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# Preschool children estimating lengths – the role of standard units as relevant prior knowledge

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*Length estimation is a relevant ability in our everyday life. Especially during the Corona Pandemic, estimating a distance of 1.50 m is crucial for our health and social life. Standard units play a central role in many length estimation processes that occur in our everyday surrounding. However, standard units as well as approaches to length estimation are generally discussed with children in elementary school while children already face situations already in their preschool age. This research addresses the question of how preschool children deal with length estimation. In a study with 189 preschool children who were about to start elementary school, we assessed their length estimates and analyzed the role of standard units in this context. It shows that standard units are used by the majority of kids to estimate lengths. However, their estimates are far from precise but the relation between the measures they name is mostly correct.*

*Keywords: Length estimation, estimation strategies, preschool education, prior knowledge.*

## Introduction

Estimating lengths as precise as possible is one main ability that becomes relevant in various situations of our everyday life. Especially in traffic there are many situations that require length estimation – the GPS system gives directions in standardized measures or there are zones with reduced speed for a given length. Children get in touch with length estimation in several situations as well. Reading books about dinosaurs holds several situations about length estimation as well as length comparison: the tooth of a Tyrannosaurus Rex was about 15 cm long which is about as long as a banana while the dinosaur itself could reach a length up to 12 m. In these books there are several pictures showing the relation of lengths between the dinosaurs and objects that are known from today. Another example situation where children are faced with length estimation is when they are baking cookies with their parents who have to place the cookie dough in a certain distance from one another or when they are travelling and they ask their parents how much longer they have to drive, they might get a response about the remaining kilometers. These examples are just a small selection of all the situations where children meet length estimation requirements. The majority of these situations also hold standardized units of lengths. Many of these situations already occur before the children start their school career meaning that they necessarily start school with prior knowledge about how to estimate lengths and a variation of intuitive approaches. As the examples already show, there are different estimation situations that require different approaches: Estimating a length in a standardized measure – such as: how many centimeters is this sheet of paper long? – requires knowledge about the standard unit centimeter, about scaling, conceptions about this specific measure as well as knowledge about the measuring process. In contrast, when asking children about what is as long as a Tyrannosaurus Rex, they might just provide a reference object (e. g. a truck) while they do not necessarily need prior knowledge about standard units but rely on direct comparisons instead. If

precise concepts about standard units are still missing, this will affect the length estimation's approach as well as its success. It is an open question, to what extent preschool children use standard units for length estimation and how their estimation results are affected if their concepts of these units are still imprecise. With this research we want to contribute information about this point of children's early years' mathematics in this field.

## **Measurement estimation**

Measurement estimation is defined by Bright (1976, p.89) as "the process of arriving at a measurement or a measure without the aid of measuring tools. It is a mental process though there are often visual or manipulative aspects to it". In the process of estimating measures such as lengths, there is specific knowledge and certain skills (such as strategies) that become relevant in order to reach a precise estimation (D'Aniello et al., 2015; Sowder, 1992; Joram et al., 1998). This is supposed to be knowledge about physical measurement (Joram et al., 1998), types of measures or basic conceptual knowledge that can be used as a reference to estimate (Sowder, 1992). Coming back to the dinosaur example from the beginning, children have to know the length of an adult man in order to estimate the height of a dinosaur that is pictured right next to him. In this case, the knowledge about a man's height functions as a reference in the estimation process. Alternatively, they might have a precise concept about a specific standard unit (such as meter) and use this knowledge to estimate the length of a man by counting how many units fit the length of the person. In these examples, we described the two main strategies that are differentiated in the literature for length estimation (1) benchmark comparison and (2) unit iteration (Joram et al., 1998)<sup>1</sup>. Empirical research shows that students from higher grades estimate lengths and areas more precisely than students from lower grades (Huang, 2014; Desli & Giakoumi, 2017).

This theoretical part about the process of length estimation shows the relevance of knowledge about standard units (e. g. the strategy unit iteration can only be used if standard units are known and concepts about their lengths available). In addition, if children encounter length estimation in their everyday life, the estimate will generally be given in a standard unit. Therefore, we put a focus on the role of prior knowledge about standard units for length estimation in this research.

### **(Prior) knowledge about standard units in the process of length estimation**

Measurement generally bridges two mathematic fields: geometry / spatial relations and real numbers (Clements, 1999). Geometrical objects are assigned with a size indication. This consists of a number and a unit. Understanding the various concepts belonging to length measurement (including the meaning of size indication and unit) is a central part of mathematics education. Referring to the developmental theory of measurement by Piaget et al. (1960), it is generally taught in several steps, starting with direct comparisons followed by indirect comparison with a non-standardized unit and, finally, with a standardized unit. This approach is generally chosen in order to reveal children the need for standard units (Clements & Stephan, 2004). However, there is critique about this approach

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<sup>1</sup> There is a third approach classified in the literature – the decomposition / recomposition approach – which is also based on either unit iteration or benchmark comparison (e. g. Joram et al., 1998).

(Clements, 1999) because children (already in preschool) prefer to use standard units comparing objects and they are even more successful using standard units to measure as compared to measuring with non-standard units (Boulton-Lewis et al., 1996, Kotsopoulos et al., 2015). Empirical studies show that preschool children are already able to use standard units. Boulton-Lewis et al. (ibid.) observed that they used it even if they do not understand it. In this regard, Barret et al. (2011, p. 638) summarize that “students’ unit concepts are rarely well formed. Although students may use unit labels to name a quantity, they often do so without being able to show the meaning of the relevant unit.” Therefore, we find some evidence that preschool children already have prior knowledge about standard units although – as Boulton-Lewis et al. (ibid.) point out – they do not understand them and may not develop conceptual understanding about their lengths and relations to one another yet. However, this understanding may become relevant in the process of length estimation. As Desli and Giakoumi (2017) analyze, third as well as fifth grade students estimated more successful if they used nonstandard units instead of standard units. Therefore, we aim to analyze how this (however incomplete) prior knowledge about standard units may affect their length estimation efforts.

Bringing the ideas about length estimation and unit concepts together with regard to preschool children and their previous experiences, it can be hypothesized that they are missing relevant knowledge in certain situations. Overall, neither length estimation nor standard units were systematically discussed at that point of their education, meaning that they might choose intuitive approaches that they either observed in their surrounding or developed during that situation. For example, due to the fact that they generally did not yet discuss the different standard units and their relations to one another, providing length estimations in a standard unit may not be possible. Therefore, this study focuses on the question of how preschool children deal with different situations of length estimation and what role standard units play in these approaches.

## **Research questions**

As the literature review indicates, preschool children prefer to use standard units for measurement purposes. However, there is some evidence that their unit concepts as well as their conceptual knowledge about these standard units are still incomplete but may be relevant in order to estimate lengths in a standard unit. Therefore, we focus the following two research questions:

1. To what extent do preschool children use standardized units to estimate lengths?

In order to gain information about how the preschool children’s understanding of standard units affects their length estimates, we additionally focus on the estimations’ accuracy. If children only estimate a number and add a standardized unit that they know about, their estimates will be somewhat imprecise. Therefore, the second research question focuses this aspect.

2. How precise are the preschool children’s length estimates?

## **Methodological approach**


In order to analyze the role of standard units in preschool children’s length estimates, we conducted short interviews with standardized questions in Kindergarten with those children who were about to start elementary school only a couple of months later (school enrollment in summer 2021).

## Interview design and administration

Four trained student teachers who interviewed the children individually conducted the interviews. Children's responses were assessed on a documentary sheet. The student teachers worked with a standardized interview script that held text about what the students should say as well as specifications about what kind of gestures they should use and what kind of material is given. The interview contained a couple of objects whose lengths should be estimated by the children. The children were asked in each of the five cases to estimate how long the object might be. The questions were intentionally held as open as in order to not limit children's responses and / or strategies. Of course, all the to-be-estimated-objects were standardized in their lengths as well.

In total, the children were asked to estimate the lengths of five objects. For item construction, we followed the suggestions by Heinze et al. (2018) and varied the different characteristics regarding the To-Be-Estimated-Object (TBEO)<sup>2</sup>, e. g. its physical presence, accessibility, size and need to construct a representation. Table1 shows one example item of the interview.

**Table 1: Task example and excerpt from the interview guide**

	<b>Material</b>	<b>Material treatment</b>	<b>Question / verbal instruction</b>
T2 <sup>3</sup>	Green brick (see picture) 	Place the green brick in front of the child.  Move your finger along the longest side of the brick.	What do you think: How long is this brick?  <i>Wait for the response.</i>

The other four items were constructed similarly. In T1, children were asked to estimate the length of a rope, T3 asked the children to estimate the height of a carton of milk, children were asked to estimate the height of the door in their kindergarten classroom in T4 and, finally, were asked to draw a line as long as a piece of toilet paper in T5.

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<sup>2</sup> Benchmarks were intentionally not given (dimensions 5 & 6 by Heinze et al., 2018) and it was not prescribed whether children should estimate in a standard or non-standard unit (dimension 7) in order to leave the preschool children the choice what kind of approach they want to choose (they might provide an estimate in a standard unit but might as well just name an object that is about as long as the TBEO). However, the dimensions regarding the TBEO were varied in the five questions.

<sup>3</sup> T2 means second task. In tables 2 and 3, all entries in the line called 'T2' refer to this task.

## Sample size and characteristics

In total, 189 preschool children participated in the study. All of these children were about to start school only about two months after the interview. The children attended 14 different Kindergartens in the north of Germany – more precisely in the areas of Mecklenburg-Vorpommern and Schleswig-Holstein. 78 of the children are girls (41.3 %) and 111 boys (58.7 %). The children were averagely 5.99 years old – there were 171 children in the age of 6 years, only nine kids were five years old and seven children were seven years old<sup>4</sup>.

## Data analysis

In order to evaluate the amount of children who use standardized unit in length estimation (research question 1), we determined the absolute and relative frequency of children in our sample that gave an answer holding a standardized unit. In order to analyze the role of standard units in children's length estimation processes (research question 2), and how precise children's concepts about standard units are, respectively, we analyzed the accuracy of children's estimates. For this second analysis, we only used the sub-sample of those children who estimated in a standardized unit because we cannot evaluate whether estimates such as 'the brick is as long as my hamster at home' is true without measuring the length of the hamster. In order to analyze the estimates' accuracy, we used the deviation of the child's estimate and the actual length of the TBEO. We clustered the children's results in the intervals that are generally chosen in research to code the accuracy of length estimates: if the estimate deviates equal or less than 10 % from the actual length, we consider the estimate as very precise. A deviation of  $10 \% < x \leq 25 \%$  is still somewhat precise while a deviation between  $25 \% < x \leq 50 \%$  is acceptable. If the estimate deviates between  $50 \% < x \leq 100 \%$  from the object's actual length, it can be considered imprecise. Finally, if the estimate deviates more than 100 %, the estimate is very imprecise. In order to gain additional information about the children's approaches and estimation accuracy, we analyzed whether the numerical values fit the right relation to one another. In this regard, we converted all given estimates into meters and evaluated whether the children gave adequate order relations between the measures.

## Results

Regarding the four questions that required the children to estimate a length<sup>5</sup>, it shows that a majority of the preschool kids use standard units for their length estimates. Due to the fact that a systematic introduction to length units occurs generally in second grade in Germany (Franke & Ruwisch, 2010), these results give another indication that children start with a great variety of prior knowledge in this field. Table 2 shows the amount of children using standard units for their length estimates, and what kind of unit they chose, respectively.

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<sup>4</sup> For two children, the age is missing.

<sup>5</sup> In T5, children were asked to draw a line that is about as long as a piece of toilet paper. Therefore, standard units were not named here.

**Table 2: Amount of children using standardized units for their length estimates**

	Amount of estimates given in mm	Amount of estimates given in cm	Amount of estimates given in m	Amount of estimates given in km	Total amount of estimates given in a standard unit
<b>T1</b> (length of TBEO: 1.3 m)	1 (0.5 %)	18 (9.5 %)	119 (63 %)	6 (3.2 %)	144 (76.2 %)
<b>T2</b> (length of TBEO: 6.2 cm)	4 (2.1 %)	43 (22.8 %)	90 (47.6 %)	5 (2.6 %)	142 (75.1 %)
<b>T3</b> (length of TBEO: 20 cm)	4 (2.1 %)	41 (21.7 %)	87 (46 %)	6 (3.2 %)	138 (73 %)
<b>T4</b> (length of TBEO: 2 m)	2 (1.1 %)	17 (9.0 %)	118 (62.4 %)	6 (3.2 %)	143 (75.7 %)

**Table 3: Accuracy of preschool children's length estimates in T1 – T5**

	Actual length of the TBEO	Range of the children's responses	Children with a difference < 10 %	Children with a difference 10-25 %	Children with a difference 25-50 %	Children with a difference 50-100 %	Children with difference > 100 %
T1	1.3 m	<i>Min</i> =10 cm <i>Max</i> =100 km	0	44 (30.6%)	0	32 (22.2 %)	68 (47.2%)
T2	6.2 cm	<i>Min</i> = 0.5 cm <i>Max</i> = 14 km	3 (2.1%)	6 (4.2%)	14 (9.9%)	18 (12.7 %)	99 (69.7%)
T3	20 cm	<i>Min</i> =0.8 mm <i>Max</i> =10000 km	6 (4.3%)	6 (4.3%)	6 (4.3%)	31 (22.5 %)	89 (64.5%)
T4	2 m	<i>Min</i> =10 cm <i>Max</i> =100 km	15 (10.5%)	3 (2.1%)	15 (10.5%)	25 (17.5 %)	85 (59.4%)
T5	12 cm	<i>Min</i> =2.5 cm <i>Max</i> =25 cm	7 (3.7%)	26 (13.8%)	29 (15.3%)	8 (4.2%)	70 (37%)

However, if we focus on the accuracy of the estimates (table 3), it shows that only a small amount of children reached an estimate that differs less than 50 % from the actual length of the object. It shows that the estimate of the majority of the children (generally about half of the sample) varies more than 100 % of the actual length while only some children estimated very precisely (discrepancy < 10%).

These results show that preschool children are aware of standard units and use them in length estimation situations to provide the lengths of different objects. However – as table 2 shows – they may not choose the different length units most appropriately and their length estimates are not very precise (table 3). This may indicate that they do not yet have precise conceptions about the standard units. The analysis about the order relation showed that 161 kids (79.3 %) gave all four estimates in the correct order (however we did not check for proportional differences). This indicates that preschool children are already able to assign objects with numbers that are given in the correct order relation. However, they may just add the name of a standard unit without really understanding their meaning.

## **Discussion**

Since children get in touch with lengths as well as situations of length estimation in their everyday life, it can be assumed that children bring a lot of prior knowledge and skills in this field when they start elementary school. It is crucial for teachers to be aware of these preconditions in order to start efficient learning processes. The results of this study give hints about these starting points. It becomes apparent that the majority of preschool children provides an estimate using standard units, however, the estimates are often not very precise. This is in line with previous research about students' estimation accuracy and its connection to students' age (e. g. Huang, 2014; Desli & Giakoumi, 2017). However, if we focus on the relation between the estimated values, almost all children's estimates are in a correct order relation to one another. This may indicate that children know about standard units and use them in length estimation contexts but their understanding of the units is still incomplete (as indicated by the imprecise estimation). Therefore, the majority of children uses standard units to estimate lengths even though their understanding is not yet sufficient. These results closely connect to the findings of Boulton-Lewis et al. (1996). Supposedly, children merely estimate a real number and simply add a standard unit that they have heard of before.

Of course, there are limitations to this study. From our results we cannot reconstruct what exactly the children thought in their process of length estimation. We can only hypothesize their approaches from our results. In addition, the Kindergarten teachers may have talked with the kids about standard units before we arrived in their classes (even though we asked them not to do that) because the amount of kids using standard units is amazingly high.

Overall, we suggest that students' prior knowledge should be closely considered as a starting point of learning processes. Further research should provide evidence about how different starting points combined with different learning approaches lead to specific learning outcomes in the field of length estimation.

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