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Didactic transposition of natural numbers in the first year of compulsory schooling: a case of comparative curricula analysis

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The first year of school has recently become mandatory in Denmark, and for the 5 to 7 age group, a national curriculum for mathematics has been designed. This study aims to present an analysis of the didactic transposition of natural numbers in the first year of school to foster reflection on the national curriculum design. The focus is on the didactic transposition from scholarly knowledge into knowledge to be taught occurring in the national curricula of three countries: Denmark, Sweden, and Australia. The methodological approach is curriculum mapping using a table. The results show that the main elements of scholarly mathematics related to natural numbers on a sector level are identified in all three curricula; still, analysis on a theme level shows significant differences.

Keywords: Didactic transposition, the first year of compulsory schooling, comparative curricula, natural numbers.

Introduction

Guidelines have existed for many years in early education, but curricula for preschool on a national level in OECD countries are a newer creation (Samuelsson et al., 2006). In Denmark, the first year of school, which forms a transition year, changed from voluntary to mandatory in 2009. In 2014, the Danish Ministry of Education presented a national curriculum for compulsory schooling (Retsinformation, 2014). One subject/competence area was mathematics. Play constitutes a critical element in teaching, and teaching should be based on several subjects/competence areas simultaneously, e.g., mathematics and commitment and community. Comparing this national curriculum with other international curricula may provide a broader insight into international perspectives on the knowledge to be taught designed for early years mathematics. This study aims to analyse the didactic transposition of natural numbers from scholarly knowledge into knowledge to be taught occurring in three national curricula covering the first year of compulsory schooling. On this background, the research question is: How can scholarly mathematics be used to analyse aims related to natural numbers for the first year of compulsory schooling in the national curricula from three different countries: Denmark, Sweden, and Australia?

Theoretical framework

The framework is built on the theory of didactic transposition and central theories of young children's development of different aspects of natural numbers.

Didactic transposition

The theory of didactic transposition is anthropological as it considers knowledge as “a changing reality embodied in human practices taking place in social institutions” (Chevallard & Bosch, 2014, p. 173). A systematic epistemological and institutional approach is employed to study these knowledge activities as they undergo a process from produced, designed to be taught, actually taught in school, and learned by students. When bodies of knowledge developed in one social institution are

transposed to another social institution, they undergo a transformation, deconstruction, and reconstruction to adapt to their new social institution (Chevallard & Bosch, 2014). Chevallard and Bosch (2014) list four social institutions involved in the didactic transposition: Scholarly knowledge, knowledge to be taught, taught knowledge and learned knowledge. The scholarly knowledge, e.g., aspects of natural numbers, is the starting point of the process. Chevallard and Bosch describe scholarly knowledge as "...generally produced at universities and other scholarly institutions, also integrating elements taken from a variety of related social practices" (2014, p. 171). The transposed work from scholarly knowledge to knowledge to be taught is chosen and decided by actors who belong to the "noosphere," meaning "the sphere of those who "think" about teaching" (Chevallard & Bosch, 2014, p. 170). These actors can, for example, be researchers, employees in the ministry of education, and teachers/educators. A national or local curriculum and school textbooks governed the knowledge to be taught. The teacher's task is to transform the knowledge from national/local curriculum and textbooks into taught knowledge. Finally, the last part of the transposition is learned knowledge acquired by students. This study focuses only on the transposition of the scholarly knowledge into knowledge to be taught, also named the external didactic transposition (Bosch & Winsløw, 2020), and more specifically, the transposition of knowledge concerning natural numbers.

Natural numbers

Natural numbers constitute a broad research field in mathematics education; therefore, the total content elements on natural numbers of the three chosen curricula for this study have given a direction of a relevant theoretical framework. Scholarly knowledge on young children's development of natural numbers carried out by Fuson (1998) and Gelman and Gallistel (1978) is considered to be an essential part of the theoretical framework.

Historically, the Hindu-Arabic system of numerals, widely used today, has developed over many years from the tally system as a unary system to the positional numerical systems built around the base 10 (Sun et al., 2018). Conceptual development progress of the numeral systems can be grouped: "the tally system, additive system, multiplicative-additive system, and decimal place value system" (Sun et al., 2018, p. 96). This study concentrates primarily on the first group: the tally system connected with young children's learning of the system. A central origin in scholarly mathematic knowledge for young children, considering natural numbers, can be found in the notions of cardinality and ordinality.

Early number consists of a network of inter-related skills and knowledge, broadly divided into cardinal and ordinal aspects, that is, those concerned with number as a representation of quantity, and those concerned with number as a representation of position in a sequence. (Bruce & Threlfall, 2004, p. 3)

Concerning ordinal aspects, Siegler (2009) states that the most basic ordinal concepts are more and less. The cardinal aspects of number include determining quantity by counting or by subitising. Subitising is the ability to instantly recognise the cardinal value of a small set of objects without counting them (Bruce & Threlfall, 2004; Fuson, 1988; Siegler, 2007). When children are 3-4 years old, they become proficient at establishing the cardinal value of a set by counting (Siegler, 2007). However, children saying a number sequence or string is not necessarily bound to cardinality or

ordinality. Fuson (1988) states that children can repeat number sequences only as a verbal activity, independent of cardinality and ordinality. There must be a shift from counting to numerical meaning for children counting to be cardinal or ordinal. Five developmental stages in young children's ways of counting are outlined: The number sequence: 1) as a string 2) as an unbreakable list (one, two, three ...) 3) as a chain that can be broken 4) as a numerable chain 5) as a bidirectional chain where the forward- and backwards- counting is related to each other.

Decomposition and composition or part-whole number knowledge are stressed to be fundamental to developing a deep understanding of arithmetic. Both cardinal and ordinal situations are related to numerical sequence, i.e., when a child knows that there are $n-1$ entities in the cardinal set, which precedes the n^{th} ordinal entity (Fuson, 1988)

Gelman and Gallistel (1978) identified a set of principles that preschool children at age 5 possess when counting. The first three principles, one-one, stable order, and cardinality, focus on how the process of counting is carried out. The last two principles, abstraction and order irrelevance, focus on what to count and define what can be counted.

To communicate effectively using numerals, children need to know three numerical representations, i.e., number words, numerals, and nonsymbolic quantities and understand each representation and how a representation maps to other representations (Hurst et al., 2017).

Method

Curriculum mapping

Curriculum mapping refers to the method for developing and employing a curriculum map "A curriculum map is a visualisation of relationships within and between a curriculum or curricula" (Greator et al., 2019, p. 3). This study follows the 6 key stages of the curriculum mapping method for comparability research: "1. Define study aims and use; 2. Decide which curricula will be considered; 3. Determine the curriculum features that will be the basis of comparison; 4. Collect relevant documentation and sources of data; 5. Extract data and input it into the standard instrument; 6. Consolidate findings through visual representation" (Greator et al., 2019, p. 5). However, changes concerning key stage 3 are made as the features that will be the basis for this study are not solely the curricula but scholarly mathematics/theoretical framework. As stated in the theoretical framework section considering natural numbers, the content of the table is not designed independently from the three curricula to be analysed. In this study, the 6 key stages are covered in the paper sections: key stage 1, introduction; key stages 2 and 3, selection and context; key stages 4, 5 and 6, results. Based on the theoretical framework, a table has been designed.

Selection and context

This study analyses aims related to natural numbers learning for young children in the national curricula from three different countries Denmark, Sweden, and Australia. The selection of those three countries' curricula is based on similarities and differences in their early childhood education. The school system in Denmark and Sweden is similar about a transition year from kindergarten to school and a strong cultural emphasis on play. Australia differs from Denmark and Sweden as the country follows a British school tradition focusing on formal learning and less on play (OECD, 2006). The

data collection comprises what three curricula say on natural numbers for age 5-7 years, from Denmark, Sweden, and Australia. Natural numbers related to other content areas of a curriculum, e.g., algebra/pattern and measurement, are not included in the analysis of the study. Mathematical processes such as, e.g. proficiency strands of problem-solving are included if they involve natural numbers. The Danish national curriculum includes learning objectives and teaching instructions (Danish Ministry of Education, 2019). The Swedish national curriculum consists of core content and competencies (Skolverket, 2018). The Australian national curriculum includes content descriptions and proficiency strands related to natural numbers (Australian Curriculum, Assessment and Reporting Authority, 2016). Children must have begun compulsory schooling in all three countries by turning six years of age. In Denmark and Sweden, the children are five-and-a-half to six years of age when they start compulsory schooling. However, some children start compulsory schooling in Australia when they are four and a half years of age. In the three countries, the children attend the class for one year. The terminology used to show the first year of compulsory schooling in Australia is "foundation year," in Denmark "kindergarten class" and in Sweden "preschool class". In the three countries, the teachers or the educators below first grade are generalists with 3½-4 years of education at bachelor a level.

Terminology

Following Bosch and Winsløv (2020), terminology of levels has been applied to organise this table appropriately concerning the data collection (curricula). The levels are domain, sector, and theme. The mathematical domain is natural numbers; the sectors are cardinal aspects; ordinal aspects; numerals as names. The themes are subsectors, e.g. the theme one-one correspondence is a subsector of cardinal aspects. The table thus also forms a reference model with an explicit description of relevant scholarly knowledge of the domain natural numbers, which is used as a reference for analysing the presence or absence concerning the content of natural numbers on sector and theme levels. Explicit presence is marked in the table with a code, and an empty box shows absence. Based on the table and code extracts, the three curricula are shown. Finally, a comparison regarding similarities and differences is made.

Results

Table 1 shows the mapping of the analysis of the three curricula with a code: a letter and a number indicating the presence of scholarly knowledge/theory in the form of theme and the belonging sector in the current domain.

Table 1: Comparative curricula analyses – presence and absence of sectors and themes

Domain: Natural numbers				
Sector	Theme	Danish curriculum	Swedish curriculum	Australian curriculum
Cardinal aspects	The one-one correspondence	D1	S1	A1
	The stable order	D1	S1	A1
	The cardinality		S1	A1
	Order irrelevance			
	Abstraction			
	Subitising			A2
	Composing and decomposing (whole-part)		S1	
	Early arithmetic	D2		A3
Ordinal aspects	The ordinal property “more numerous” (more and less)	D3		A4
	The relative magnitudes of numbers	D4	S2	A4
Numerals as names	Saying the number names	D5	S1	A1
	Connect number names, numerals, and quantities	D6		A5

Below are content extracts from the three curricula, informing the coding decision shown and supplemented with an explanation, if necessary. The content extracts are listed in the native language and translated into English where necessary. The native content extracts are stated in *italics*.

D1) *Har viden om metoder til antalsbestemmelse*/knows methods for counting. Methods for quantification are not explicit in the curriculum, but the first two principles of counting, as one-one correspondence and stable order, are fundamental. Regarding Fuson (1988), children may count but do not make a cardinal integration; therefore, the third principle (cardinality) is not coded.

D2) *Regnehistorier, der rummer et problem, som eleven skal regne på*/problem solving with early arithmetic in a real-world context.

D3) *Bruge forklaringer med ord som større og mindre, når der arbejdes med antalsbestemmelser*/uses explanations, which include terms as more and less when working with number sets.

D4) *Har viden om talsymbolerne og deres ordning*/knows the number symbols and their order.

D5) *Kan læse etcifrede naturlige tal*/can read single-digit natural numbers.

D6) *Forstå sammenhængen mellem mængde, antal, talord og talsymbol*/understands the relationship between quantity, number, number word and number symbol.

S1) *Naturliga tal och deras egenskaper och hur de kan användas för att ange antal and ordning*/Natural numbers and their characteristics and how they can be used for counting and order. Concerning natural numbers, the Swedish curriculum only contains 24 words. The sentence of cardinality is interpreted to include the three principles of how to count one-one correspondence; stable order; cardinality (Gelman & Gallistel, 1978).

S2) *Del av helhet och del av antal*/part of a whole and part of a quantity.

A1) *Establish understanding of the language and processes of counting by naming numbers in sequences, initially to and from 20, moving from any starting point. A point marks a position but has no size. Understanding that numbers are said in a particular order, and there are patterns in the way we say them.*

A2) *Subitise small collections of objects. Using subitising as the basis for ordering and comparing collections of numbers.*

A3) *Represent practical situations to model addition and sharing. Estimating and calculating with whole numbers.*

A4) *Compare, order and make correspondences between collections, initially to 20, and explain reasoning. Comparing and ordering items of like and unlike characteristics using the words “more,” “less”... and giving reasons for these answers. Understanding and using terms such as “first” and “second” to indicate ordinal position in a sequence.*

A5) *Connect number names, numerals and quantities, including zero, initially up to 19 and then beyond. Estimating and calculating with whole numbers. Understand and use number in context.*

A comparison of the three curricula regarding central similarities and significant differences is outlined. Regarding similarities, all three sector levels (cardinal aspects, ordinal aspects, and numerals as names) were identified in the three curricula. This means that the main elements of natural numbers concerning scholarly mathematics are preserved in all three curricula or the knowledge to be taught. None of the curricula has explicitly included scholarly mathematics of order irrelevance and abstraction on the theme level. Only the Swedish curriculum has explicitly included the theme composing and decomposing. Only the Australian curriculum has explicitly included the theme subitising. The Australian curriculum has explicitly included more themes (9) than the Danish curriculum (7 themes) and the Swedish curriculum (6 themes). The themes indicate that the

Australian curriculum is more detailed and precise and goes closer to practice than the Danish and Swedish curricula. Concerning the levels of didactic transposition regarding the content, the Danish and Swedish national curricula tend to be organised at the sector level. In contrast, the Australian national curriculum tends to be organised at the theme level.

Discussion and conclusion

The present study sought to answer the research question on how scholarly mathematics can be used to analyse aims related to natural numbers for the age 5-7 years in the national curricula from three different countries. The study shows that the main elements of scholarly mathematics related to natural numbers on the sector level are identified in all three curricula. Analysing and identifying the aims of the curricula elements related to natural numbers on theme level call for some validity considerations because the designs of the three curricula differ. A significant difference is the detailed design of the Australian curriculum versus the compact design of the Swedish curriculum, while the design of the Danish curriculum is in between. According to Greateorex et al. (2019), the level of detail of curricula data varies, which can influence the usability of data. In the present study, the varying degree of detail makes the selection criteria in the analysis "explicit presence" challenging. The challenge is addressed by informing the coding decision, e.g., the Swedish curriculum where the sentence of cardinality is interpreted to include the three principles of how to count (S1). Furthermore, this study is constrained to the national curricula, while other documents of the knowledge to be taught, e.g., school textbooks, may elaborate on these content areas.

When the scholarly knowledge of natural numbers is transposed into three national curricula, the transformation, deconstruction, and reconstruction (Chevallard & Bosch, 2014) of each curriculum are linked to the educational culture and tradition of the current country (Greateorex et al., 2019). Australia has a longer curriculum tradition within early childhood education than Denmark and Sweden have. The contrasting can foster reflection on the two relatively compact Scandinavian curricula, especially considering the next step of the didactic transposition from knowledge to be taught into taught knowledge. It is the teacher's task to transform the knowledge from the national curriculum into taught knowledge, and it is an open question to what extent she needs scaffolding.

The present study has made some of the cultural differences of the didactic transposition of natural numbers on the national curricula level transparent. Thus, the curricula analyses have provided reflections on different ways to select and design knowledge of natural numbers to be taught in early childhood. The study could be supplied with an analysis of school textbooks considering natural numbers. Also, the study of the national curricula could be extended to include other content areas and mathematical processes. In a questionnaire for educators in the Danish kindergarten class, I will follow up on how often different mathematics skills, including natural numbers, are taught in the classes.

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