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Struggling with the problem of complexity or how to (not) arrive at a general theory of what happens in mathematics classrooms

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For about half a century now, it has been known and widely discussed that the field of study of mathematics education research is extremely complex. In mathematics classrooms, mathematics emerges from a complex interplay between bodily, cognitive, and social processes; and the crucial question for mathematics education research is: How can we develop methodical and theoretical means to ‘tackle’ these complexities? Given this challenge, which we will call the complexity problem, this paper aims at two things: First, to indicate at what point we think two well-known theories, namely Ernst von Glasersfeld’s Radical Constructivism and Anna Sfard’s Theory of Commognition fail at the complexity problem; and second, to attempt to point out an alternative route to a general theory that might be able to account for the complex dynamics between the bodily, cognitive, and social dimension of mathematics classrooms.

Keywords: complexity, transdisciplinarity, the social turn, commognition, radical constructivism

Introduction: The complexity problem

For about half a century now, it has been known and discussed that the field of study of mathematics education research is “characterized by an *extreme complexity*” (Steiner, 1985, p. 11). In order to provide some evidence for this thesis, it is, we think, helpful to take a look at the ‘prototypical’ object of study – the mathematics classroom – and ask: What does actually go on in there?

First of all, in every mathematics classroom there is communication that takes place. Teachers and students talk about mathematical topics, about mathematical problems and their solutions, and of course also about extra-mathematical issues (e.g. about an upcoming class trip). Regardless of how much the *particular forms* of communication may vary from one classroom to another, one will always observe *some* sort of communication processes between teachers and students. So, the first aspect that must be considered when answering the question above is communication – or, more general: *the social dimension* of mathematics classrooms. Now when social processes unfold, at the same time, numerous cognitive activities take place. Both teachers and students are presumably thinking, imagining, expecting, or fearing something when participating in classroom communication. Thus, besides the social dimension, there is also *the cognitive dimension* of mathematics classrooms that must be taken into account. Finally, a third area also comes into play: *the bodily dimension* of mathematics classrooms. Whether teacher or student, everyone participates with his or her body in the classroom – be it sitting, standing, walking or lying – and, for instance, reacts completely different to social events on the neuronal level than on the cognitive level.

So, one is always confronted with the *simultaneity* of at least three different kinds of processes – namely, the simultaneity of bodily, cognitive, and social processes – that are involved in one way or another when mathematics is constituted in the classroom. Now the crucial point about these three

kinds of processes is that their simultaneity rules out the possibility of one of them being considered 'the' cause of the other two. It is rather that each of the three dimensions obeys its own logic. Each dimension (and that justifies the use of this metaphor) is able to vary independently from the other two dimensions. And yet it is precisely this complex web of bodily, cognitive, and social processes that one inevitably encounters in the study of mathematics classrooms.

This brief description of what happens in mathematics classrooms leads to a more general problem, which lies at the core of research in mathematics education: How can we, as mathematics education researchers, handle the extreme complexity of our field of study? How can we design theoretical and methodical tools which can account for the complex relationships between the bodily, cognitive, and social dimension of mathematical activity in its many facets and forms? It shall be this fundamental challenge to research in mathematics education which we will call *the complexity problem*. Once the complexity problem has been identified, it becomes apparent in what a difficult situation research in mathematics education finds itself in. For to arrive at a theory capable of accounting for the complexities of mathematical activities in educational contexts, the analytical tools of psychology, sociology, or biology are not sufficient. If we ask, for instance, for the conditions of success of a teaching-learning process in a mathematics classroom, we ask about a process that lies perpendicular to the boundaries between these three disciplines: Mathematical teaching-learning processes are processes that require cognitive activities on the part of both students and teachers, and these activities, in turn, depend on neural activities and other bodily processes. If one now continues and asks how teachers can teach and students can learn what they are taught, one encounters the mediating function of communication. Teachers affect the cognitive activities of their students and students affect the cognitive activities of their teachers by participating in communication. To ask about the conditions of success for such complex processes, thus, means to choose a unit of analysis that goes beyond the respective 'fields of responsibility' of the three disciplines. Because here it is about the question of the possible *relationships* between the social, cognitive, and bodily dimension of classroom events.

But how do we deal with this situation in our field? Is the complexity problem already solved? Are there theories that are not only able to describe either the bodily, the cognitive, or the social dimension, but all of them together with their interrelationships? And on what grounds can such a general theory be built? These are precisely the questions we would like to explore in this paper.

From the cognitive to the social dimension and back again

We begin with a brief look at the history of mathematics education research and discuss how two theories – Glasersfeld's *Radical Constructivism* (1) and Sfard's *Theory of Commognition* (2) – struggle with the complexity problem. By choosing these two much-discussed theories, we aim to show that the complexity problem is indeed of key importance for research in mathematics education. Thereby, we will focus on one aspect of the complexity problem, namely, how to theorize the relationship between the cognitive and the social dimension of teaching and learning in mathematics classrooms.

(1) *Radical Constructivism* is based on the assumption that only the mind of an individual can be considered as the 'bearer' of knowing and knowledge. To Glasersfeld, all processes of knowing –

and thus also: all knowledge structures – must be located in the mind of an individual. But how does one get from this focus on cognitive activities of the individual to the social dimension? Against all critics who accuse Piaget of not paying attention to the social dimension, Glasersfeld argues:

But the child's experiential world also comes to contain other people, and the almost constant interaction with them is an even richer source of perturbation and consequent accommodations. Piaget has stressed many times that the most frequent cause of accommodation is the interaction, especially linguistic interaction, with others. Yet he is often criticized for not having taken into account the social component. (Glasersfeld, 2003, p. 66)

If a child calls a rectangle 'square', then it is corrected in communication; and this communicative correction then leads in many cases to a *perturbation* (i.e. to a disappointment of its expectation that the word 'square' fits these environmental circumstances) and then perhaps to an *accommodation* (i.e. to an alteration of its knowledge base through an adjustment of this expectation). Thus, without a doubt, it is possible to account for the social dimension from the standpoint of Radical Constructivism. But the social is considered only insofar as it appears on the 'screen' of an individual mind. Social events can be investigated for their causal contributions on the individual's cognitive development. They have the character of an *external* 'source of perturbation', which may then lead (or not lead) to an *internal* reorganization of what an individual knows and believes. Hence, Radical Constructivism is and remains a purely psychological perspective on the social. The 'unit of analysis' is the individual knower and his or her processes of knowing.

It was *this* unit of analysis that a number of researchers objected to in the second half of the 1980s. The representatives of this new 'movement', which Stephan Lerman called 'the social turn in mathematics education research' a decade later, no longer had in mind the study of the social from the perspective of an individual mind.

The *social turn* is intended to signal something different; namely, the emergence into the mathematics education research community of theories that see meaning, thinking, and reasoning as products of social activity. This goes beyond the idea that social interactions provide a spark that generates or stimulates an individual's internal meaning-making activity. (Lerman, 2000, p. 23)

From this standpoint, the social should thus be understood from within itself. To account for the fact that the social obeys its own logic, the unit of analysis was shifted from the cognitive to the social dimension. Although familiar concepts such as knowing, learning, or meaning continued to be used, these concepts were no longer tied to the mind of the individual, but were located in the social. The question was no longer, 'How do social events influence the constructions of an individual mind?'. But rather, 'How is the individual mind participating in the social?'. Researchers of the social turn began to study cognitive activities from the perspective of the social, and the crucial question then became whether the individuality of the individual could still be accounted for in this way:

A major challenge for theories from the social turn is to account for individual cognition and difference, and to incorporate the substantial body of research on mathematical cognition, as products of social activity. (Lerman, 2000, p. 23)

The challenge for Lerman, then, is to make the insights of psychologically inspired theories accessible again from the sociological perspective without having to return to a primacy of the individual mind. The question that has emerged from the social turn is therefore: If one apparently cannot get from a theory of mind to a substantial theory of the social, does it perhaps work the other way around?

(2) It is, we believe, Anna Sfard who has most decisively pursued this question with her *Theory of Commognition*. Sfard's theoretical program can be read as a systematic exploration of the question of how far one can get with a theory of mind when starting from a theory of the social. In the introduction to her book, Sfard writes:

In this book [...] thinking is defined as the *individualized version of interpersonal communication* – as a communicative interaction in which one person plays the roles of all interlocutors. The term *commognition*, a combination of *communication* and *cognition*, stresses that interpersonal communication and individual thinking are two facets of the same phenomenon. (Sfard, 2010, p. xvii)

The key word for a proper understanding of this theoretical program is: 'defined'. Sfard does not claim that there is no need to distinguish empirically between thinking and communication, between the cognitive and the social dimension. Rather, her thesis is that one can *define* the *concept* of thinking by means of the *concept* of communication. Sfard does not make a statement about the empirical world, but about the theoretical description of it. She claims that one can get to a substantial theory of mind from a theory of the social. For this, Sfard argues, one can start with a concept of communication and then characterize thinking as self-communication—more precisely: as an 'individualized version of communication in which one person plays the role of all interlocutors.' But it is precisely the extraordinarily clear way in which Sfard presents her theoretical program that also reveals a potential weakness: Thinking is *by definition* declared to be a special case of communication. If one can now come up with an example of a cognitive function that is different *in type* from communication in Sfard's sense, then the theoretical program is, at best, incomplete. Sfard herself states: "In my case, no such instance comes to mind" (Sfard, 2010, p. 82). In contrast, we believe that the *cognitive function of perception* can be deployed to generate such a counterexample. Sfard admits that she is "prepared to compromise and leave the more primitive form of perceiving, that which leads to immediate instinctive reactions, out of the realm of thinking" (Sfard, 2010, p. 82). This move, however, which may seem 'generous' at first glance, undermines, we believe, the whole theory of mind. For it is precisely the primitive form of perceiving in which all higher cognitive functions – especially all those functions that presuppose the use of signs, such as all sorts of mathematical thinking (Duval, 2006) – are ultimately founded.¹ Although a comprehensive development of the counterexample is beyond the scope of this paper (see Lensing, 2021, pp. 43–52), we will at least hint at how the argument runs: Sfard defines a communicational action recursively as an action A which is followed by an action B so that A is interpreted by B as being an action *about* an object (cf. Sfard, 2010, pp. 86–89). A communication is thus seen by Sfard as an operation that

¹ It was probably Edmund Husserl (1992) who showed most impressively in *Experience and Judgement* that every theory of logical and mathematical thinking finds its ultimate support in a theory of experience (cf. §1-14, especially: §10).

always processes a distinction. Action A must be 'seen' by action B in a dual way, namely as a communication about a certain object. In a second step, it is then argued that elementary perceptual processes are not processing distinctions. For example, one sees a figure in front of a ground and not the distinction between figure and ground. It is then concluded that the cognitive function of perception cannot be characterized as a specific mode of communication in Sfard's sense.

The social turn had emerged in mathematics education research around the problem that one could not get from a theory of mind to a substantial theory of the social. However, as we have argued in this section, more recent attempts to travel the same route in opposite direction do not seem to be completely successful either. But if there is no clear evidence so far that the path is viable in either direction, what should we do then? Should we consider ourselves defeated by the complexity problem and accept that, while we may well be able to study certain *aspects* of what happens in mathematics classrooms, we will probably never make the *whole picture* accessible to theoretical description?

We believe that there is in fact another way out. It comes into sight if one asks: What do the two aforementioned attempts at bridging the gap between the cognitive and the social dimension have in common? Both, Glasersfeld as well as Sfard, begin their theoretical endeavors by aligning themselves with a particular discipline: psychology in Glaserfeld's case, sociology in Sfard's case.² While Glasersfeld takes a psychological perspective and thus does not get through to the social dimension, Sfard starts from a sociological perspective and does not get a grip of the cognitive dimension. And it is this beginning, the alignment of one's theoretical program to one particular discipline, be it psychology or sociology, that we believe to be undermining the possibility of arriving at a general theory. To put it bluntly: *Developing a general theory that can actually 'grasp' the complexities of mathematics classrooms must be considered as a transdisciplinary endeavor from the very beginning.* However, such a theory can only be developed from a theoretical standpoint that is not associated with any of the classical disciplines. But how can this be possible? What ways of theorizing might lead to such a transdisciplinary theory?

The formal method: Following the example of modern mathematics

In order to address those questions, we believe it is helpful to take modern mathematics as an example and adopt a theorizing strategy that has revolutionized the field of mathematics over the last two centuries. We will call this theorizing strategy the *formal method*. Now what is the formal method and how can it be employed for theorizing in mathematics education research?

In his or her undergraduate studies, every mathematics student learns concepts from abstract algebra, order theory, topology, measure theory, and so on these days. These theories provide concepts (e.g. the algebraic concepts of group, ring, and field) that allow for relating the seemingly most remote

² That we make this assignment arises from the unit of analysis that the two theorists choose as their starting point: While Glasersfeld chooses the individual knower and his or her processes of knowing as his unit of analysis (= psychological perspective), Sfard starts from the concept of communication and defines it in such a way that communication can only emerge through a certain kind of recursive linking of the actions of at least two different individuals (= sociological perspective).

areas of mathematics (e.g. permutations, geometric transformations, and numbers). Although these *new concepts* have become part of the standard repertoire of a mathematician, surprisingly, it is rarely stated explicitly from which *new way of theorizing* those concepts have actually emerged.

It was most likely Edmund Husserl who, in 1900, in the first volume of his *Logical Investigations*, first provided a satisfactory answer to the question of what was new about these revolutionary developments in mathematics from a methodological standpoint. Husserl argued that the key idea was to *formalize* existing mathematical theories *in their entirety*. Thus, instead of investigating mathematical domains consisting of entities with a determinate content (e.g. numbers and their operations or geometrical transformations and their compositions), mathematicians proceeded to investigate their purely formal counterparts. That is to say, they no longer investigated *particular* mathematical domains, but rather *general* ‘domain-forms’ (Husserl, 2012, §70): In this sense, abstract algebra is no longer concerned with *particular* operations on *particular* kinds of numbers. Rather, it is concerned with the question of what can be said about mathematical domains of a particular *form*, namely about all those domain-forms in which certain operations on objects are defined by their operational laws. When the addition symbol ‘+’ is used in group theory, it does not stand for the operation of addition, which composes two numbers giving their sum, but for *any* operation that satisfies the group axioms. What these operations and objects on which they are performed then actually look like remains completely indeterminate in terms of content. Not the content of the objects and operations is determined, but only certain conditions for their form are demanded. A theory in abstract algebra, such as group theory, is actually a *theory-form*. It is freed from all numerical or geometrical content. By means of formalization, these contents are “converted into indeterminates, modes of the empty ‘anything-whatever’” (Husserl, 1969, §29). And it is this particular way of theorizing, this ‘conversion’ of a theory (or of a whole class of theories) with a determinate content into its (their) corresponding theory-form which we want to call the *formal method*.

With this in mind, it is no longer difficult to understand the networking power of ‘theories’ such as abstract algebra or topology. Since these ‘theories’ are theory-forms, they do not really consist of concepts, theorems, and proofs, but of concept-forms, theorem-forms, and proof-forms (cf. Husserl, 1969, §29). And since a theory-form is precisely something that many theories can have in common, it seems quite natural that one can study connections between remote mathematical domains this way. But what, one may now ask, does any of this have to do with research in mathematics education?

Towards a transdisciplinary approach to research in mathematics education

The crucial point is that a proper understanding of the formal method suggests an entirely new approach to theorizing in mathematics education research. Rather than concluding from the striking similarities between cognitive and social processes (e.g., both kinds of processes utilize signs, process meanings, and refer to all sorts of objects through these very meanings) that they are essentially “two facets of the same phenomenon” (Sfard, 2010, p. xvii), our proposal is that they are in fact two quite different phenomena that share a common form. And we hence believe that it may be better to start from an appropriate *theory-form* to explain similarities *and* differences between these processes. In order to avoid a potential misunderstanding right away: Our proposal here is not to apply mathematical theories in the human sciences, but rather to adopt a particular way of theorizing,

namely the formal method, and apply it for theorizing in these fields. Fortunately, such a project does not need to be started from scratch. By formalizing and enriching the theory of autopoietic systems, which was first developed by Maturana and Varela as a *biological* theory (Maturana, 2002), Niklas Luhmann was able to arrive at a general theory of autopoietic systems (or more correctly: at a general theory-form of autopoietic system-forms). And just as the formalized complement of the concept of addition in group theory is no longer about numbers, the formalized complement of the concept of autopoiesis in this general theory is no longer about organic processes. Rather, the formalized version of the concept (= its concept-form) merely captures a certain self-referential form of (re-)production: “We want to call systems *autopoietic*, which produce and reproduce the elements of which they are composed by the elements of which they are composed” (Luhmann, 1985, p. 403, translated by F.L.). The criticism of an illegitimate transfer of biological concepts to social and cognitive phenomena is hence unfounded, because such a transfer simply does not take place. Instead what Luhmann does is formalizing the biological concept of autopoiesis. He first converts all biological terms into empty forms (‘system’, ‘element’, and so on) and then asks for the concrete modes of reproduction that characterize social and cognitive systems. In this way, Luhmann is able to conduct a systematic study of similarities *and* differences between organic, cognitive, and social systems and arrives at a *theory of mind* (Luhmann, 1985) as well as a *theory of the social* (Luhmann, 1995). And it is this work that Lensing (2021) takes as a point of departure to develop the outlines of a general theory which can account for the bodily, cognitive, and social dimension of mathematical activity and model such phenomena as mathematical teaching-learning processes within a single theoretical framework.

Since a detailed exposition of this theoretical program hardly seems possible on the few lines we have left, we would like to conclude this paper with a brief indication of how the formal method may operate for mathematics education research. For this purpose, we have chosen a concept that is used quite frequently in our field, but almost never explicitly discussed with respect to its conceptual content: the concept of *structure*. In Lerman (2020), for example, authors speak of ‘knowledge structures’, ‘mental structures’, ‘cognitive structures’, but also of ‘social structures’, the ‘structure of classroom discussions’, ‘structures of power and control’, and many more. What is striking about this list is the ‘parallel terminology’ that cuts across various disciplines: Psychology deals with cognitive or mental structures, sociology examines all sorts of social structures, and mathematics education research is concerned with both areas and how the formation of structures in one area might have a bearing on the formation of structures in another (i.e., how certain structures of communication in mathematics classrooms affect the formation of students’ cognitive structures). These considerations indicate that it might be of great value to our field to gain possession of a concept of structure that is not bound to any of the disciplines relevant to mathematics education research, a concept that, due to its formality, can be employed in both the cognitive as well as the social sphere.

But how do we actually get to such a formal level of analysis? The answer is: by starting off at the empirical level and then employing the formal method to work our way up. We may start, for instance, with the well-known example of the ‘IRF-Pattern’ (Initiation – Response – Feedback) and the observation that this social structure constraints the possibilities of the occurrence and linking of communications in the classroom. The pattern thus structures communication events as they occur in the classroom in two respects: communications should be of certain types (initiation, response,

feedback) and they should be linked in a certain way (first, initiation, then, response, and so on). If we now continue and include other kinds of social structures, such as norms, roles, social positions, etc., we can see that the way in which these structures exceed constraints on social events varies greatly from case to case, but that all instances of social structure at least agree in that there is some sort of constraints: A structure of a social system, quite generally, constraints the possibilities of the occurrence and linking of social events in that system. With this step we still reside in the realm of social theory and the next step of formalization would then be to abandon the qualification of events as social events and thus elevate our ‘definition’ of structure to the transdisciplinary level of a general theory of autopoietic systems: A structure of an autopoietic system constraints the possibilities of the occurrence and linking of elements in that system.

Clearly, this brief example is only a very rough sketch of how the formal method may be employed for the purposes of theorizing in mathematics education research. Yet we hope that it will at least motivate the thesis that this method allows for theorizing that yields to a theory that is beyond all disciplinary boundaries. And it is the transition to this transdisciplinary level of theory that we believe provides a possible starting point for solving the complexity problem.

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