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# **“It all depends on the sign of the derivative”: A praxeological analysis of the use of the derivative in similar tasks in mathematics and mechanics.**

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*Calculus and mechanics are closely connected disciplines. However, students are not always able to establish links between them or use knowledge from one to solve problems in the other. In this paper, we study students' solutions of two tasks: a familiar mechanics task and a similar yet unfamiliar calculus task. Our results indicate that while students had acquired praxeologies from mechanics suitable to the mechanics task, they had difficulties transferring these praxeologies to the task presented in a calculus context.*

*Keywords: Calculus, mechanics, anthropological theory of the didactics (ATD), derivative task, college mathematics.*

## **Introduction and research problem**

Calculus and physics have been closely linked since the birth of the former: the emergence of the notion of derivative is tied to the formalisation of velocity and acceleration as concepts. Taşar (2010) points out that “in the educational literature [velocity and acceleration] emerge as nodes of difficulties”, but also that “students experience great difficulty when they need to transfer knowledge between these two domains” (p. 142).

International literature has identified a number of difficulties that students face in grasping the notion of derivative (e.g., Montoya Delgadillo et al., 2018; Orton, 1983). Taking into account different representations of the derivative (graphical, verbal, symbolic, paradigmatic physical, and others), Zandieh (2000) observes in particular that if a student understands the derivative in one context, this does not mean that they can solve tasks in another.

The notion of “transfer” has been studied in the literature, initially from a cognitive perspective examining the transfer of knowledge from one situation to another, and more recently from the perspective of the *Actor-Oriented Transfer* (AOT) (e.g., Lobato, 2012; Roorda et al., 2015). With AOT, transfer is defined as “the influence of a learner’s prior activities on his activity in novel situations” (Lobato, 2012, p. 233). From a cognitive perspective, the fact that students fail to intuitively transfer knowledge from mathematics to physics has been highlighted in the physics education literature (e.g., Christensen & Thompson, 2012; Planinic et al., 2019). As for mathematics education, our literature review of the main international journals identified only two studies that focus explicitly on the links students make between each domain. In the first study, Marrongelle (2004) provides evidence that students can use physics to solve calculus problems. In the second one, Roorda et al. (2015) use AOT to study the transfer of procedures between mathematics and physics in both directions (from the traditional cognitive perspective, transfer is “usually expected to occur from mathematics to physics” [Planinic et al., 2019, p. 235]). In Roorda et al. (2015), the participants

did not tend to explicitly mention links between the formulae learned in physics and the results obtained through symbolic differentiation in mathematics. However, they were able to establish further relationships between the procedures learned in both courses over the long term.

Hitt and González-Martín (2016) and Rasmussen et al. (2014) state that research on the teaching and learning of the derivative in relation to physics is still scarce, calling for more studies in that area. In particular, they highlight a lack of research in mathematics education examining teachers' and students' practices at the intersection of both disciplines. To address this issue, in her PhD, the first author of this paper examines practices related to the use of derivatives in calculus and mechanics courses at the college level. In this paper, we present some preliminary results of this ongoing work. Our research is divided into three main stages: (1) a praxeological analysis of calculus and mechanics textbooks (Hitier & González-Martín, 2021); (2) an analysis of teaching practices, through interviews with mathematics and mechanics teachers and in-class observations of differential calculus and mechanics courses; (3) an analysis of student practices using a questionnaire and task-based interviews. In order to shed light on the complex relationship between the two disciplines, our research aim is to study the similarities and differences between the way the notion of derivative is used in mathematics and mechanics courses, and the resulting impact on students' learning.

In the first stage, we observed that while the different representations of the derivative considered by Zandieh (2000) appear in the mechanics and calculus textbooks analysed, the practices studied do not seem to consider the difficulties inherent in shifting among these representations. We also observed that the practices employed in the context of one-dimensional motion were different in calculus and mechanics, relying on the limit definition and differentiation formulae in calculus, while applying given formulae in mechanics (Hitier & González-Martín, 2021). In this paper, we provide preliminary results from the third stage of our research, focusing on two tasks included in the students' questionnaire (see Methodology). We aim to answer the following research question: How do the praxeologies that students use when solving an unfamiliar calculus task relate to the praxeologies they use when solving a similar but familiar task in a mechanics context?

We note that in the international literature, certain studies remove the physics context from mechanics tasks, using a graphical representation to create calculus tasks (e.g., Christensen & Thompson, 2012). Their results “suggest that students have difficulties conceptualizing mathematics tasks that are common to the ways in which [they] ask questions in physics courses” (p. 5). However, we have found no example where essentially the same task is presented both with its physics context and without it. The tasks discussed in this paper do exactly this, and it is in that sense that we consider them to be “similar”.

## **Theoretical framework**

As stated above, we are interested in analysing practices related to the use of derivatives in two different courses (calculus and mechanics), in particular when solving a similar task in each discipline. We draw on the Anthropological Theory of the Didactic (ATD—Bosch et al., 2020), which considers human activities as institutionally situated. In ATD, knowledge is seen as embedded in practices which are conceptualised through the key notion of *praxeology*. Praxeologies are formed by a quadruplet  $[T/\tau/\theta/\Theta]$ , where  $T$  refers to a type of task to perform,  $\tau$  to a technique that allows the

completion of the task,  $\theta$  to a rationale (called “technology”) that explains and justifies the technique, and  $\Theta$  to a theory that includes the rationale. These four components form two blocks:  $[T/\tau]$  is the practical block that describes tasks and ways to solve them (*know-how*), and  $[\theta/\Theta]$  is the knowledge or *logos* block that describes, explains, and justifies what is done.

Additionally, ATD distinguishes between the *knowledge to be taught* (i.e., predetermined praxeologies that appear, for instance, in textbooks), the *knowledge actually taught*, and the *knowledge actually learned* (Bosch et al., 2020). In our analyses, we refer to our study of calculus and physics textbooks (identified as *knowledge to be taught*) and connect it with the students’ responses (as evidence of *knowledge actually learned*). Finally, we consider a task to be familiar when it is part of the *knowledge actually taught* (it is also generally part of the *knowledge to be taught*).

## Methodology

Our research is taking place at a large Canadian college (College A hereinafter). In Québec, students attend colleges after finishing high school and before entering university. Pre-university science programs are four-term, two-year programs. At College A, science students usually take their differential calculus and mechanics courses in their first term. On top of the *Regular Science* program (R), College A offers an *Enriched Science* program (E) featuring extracurricular activities, such as weekly seminars. Enrolment in the *Enriched Science* program is primarily based on motivation and interest. College A also offers two other programs (O): an *Explorations Science* program (consisting mostly of remedial courses) and continuing education classes at night or on the weekend.

During the fall of 2020, all courses were taught online due to the Covid-19 pandemic. That term, the college established two cohorts of students: one Enriched (EP, 37 students) and the other Regular (RP, 35 students). Each cohort attended calculus and mechanics courses together. A number of other students were not assigned to specific cohorts. The cohorted students followed the same curriculum as the non-cohorted students, with the calculus and mechanics teachers working in collaboration and sometimes attending each other’s classes. At the end of the term, between the final classes and the beginning of the examination period, we sent an online questionnaire to all science students at College A (approximately 1,200 students). We chose to use an online questionnaire despite its limitations, due to the pandemic and the fact that students were not physically present at the College throughout the 2020–2021 academic year.

The questionnaire was divided into three main sections. The first included questions concerning the student’s profile (the name of their program, whether they were attending a calculus and/or a mechanics course, etc.). The second section consisted of three contrasting questions addressing the students’ views on calculus and mechanics, inspired by Halloun (2004)’s Inventory of Basic Disposition. The third section included seven questions containing tasks to be solved, focusing on either calculus or mechanics. Among them were two pairs of similar questions, with each pair containing one calculus and one mechanics question. In this paper, we focus on one of these pairs (Question 4 and Question 7, see Figure 1).

Question 4 is considered a familiar question in mechanics. As our textbook analysis (Stage 1 of our research project) revealed, such tasks are part of the praxeologies developed in mechanics courses.

Question 7 is similar to Question 4: the velocity  $v$  is replaced by a function  $f$ , so that the acceleration  $a$  becomes the derivative  $f'$ . However, this “translation” comes with a few adjustments. We considered the absolute value of  $f$  in Question 7 for two reasons. First, *speeding up* and *slowing down* refer to the variation of speed, which, in one-dimensional mechanics (the content considered in our study), is the absolute value of the velocity. Second, without the absolute value, Question 7 could become a familiar task in differential calculus, likely prompting students to use learned calculus praxeologies. Another important difference: to avoid an explicit connection between both questions, we used different terms: “not moving” for Question 4 and “constant” for Question 7. We are also aware that “not moving” in mechanics means  $v = 0$ , whereas  $f$  constant allows for more possibilities for  $f$ . We did not deem this difference to be crucial in the resolution of the task.

Question 4	Question 7								
<p>An object, with velocity <math>v_x</math> and acceleration <math>a_x</math> is moving on a straight line.</p> <p>In each of the situations, state whether the object is <i>speeding up</i>, <i>slowing down</i>, <i>not moving</i>, or there is <i>not enough information to decide</i>.</p> <table border="1" style="width: 100%; text-align: center;"> <tr> <td><math>v_x &gt; 0</math> and <math>a_x &gt; 0</math>.</td> <td><math>v_x &gt; 0</math> and <math>a_x &lt; 0</math>.</td> <td><math>v_x &lt; 0</math> and <math>a_x &lt; 0</math>.</td> <td><math>v_x &lt; 0</math> and <math>a_x &gt; 0</math>.</td> </tr> </table> <p>For each of the situations, explain your reasoning.</p>	$v_x > 0$ and $a_x > 0$ .	$v_x > 0$ and $a_x < 0$ .	$v_x < 0$ and $a_x < 0$ .	$v_x < 0$ and $a_x > 0$ .	<p>In each of the situations, state whether <math> f(x) </math> is <i>increasing</i>, <i>decreasing</i>, <i>constant</i>, or there is <i>not enough information to decide</i>.</p> <table border="1" style="width: 100%; text-align: center;"> <tr> <td><math>f &gt; 0</math> and <math>f' &gt; 0</math>.</td> <td><math>f &gt; 0</math> and <math>f' &lt; 0</math>.</td> <td><math>f &lt; 0</math> and <math>f' &lt; 0</math>.</td> <td><math>f &lt; 0</math> and <math>f' &gt; 0</math>.</td> </tr> </table> <p>For each of the situations, explain your reasoning.</p>	$f > 0$ and $f' > 0$ .	$f > 0$ and $f' < 0$ .	$f < 0$ and $f' < 0$ .	$f < 0$ and $f' > 0$ .
$v_x > 0$ and $a_x > 0$ .	$v_x > 0$ and $a_x < 0$ .	$v_x < 0$ and $a_x < 0$ .	$v_x < 0$ and $a_x > 0$ .						
$f > 0$ and $f' > 0$ .	$f > 0$ and $f' < 0$ .	$f < 0$ and $f' < 0$ .	$f < 0$ and $f' > 0$ .						

**Figure 1: Question 4 and Question 7 of the online questionnaire**

Of the students who received the invitation to take the online questionnaire, 179 accessed it and 62 answered at least one of the questions in Section 3. Question 4 was answered by 27 students; of these 27 students, 23 also answered Question 7, while four students skipped Question 7 but completed the rest of the questionnaire. To preserve anonymity, we identified the participants with numbers, preceded by letters corresponding to their academic profile (“N” for students who had already passed differential calculus and mechanics, “O” for Explorations Science or continuing education, “R” for students in the Regular program, “E” for students in the Enriched stream, adding “P for students in a paired group). We conducted a thematic analysis of the students’ answers to each question, identifying the main elements of their praxeologies (mainly, their techniques and the rationales provided). We then cross-referenced the categories that emerged from our analysis of each question.

## Data analysis

### Question 4

Only a few participants had difficulties answering this question correctly (see Figure 2 or [Figure S1](#) for a more detailed distribution of the answers). Consistent with our analysis of textbooks, the data indicates that this content is part of the *knowledge to be taught*: the students’ praxeologies match the textbook praxeology, and we observe that the students use rationales present in their textbooks. For instance, one of the textbooks used in mechanics at College A (Serway & Jewett Jr, 2014), invites the reader to “think about the signs of velocity and acceleration by imagining a force applied to an object and causing it to accelerate” (pp. 32–33). Some examples of rationales are:

when  $a$  and  $v$  are the same sign, the object is speeding up. If they are opposite signs, the acceleration is against the movement, which means that the object is slowing down. (EP6)

when acceleration and velocity have the same sign then the object is speeding up and when they

are opposite signs (or their vectors are in the opposite direction) the object is slowing down. (N1)

We note that these rationales were also present in the mechanics teaching practices we observed in class. In this case, the *knowledge to be taught* also seems to be the *knowledge actually learned* by most of the participants.

### Question 7

In contrast with the results of Question 4, the results of Question 7 (see Figure 2 or [Figure S2](#) for a more detailed distribution of the answers) confirm that this is an unfamiliar question in calculus, with participants displaying greater difficulty in answering correctly. With the exception of one student who mentioned the first derivative test (RP13), the rationales used to support correct answers are common to rationales for Question 4 (referring to the sign or using the idea of (co)variation). Many participants develop techniques related to the use of rules based on the signs of the derivative, but only a quarter of them do so successfully. This could be because they have memorised the rules without truly understanding their meaning (which could in fact be understood using knowledge from mechanics). The majority of the students who use rules based on the signs of the derivative refer to the relationship between the sign of the derivative and the variation of the function (part of the *knowledge to be learned* in calculus), without clearly considering the absolute value of the function  $f$ . For example, RP5 states:

If  $f'(x)$  is bigger than 0, then it is increasing. If  $f'$  is smaller than 0, it is decreasing. The sign of  $f$  does not matter.

R3 goes further in her explanation, linking the sign of the derivative with the slope of the graphical representation:

[...] This is because a function will have a positive slope when it is increasing, and therefore a positive derivative. The opposite applies for decreasing.

Here also, the sign of the function, and therefore its absolute value, is not taken into account in the student's reasoning.

### Cross-referenced results of Questions 4 and 7

Figure 2 provides an overview of the answers to Questions 4 and 7. To simplify the coding, each response was coded in only one category. We can see that of the 24 students who answered Question 4 correctly, only seven answered Question 7 correctly. Additionally, none of the students who failed to answer Question 4 correctly was able to answer Question 7 correctly.

In both questions, when students refer only to the sign(s) or to vector direction (e.g., EP5, O2), the rationale is relatively poor, as it comes down to the technique itself. For instance, EP3 wrote: "In mechanics, we saw that whenever the acceleration is of opposite sign to the velocity, the object was slowing down". We observe that this type of rationale, typical of mechanics courses, allows students to be successful in the familiar task (Q4), but it mainly leads to failure in the unfamiliar task (Q7). These findings are consistent with those of our textbook analysis (Hitier & González-Martín, 2021), as well as with other studies on the use of calculus in other disciplines, such as engineering. Many results that depend on calculus are proved once and then taken for granted, indicating that students

may learn the explanations by heart without fully understanding them (e.g., Faulkner et al., 2020; González-Martín, 2021; Hitier & González-Martín, 2021).

Question 7	Question 4							Total
	✓ Idea of variation	✓ Idea of acting on movement	✓ Referring to vector directions	✓ Referring only to the signs of $v$ and $a$	✦ "Not enough information" in the case $v < 0$ & $a > 0$ *	⚡ Depending only on the sign of $a$	✗ "Not enough information" for all cases*	
✓ Idea of (co)variation, taking the absolute value into account	R2	O3						2
✓ First derivative test, taking the absolute value into account			RP13					1
✓ Referring only to the signs of $f$ and $f'$ (correct)			N1	EP3 R4 RP7				4
✦ Referring only to the sign of $f'$ (correct-inconsistent)						E1		1
✦ Referring only to the sign of $f'$ (incorrect-inconsistent)				EP5				1
✦ Constant in the case $f < 0$ and $f' > 0$			N2					1
⚡ Referring only to the sign of $f'$ (incorrect)		RP5 RP6 EP6	O2 RP8 RP14	N3 EP7				8
⚡ Graphical argument		EP2	R3 RP11					3
⚡ Concavity mixed with variations				O1				1
⚡ Unclear explanation		RP10						1
-- No answer				RP9 RP12	EP4		R1	4
<b>Total</b>	1	5	9	9	1	1	1	27
			24		1	1	1	

\* No rationale given in the response.

**Figure 2: Summary of responses for Question 4 and Question 7**

We also note that although the students were not asked to make explicit connections between Questions 4 and 7, two students (R2 and, to a lesser extent, RP7) did so. For Question 4, R2 and O3 provided the most detailed rationales. Remarkably, R2 does not use the vocabulary *speeding up* or *slowing down* but provides explanations in terms of the variation of the absolute value of the velocity (the only participant to do so). Below is an excerpt of her rationale:

for i and iii: since both  $v_x$  and  $a_x$  are in the same direction, there is no change in direction, so there is no decrease of the absolute value of velocity. [...]

iv: the absolute value of the velocity will decrease until zero, then the velocity will increase in the positive direction. (R2)

Her ability to rephrase Question 4 in terms of the variation of the absolute value of the velocity might have allowed her to identify the link between the two questions. Regarding Question 7, her rationales are:

i and iii: since  $f$  and  $f'$  have the same sign, the abs value of  $f$  will continue to grow;  $f'$  will not subtract from the absolute value of  $f$ .

ii and iv: since  $f$  and  $f'$  have different signs, the abs value of  $f$  will decrease until it reaches zero. After that point,  $f$  will increase in the direction of  $f'$ . (R2)

In her explanation, we see the expression " $f'$  will not subtract from the absolute value of  $f$ " as hinting at the "idea of acting on movement" that we found in the other students' rationales for Question 4.

We believe that this provides evidence, as observed by Roorda et al. (2015), that students are able to establish certain links between the knowledge learned in the calculus and the physics courses.

## Final remarks

In this paper, we analyse students' solutions to two tasks: a familiar mechanics task and a similar yet unfamiliar calculus task. As in the work of Marrongelle (2004), we observe certain students using praxeologies from mechanics to solve the calculus task. However, those students make up only a small fraction of the study's participants. Even if we consider that the students who provided correct answers to Question 7 (*referring only to the sign of  $f$  and  $f'$* ) might also have seen the link between the two questions without mentioning it explicitly, they still represent less than a quarter of our participants. In fact, our analyses tend to indicate that, for the calculus task, most students develop techniques based on results from calculus without giving any real sense to them. These techniques allow students to solve familiar calculus tasks but seem to lead them to failure when faced with unfamiliar tasks. Connecting these results with our textbooks analyses (Hitier & González-Martín, 2021), we see important implications for the teaching of calculus and mechanics. Praxeologies in calculus seem to foster the development of practices whereby results and formulae are learned without necessarily connecting them with a physical meaning or an interpretation. On the other hand, although derivatives are used in mechanics, the praxeologies rely heavily on given formulae and the link with calculus is obfuscated. Figure 2 shows that the small number of successful rationales in mechanics does not translate to successful rationales when solving a similar calculus task. This sheds some light on students' difficulties in transferring knowledge between mathematics and physics, as discussed in the introduction. These difficulties may be related to praxeologies in both disciplines, which do not establish explicit connections between the two. This is consistent with Planinic et al.'s (2019) view that "students' almost exclusive reliance on formulas in physics presents [...] an important obstacle for the development of students' deeper reasoning in physics and sometimes even an obstacle for the application of their already existing knowledge and reasoning developed in other domains" (p. 243). In this vein, Taşar (2010) also noted that the transfer of knowledge between the two domains seems problematic. We believe our results help to reveal institutional reasons for these difficulties in transferring knowledge between the two disciplines, as a consequence of institutional choices to organise separate praxeologies in each one.

In the previous paragraph, we mentioned the possibility that students may have seen the link between Questions 4 and 7 without mentioning it. The fact that the questionnaire was completed online could have affected the students' ability to develop their rationales. We acknowledge this limitation and we hope to gain further insights through the analysis of the students' interviews. This analysis, as well as other parts of our study, will be the focus of future publications.

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