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# Coordinating conceptual frameworks for an in-depth understanding of knowledge when teaching mathematics

Malin Gardesten

Linnaeus University, Sweden; [malin.gardesten@lnu.se](mailto:malin.gardesten@lnu.se)

*This paper is methodological and theoretical, focusing on how teachers' knowledge in action and interaction when teaching mathematics can be explored. The two conceptual frameworks adopted, The Knowledge Quartet and the Pedagogical Relational Teachership, are coordinated, as they grasp disparate aspects of knowledge in action and interaction when teaching mathematics. An empirical example is given to illustrate how these two conceptual frameworks can be adopted in coordination with each other. The results show that the coordination of these two conceptual frameworks used as a methodological framework can contribute to a deeper understanding of the characteristics of teachers' pedagogical content knowledge and relational abilities when teaching mathematics.*

*Keywords: Mathematics teaching, networking frameworks, relational abilities, teachers' knowledge.*

## Introduction

This paper is methodological and theoretical, focusing on how teachers' knowledge in action and interaction when teaching mathematics can be explored. This exploration is based on a classroom study conducted in a Swedish Grade 5 class. It is part of a research project intended to contribute to a more profound understanding of inclusive mathematics education regarding the connections between teachers' knowledge and relational abilities when teaching mathematics. Thus, this paper focuses on the use of a methodological framework based on two conceptual frameworks rather than interpreting the implications for mathematics education in the classroom.

Previous studies have shown that relational leadership promotes inclusive mathematics education (Schmidt, 2015). Furthermore, Roos (2019) concluded that inclusive mathematics education requires the teacher to have mathematical, didactic, and pedagogical skills as well as relational competencies in seeing each student as a person and understanding their needs. Roos' conclusion concurs with that of Ernest (2019). The latter stated that teaching mathematics involves the responsibility to treat students with care and respect and teach mathematics in a manner that benefits students effectively. Additionally, Valero (2005) pointed to the need to view the student as a whole human being, not just as a cognitive subject of mathematical thinking, to fully understand the student's intentions when acting in social mathematics teaching and learning situations.

Several analytical models deal with mathematics teaching, for example, Mathematical Knowledge for Teaching (Ball et al., 2008), The Teaching Triad (Jaworski, 1992; Potari & Jaworski, 2002), and The Knowledge Quartet (KQ) (Rowland, 2013). Ball et al. (2008) aimed to elaborate theoretically on the concepts of subject matter knowledge and pedagogical content knowledge (Rowland, 2013). Potari and Jaworski (2002) acknowledged sensitivity to students as one of the three keystones, making their framework useful as an analytic tool. However, they gave no further descriptions of how teachers act in interactions.

Based on the above, this paper is concerned with coordinating conceptual frameworks that can surface the complexity of teachers' knowledge when teaching mathematics in inclusive classrooms, considering knowledge in action and interaction with students. This complexity is addressed in this paper by illustrating two conceptual frameworks complementing each other using networking strategies (Bikner-Ahsbals & Prediger, 2010) as guiding heuristics. The two conceptual frameworks are the KQ (Rowland, 2013), which frames how mathematical subject knowledge plays out in teaching, and the Pedagogical Relational Teachership (PeRT) (Ljungblad, 2019), which frames teachers' relational abilities, are coordinated. Thus, this paper aims to illustrate how these two different conceptual frameworks may contribute to exploring teachers' knowledge in action and interaction with students when teaching mathematics in inclusive classrooms.

## **The two conceptual frameworks**

First, the two conceptual frameworks are introduced. Then the common core elements between the conceptual frameworks are presented.

### **The Knowledge Quartet**

The KQ is a conceptual framework of four categories: *foundation*, *transformation*, *connection*, and *contingency*. Each of the four categories consists of several methodological codes to be used when analyzing empirical material (Table 1), from which the four dimensions of the KQ are extracted.

*Foundation* is related to the teacher's theoretical background in and knowledge of mathematics, making it possible for the teacher to use mathematical terminology deliberately, be aware of its purpose and identify errors (Rowland, 2013). Epistemologically, foundation can be seen as propositional knowledge, involving knowledge about mathematics and the purpose of mathematics teaching (Hundeland et al., 2017). *Transformation* refers to how a teacher's foundational knowledge is transformed into action when teaching, such as demonstrating mathematical content through examples, instructional materials, and mathematical representations (Rowland et al., 2005). Epistemologically, transformation can be seen as knowledge in action, as this knowledge is situated in and visible in teaching moments (Hundeland et al., 2017). *Connection* refers to the connections made by the teacher concerning the coherence of the teaching across a shorter or longer period, for example, connections between procedures, concepts, and sequenced examples (Rowland et al., 2005). Epistemologically, connection can be seen as knowledge in action, as this knowledge is situated in and visible in teaching moments (Hundeland et al., 2017). *Contingency* refers to the teacher responding to ideas from students for which it is impossible to plan and represents deviations from the intended actions in a planned lesson, but which make the teaching meaningful for students (Rowland et al., 2005). Epistemologically, contingency can be seen as knowledge in interaction, as this knowledge is situated in and visible in teaching moments when interacting with students (Hundeland et al., 2017).

### **The Pedagogical Relational Teachership**

The PeRT is a theoretical relational perspective involving a taxonomy categorized under two themes, *tact* and *an inclusive stance* (Ljungblad, 2019), which is then further organized into smaller units of codes to be used when analyzing empirical material (see Table 2) (Ljungblad, 2021).

*Tact* is situated and improvised and can be observed in the teacher's different ways of relating to the students and their situated needs. Tact can be expressed verbally and non-verbally. The verbal expressions of tact are connected to how teachers use the tone of voice, degree of loudness, and rate of speech. Tact is expressed non-verbally through the teacher's eye contact, movements, gestures, and facial expressions. Epistemologically, tact can be seen as knowledge in action as well as in interaction, as tact is situated in the teaching and visible in the teaching as the teacher relates to the students (Ljungblad, 2016). *An inclusive stance* is connected to the teachers' sensible choices and can be observed in how the teacher takes responsibility for teaching and developing sustainable relationships within the classroom. Epistemologically, an inclusive stance can be seen as knowledge in action as well as in interaction and is both situated in and visible in teaching (Ljungblad, 2016).

### **Common core elements**

Strategies of networking theoretical approaches are research practices that may provide a more comprehensive view of the complexity of teaching and learning mathematics. Various strategies to use more than one theoretical approach have been systematized described as a continuum, from understanding others and making their theories understandable to synthesizing and integrating locally. Somewhere in the middle of the spectrum of the degree of integration is to *coordinate* theoretical approaches. Coordinating multiple theoretical approaches can contribute to a conceptual framework for understanding empirical data (Bikner-Ahsbaks & Prediger, 2010).

In this paper, coordinating two conceptual frameworks implies three consecutive analyses on the same data source, complementing each other in a methodological framework. The two conceptual frameworks are chosen to elaborate on the interactions between a teacher and a student and the teacher's action reflecting the student's responses. The teacher's foundational mathematical knowledge plays out in the teaching, referring to the dimensions of transformation, connection and contingency. Unlike, the teacher's responses to the individual student's actions refer to tact and an inclusive stance dimensions, as well as contingency. Epistemologically, the dimension of contingency intersects with the dimensions of tact and inclusive stance. They are both seen as knowledge in interaction, addressing how the teacher's mathematical knowledge and relational ability unfold in the teaching in unpredictable ways. The common core elements are based on knowledge in interaction between the teacher and the student since they are connected to a context. When teaching mathematics, the teacher's actions are influenced by the context. However, when interacting with the student, the teacher responds to the student's ideas and emotions, which means the teacher influences the context.

### **Coordinating the two conceptual frameworks**

The two conceptual frameworks were used to analyze video-recorded observations of mathematics lessons carried out in an intervention that was ongoing during a school year. The video-recorded classroom observations from this study were transcribed and coded in NVivo. In the analysis, first, the codes from the KQ (Table 1) were used to mark up the data material.

**Table 1: Methodological codes from the KQ (Rowland, 2013)**

Foundation	awareness of purpose (AP); identifying errors (IE); overt subject knowledge (OSK); the theoretical underpinning of pedagogy (TUP); use of terminology (UT); use of textbook (ATB); and reliance on procedures (COP)
Transformation	teacher demonstration (DT); use of instructional materials (UIM); choice of representation (CUR); and choice of examples (CUE)
Connection	making connections between procedures (MCP); making connections between concepts (MCC); anticipation of complexity (AC); decisions about sequencing (DS); and recognition of conceptual appropriateness (RCA)
Contingency	responding to students' ideas (RSI); deviation from agenda (DA); teacher insight (TI); and (un)availability of resources (RAT)

Then, as a second layer on the same data material, the codes from the PeRT (Table 2) were used. The sequence was first coded by me and then again by three senior researchers and two doctoral students to increase interrater reliability. Discussions were carried out among the experts involved until a consensus was reached regarding the codes used.

**Table 2: Methodological codes from the PeRT (Ljungblad, 2021)**

Tact	improvising shifts of tact (A11); seeking contact by showing interest in students' different ways of working and reasoning (A21); seeking contact with students by showing interest in the person (A22); meeting the students with respect (A31); and meeting the students in different ways in the same teaching situation (A32)
An Inclusive stance	taking responsibility for the teaching (B11); taking responsibility for the relationships (B12); listening to students when creating space for them to speak in their own way (B21); creating space for students to listen to each other (B22); showing students different possible ways to explore the content (B31); and encouraging students by showing trust in the students' ability and willingness to explore the content mutually (B32)

The third step implied an inductive analysis of students' participation in mathematics education on instances categorized into three groups: i) coded to both KQ and PeRT, ii) coded to only KQ, and iii) coded to only PeRT, exploring differences and similarities of students possibilities to participate in mathematics education. In this paper however, only the first and second deductive steps are focused on and thus not the third inductive step.

### **An empirical example using the KQ and the PeRT as lenses**

In this section, one empirical example is first introduced and coded to illustrate how the coordination of the conceptual frameworks was carried out. The empirical example is from a Swedish Grade 5 classroom with two teachers, a mathematics teacher and a special education teacher in mathematics.

This empirical example was chosen for this paper because the codes from the two conceptual frameworks sometimes overlapped. The codes are written in brackets to the right. Codes from the KQ are in *italics*, and codes from the PeRT are underlined.

First, the mathematics teacher, Felicia, introduces a task about a jogging tour to the whole class. The special education teacher in mathematics (Selma) is present during the lesson. The task describes a jogging tour that took the form of a circle. There is a picture of the circle on the task paper, where the circle's circumference (3,140 m) and radius (500 m) are written. The jogging tour has a shortcut straight across the circle, along its diameter. Subtask b is: Fatima is jogging half the distance and then taking the shortcut home. How long is Fatima's jogging round? Tanja, a student, raises her hand when the transcript below starts. The teachers consider Tanja a student in special educational needs in mathematics (SEM), as she often struggles to participate in mathematics. Selma, who is nearby, stops beside Tanja's right side, bends down, leans her forearms on the desk, and quietly asks Tanja about her thoughts on subtask b while looking at her task paper. Tanja asks about how to figure out the distance.

- 62 Selma: How are we about to figure out the half then? (A11, A21, B32, *RAT*)
- 63 (Quiet, 7 sec.) (B21)
- 64 Selma: How can we do it? (B32)
- Next, after a passage where Tanja is being silent, Selma asks:
- 67 Selma: How long is the whole [circle]? (*RSI*)
- After a wrongly read number, Tanja quickly answers correctly.
- 71 Selma: Yes. Suppose we pretend that this is three thousand. How long would the half be then? (A11, B32, *MCP*)
- 72 Tanja: Two and a half. No. (B21)
- 74 Selma: Not two and a half. Instead? What if you and I would split these three? (A11, B32, *RAT*)
- 75 Tanja: Mm. (B21)
- 76 Selma: How much do each of us get? (A11, *RAT*)
- 77 Tanja: Three and a half. (B21)
- 78 Selma: Do we get three and a half if we split three?! (A11, *IE*, *RAT*)
- 79 Tanja: No. (Giggles.) (B21)
- 80 Selma: No. (Laughs.) (A11)
- 81 Tanja: One and a half. (B21)
- 82 Selma: One and a half. And, what does it mean in meters in this task? (A11, *MCP*)
- 83 (Quiet, 2 sec.) (B21)
- 84 Selma: Instead of three thousand? (A11, *MCP*)
- 85 (Quiet, 2 sec.) (B21)
- 86 Selma: If we split, instead of meters, if there were [Swedish] crowns instead. Suppose we split three

		thousand [Swedish crowns] you and me.	( <u>A11</u> , <i>UIM</i> , <i>RAT</i> )
87	Tanja:	Mm.	( <u>B21</u> )
88	Selma:	How many [Swedish crowns] do we get each?	( <u>B32</u> , <i>UIM</i> , <i>RAT</i> )
89		(Quiet, 4 sec.)	( <u>B21</u> , <i>RSI</i> )
90	Tanja:	One and a half.	( <u>B21</u> )

Above is an example where most lines are only coded with codes from the PeRT. In these lines, the special education teacher actively listens to the student or make room for her to speak (code B21, lines 63, 72, 75, 77, 79, 81, 83, 85, 87, 90). In another line, the special education teacher acknowledges that they work together using "we" in her talk, signalling that they will complete the task together (code B32, line 64). She changes her voice tone to a joyful tone (code A11, line 80) and seeks contact, interested in the student's reasoning (code A21, line 91). These lines coded only to the PeRT could be seen as the teacher responding to the student's needs as a person.

One line is coded only to the KQ. Here, the teacher responds to the student's silence by asking an easy question to which the information for the answer is written on the task paper (code *RSI*, line 67). The line could be seen as the teacher wants the student to give an answer related to the mathematical content, as she chooses a question to which the student already knows the answer.

There are also lines with codes from both the KQ and the PeRT. Lines coded to both the KQ and the PeRT display the special education teacher shifts tact while she contingently uses her fingers as manipulatives (codes A11 and *RAT*, lines 62, 74, 76, 78, 86). In some of these lines, the teacher also shows interest in the student's way of reasoning (code A21, line 62) and acknowledges that they together, using "we" in her talk, will be able to complete the task (code B32 line 62, 74). Yet another line shows the teacher suggesting they together, "we", can pretend the number is 3000 instead of 3140 while making a connection between the procedures of dividing 3140 by 2 and dividing 3000 by 2 (codes B32 and *MCP*, line 71). Further, one line displays the teacher pointing to her three fingers when identifying a wrong answer and changing her voice to a joking, surprising tone when asking a question (codes *RAT*, *IE* and A11, line 78). More, there are lines where the teacher connects one and a half to 1500 while changing the direction of her gaze to look at the student and changing her speech rate to slow and gentle (codes *MCP* and A11, lines 82, 84). Another line implies that the teacher again gazes at the student talking about money as imagined manipulatives (three thousand Swedish crowns) while pointing at her three fingers (codes A11, *RAT* and *UIM*, line 86). Again, the teacher uses "we", asking the student about how they can split the money (B32, *RAT* and *UIM*, line 88). The teacher is silent long enough, so the student finally answers (B21, *RSI*, line 89). The lines coded to both the KQ and the PeRT could be seen as the teacher responding to the student's needs as a person as well as adapting her foundational mathematical knowledge to this particular social setting.

## Conclusions

This paper aimed to illustrate how the two conceptual frameworks used as a methodological framework can contribute to exploring a teacher's knowledge in action and interaction with a student when teaching mathematics in an inclusive classroom. The empirical example illustrates how the two frameworks make aspects beyond teachers' knowledge respective relational abilities surfaced. That is, from the dimensions of the KQ (Rowland, 2013), the special education teacher in mathematics

shows foundational mathematical knowledge connected to procedures (lines 71, 82, 84), choosing (unavailable) materials and representations (lines 62, 74, 76, 78, 86, 88), and responding to a student's silence (lines 67, 88, 89). From the dimensions of the PeRT (Ljungblad, 2021), the special education teacher uses verbal and non-verbal expressions to seek contact with the student and shows interest in her ways of reasoning. That is, the special education teacher creates space for the student to speak with her voice (lines 63, 72, 75, 77, 79, 81, 83, 85, 87, 89, 90), acknowledging her answers, although her answers are sometimes incorrect (lines 71, 82). The teacher tactfully makes jokes (line 78), supports the student, and shows trust that they two will solve the task (lines 62, 64, 71, 74, 78, 86, 88). Thus, based on the interaction with the student, the special education teacher adapts her actions to the specific student within the particular situation in time and space, using relational abilities. When using relational abilities, the special education teacher's foundational mathematical knowledge is transformed through adaptation to the specific student in the interaction (lines 62, 71, 74, 76, 78, 82, 84, 86, 88, 89). Thus, when coordinating the KQ and the PeRT, the features of a teachers' relational abilities concerning a specific student's diverse interests and prerequisites contribute to our understanding of the complex situations of mathematics teaching and how they evolve.

Earlier studies on mathematics teaching have either focused on teachers' knowledge (for example, Ball et al., 2008; Jaworski, 1992; Potari & Jaworski, 2002; Rowland, 2013) or teachers' relational abilities (Roos, 2019; Schmidt, 2015; Valero, 205) and the approaches of the KQ and the PeRT function as frameworks of their own. They both can be used to analyze mathematics teaching but grasp disparate aspects of teachers' actions or interactions with students. However, coordinating the KQ and the PeRT may contribute to an understanding of mathematics teaching in inclusive classrooms from an extended perspective. The above-illustrated two-step deductive analysis makes it possible to carry out a forthcoming third inductive analysis of the instances where the codes from the KQ and the PeRT overlap and don't overlap. This analysis will make visible if and then how teachers relational and/or mathematical knowledge influence student participation in mathematics.

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