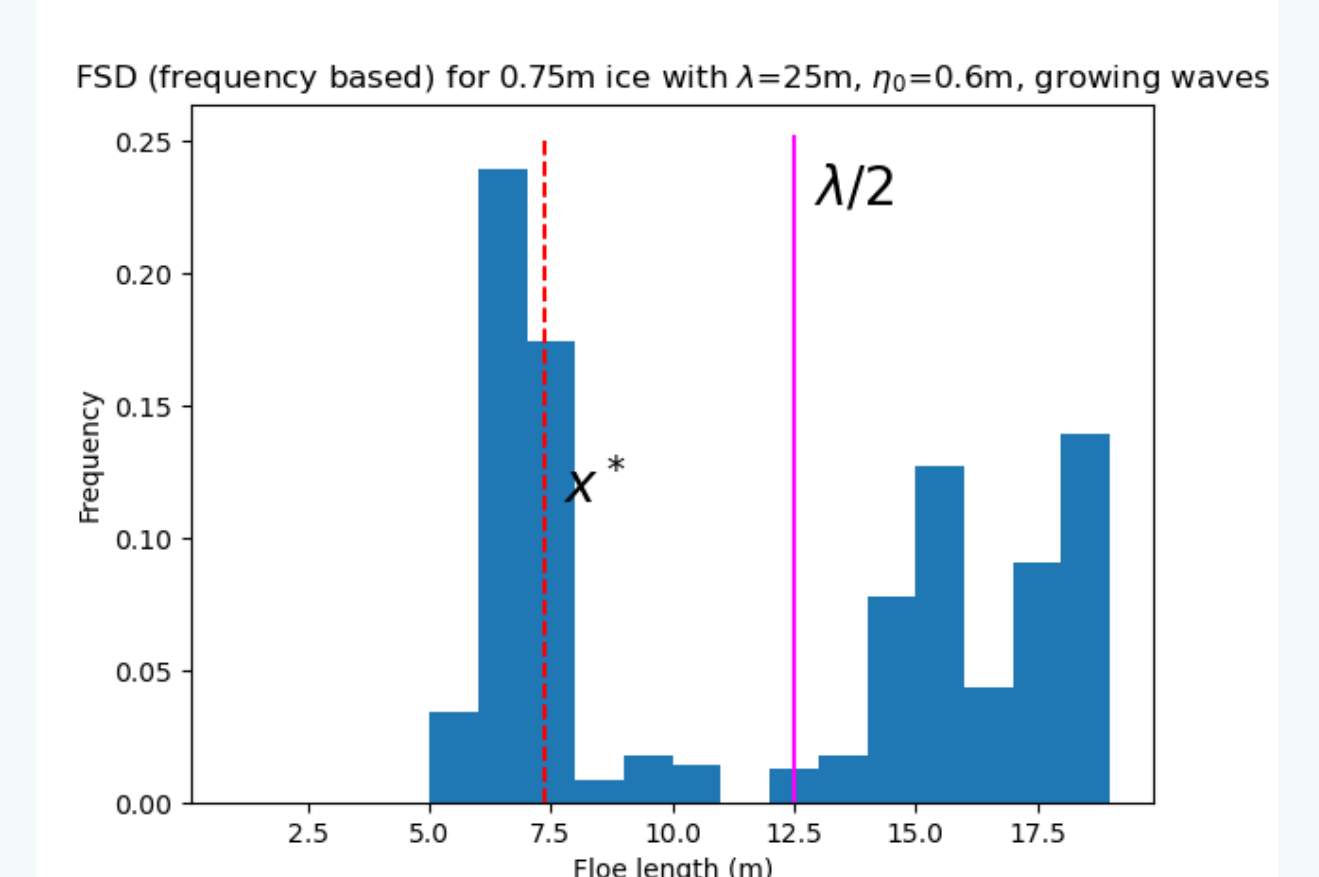
$$E_{tot}(\sigma^*) < E_{el} \quad \text{such that} \quad E_{tot}(\sigma^*) = \min_{\sigma \in I_\sigma} (E_{el,1}(\sigma) + E_{el,2}(\sigma) + k)$$

with, $I_\sigma \subset \Omega_f$, the close set of admissible fractures and $(E_{el,j}(\sigma))_{j=1,2}$ the elastic energy of the two floes created by the fracturing σ .

- Simulations forced with monochromatic waves allow the extraction of a floe size distribution (FSD).



Floes formed from the breakup of a 500m floe



Resulting FSD

- Current work using more realistic waves :

- multi-phase and multi-amplitude (spectral definition)
- wave propagation and attenuation affected by ice floe

Numerical modelling of a fracture

A fracture independent of the mesh : the extended finite element method (XFEM)

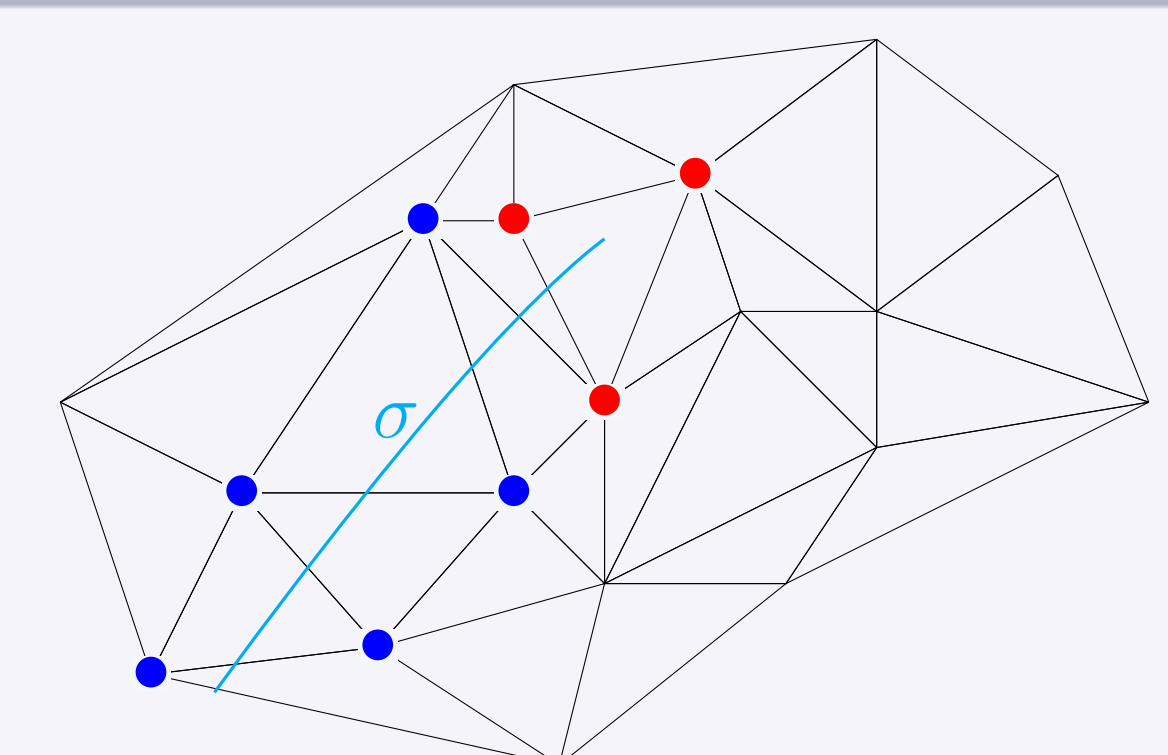
An extension of the classical finite elements especially invented to model fractures without changing the mesh : it allows considering discontinuous functions on the domain.

$$u^h = \sum_{i \in N} u_i \phi_i(x) + \sum_{j \in N_c} c_j \Phi_j(x) (H_\sigma(\nu) - H_\sigma(\nu_j)) + \left(\sum_{k \in T} \Phi_k(x) \left(\sum_{l=0}^2 b_l^k (F_l(x) - F_l(x_k)) \right) \right)$$

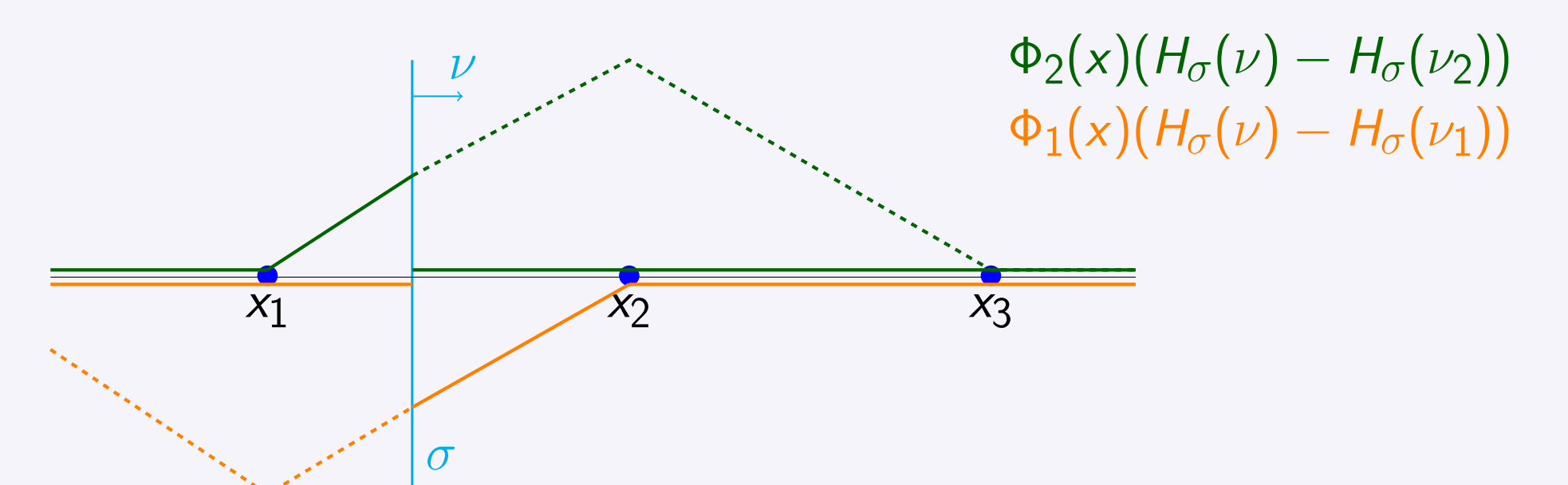
with

- N the set of nodes in the mesh, ϕ_j the classical shape functions and u_i the associated degree of freedom.
- N_c nodes which see the **crack face** but not the crack tip (blue bullets), ν is the signed distance with the crack, H the Heaviside function such that $H(\nu) = 1$ if $\nu > 0$ and $H(\nu) = -1$ else, c_j the corresponding degree of freedom.
- T is the node which see the **crack tip** (red bullets), F_l the enrichment function and the associated degree of freedom b_l^k . These terms are modelling crack tips.

XFEM version from Mohammadi (2012) guarantee the interpolation $u^h(x_j) = u_j$.



A triangular finite element mesh



An example of P_1 extended finite element in 1D

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