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Methodological Aspects in the Theory of Mathematical Working Spaces

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The theory of Mathematical Working Space (Kuzniak & Nechache, 2021), provides theoretical and methodological tools to study three main issues on mathematical work: its description, its characterization, and its formation and/or transformation. Description and characterization of mathematical work help to understand and then to transform it, especially when it is not correct. We intend here to show how focusing on these three issues can help to design didactical situations based on the theory of Mathematical Working Space (MWS).

Describing mathematical work

Describing mathematical work aims at capturing the work undertaken in an educational context. The description is based on the constructs of MWS theory and has an explanatory purpose.

Describing the development of mathematical work within a mathematical domain

In their research on the teaching of geometry, Kuzniak and Nechache (2015) used the theory of MWS to describe the different components and processes of the work as they emerged during a teaching sequence (consisting of 5 sessions) on the notion of the circle in Grade 4 - 6 lessons. In their study, they introduce the use of a “comics” as a set of MWS diagrams to give a global and quick vision of the development of mathematical work throughout a teaching sequence.

Describing the circulation of work between different mathematical domains

Derouet's (2017) research on the teaching of continued probability distributions in high school classes to describe the interactions and circulations between mathematical domains and subdomains. She used of coding (red for what is the students' responsibility, green for the teacher) combined with the MWS diagram. This enabled to highlight the various transitions between different MWS associated with three domains: Probability, Statistics, and Integral Calculus during the completion of task by students (high school).

Describing the subject's personal work

To provide an account of the mathematical work actually produced by student teachers, Kuzniak and Nechache (2021) designed an analysis method based on a division of the students' activity into episodes. Each of these episodes comprises a sequence of mathematical actions, which lead to achieve a subtask. In this view, each episode corresponds to a sub-task self-prescribed by a student and includes the sequence of mathematical actions used to solve the task. All these actions are then analysed in great detail using the theory of MWS.

Characterizing mathematical work

The objective of characterizing mathematical work is to identify certain invariants from the description of work.

Cognitive unit associated with a mathematical entity

To Kuzniak and Nechache (2015), the mathematical work is structured according to a cognitive unit associated with a mathematical entity. In this way, they differentiate the cognitive and epistemological aspects of a mathematical object. The mathematical entity associated with a mathematical object is defined as being a triplet of elements of the epistemological plane, composed of representamen or signs, artefacts, and the mathematical properties and definition associated with the object. This Cognitive Unit is generated from the interaction of the three MWS geneses: instrumental, semiotic, and discursive (Kuzniak & Nechache, 2021), which trigger the cognitive processes.

Forms of mathematical work

Kuzniak and Nechache (2021) sought to characterise forms of mathematical work developed by students preparing for a master's degree. The authors gave three criteria for assessing the process, outcome, and circulation of mathematical work in the MWS: compliance, correctness, and completeness.

Forming mathematical work

Research on the formation or transformation of mathematical work aims to build a new structure for the epistemological plane of work or to provide a new structure for the cognitive plane. The method used by Reyes (2020), to develop and form mathematical work around the notion of 'function' to make it more accessible to students, is based on task design using the theory of MWS. Reyes uses MWS theory to account for the flow of mathematical work produced by students (using the MWS diagram) and to analyse how the work was completed.

References

- Derouet, C. (2017). Circulations entre trois domaines mathématiques: les probabilités, la statistique et l'analyse. In K. Nikolantonakis (Ed). *Proc. 5th Conf. Espace de Travail Mathématique* (pp. 63—78). University of Macedonia.
- Kuzniak, A., & Nechache, A. (2015). Using the Geometric Working Spaces in order to plan the teaching of geometry. In K. Krainer (Ed), *Proceedings of the 9th Congress of the European Society for Research in Mathematics Education*. (pp. 543—549). Charles University, Prague.
- Kuzniak, A., & Nechache, A. (2021). On forms of geometric work: A study with pre-service teachers based on the theory of Mathematical Working Spaces. *Educational Studies in Mathematics*, 70 (2), 127—142. <https://doi.org/10.1007/s10649-020-10011-2>
- Reyes, C. (2020). *Enseignement et apprentissage des fonctions numériques dans un contexte de modélisation et travail mathématiques*. PhD thesis. University of Paris. <https://hal.inria.fr/tel-03211997>