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Points of contact between the ATD and the TDS: questions raised by the implementation of study and research paths

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The Anthropological Theory of the Didactic (ATD) aims to capture the complexity of real inquiries by using a type of instructional proposal called study and research path (SRP). Experimental research shows that certain notions of the Theory of Didactic Situations (TDS) appear specially adapted to analyse implementations of SRPs and point at some didactic phenomena related to the conditions needed for their implementation and management. We report this experience by first identifying some contact points between the ATD and the TDS and then describing the borrowings done. The analysis of the commonalities and specificities of both approaches helps to question the framework of the ATD and to point at some needed theoretical and methodological developments.

Keywords: Anthropological Theory of the Didactic, Theory of Didactic Situations, study and research paths, didactic contract, didactic situation, didacticity, networking.

Introduction: ATD, TDS and networking

In this paper, we are considering two of the main theoretical frameworks of what is known as the French tradition in didactics of mathematics (Artigue et al., 2019), namely the Theory of Didactic Situations (TDS, Brousseau, 1997) and the Anthropological Theory of the Didactic (ATD, Chevallard, 2015). We present a networking activity that emerged when using notions from the TDS within ATD analyses and continues by considering their differences and commonalities about the kind of tools they propose to problematise, model and develop teaching and learning processes.

In 2007, in an invited lecture at the First International Conference of the Anthropological Theory of the Didactic, Guy Brousseau established some connections between key notions of the TDS and what could be considered the analogous ones in the ATD. He analysed the differences in how both theories model human activities in their collective and dynamic dimensions, including doing mathematics and teaching and learning mathematics. His comparison involved the notions of situation, milieu, situated knowledges (*connaissances*) and didactic processes, in contrast with some elements of the praxeologies (types of tasks and techniques) and the moments of the study. Brousseau considered these notions “more as points of contact, points of articulation that allow moving from one [theory] to the other, rather than as borders” (Brousseau, 2007, p. 3, our translation). He concluded:

The ATD and the TDS complement each other. But in my opinion, it would be absurd to juxtapose them simply. In many issues, they are intertwined; they must be considered together. What

problems do they pose for each other? What answers do they offer each other? What advances do they promise together? (Brousseau, 2007, p. 22, our translation)

The relationships between the TDS and the ATD have been the object of previous research in the project of networking of theories initiated by Angelika Bikner-Ahsbals in 2005 at CERME4 (Bikner-Ahsbals & Prediger, 2014). For instance, Artigue et al. (2010) show how a teaching episode can be described from both approaches and relates the TDS analysis about the limitations of an *adidactic milieu* with the ATD one in terms of the functioning of the *media-milieu dialectic*. Ghemansi and Lecorre (2019) propose combining some TDS and ATD methods in design-based research related to the university teaching of calculus. These works and our experience as researchers are in tune with Brousseau's proposal of considering notions as contact points to question one theory from the other's perspective. This paper presents a bottom-up networking strategy that arose during the analysis of some empirical work carried out within the ATD, where the resource to some TDS notions appeared in a rather spontaneous way. This research practice nourished a reflection about the contact points between both theories, together with some specificities and differences.

Contact points between the TDS and the ATD

Since 2005, our ATD-research team has been implementing and analysing a new type of instructional proposal named *study and research paths* (SRP) based on the continued inquiry of problematic questions. In a way, SRPs include aspects of project-based and inquiry-based learning (IBL). However, they also provide new perspectives and methodologies currently not elaborated in the IBL literature (Markulin et al., 2021). Barquero et al. (2020) summarised that the implementation of SRPs faces several challenges related to the coexistence of two pedagogical paradigms in the school institutions in a very explicit way. Historically, the one that prevails is the *paradigm of visiting works* (Chevallard, 2015) and is based on the proposal of a set of bodies of knowledge – the *works* – for students to study under the teacher's guidance. The new paradigm that is struggling to emerge is the *paradigm of questioning the world*, in which the bodies of knowledge are replaced by open questions to address (or inquire). Between both extremes, some compromise situations can exist. In any case, our research shows how the conditions needed to implement a study process framed in the paradigm of questioning the world are diverse and fragile. We then identify many constraints coming from the prevailing paradigm of visiting works. In our analysis of such conditions and constraints, four points of contact between the TDS and the ATD appeared, as we said, very naturally. We first present these four points before examining each one in more detail in the following two sections.

First point of contact: new epistemological demands. One of the conditions for the normalised running of SPRs is the availability of tools to describe and manage the different types of knowledge mobilised during an inquiry. The existence of new study processes based on the paradigm of questioning the world highlights the need to deal with knowledge-related aspects, such as uncertainty, temporariness, and validation, that often remain implicit in visiting-works instructional proposals, where knowledge is conceived as static and crystallised. Brousseau was one of the first to point out the necessity for researchers in didactics to question and rebuild school mathematics organisations to avoid assuming the prevailing school epistemology as the only and appropriate one. The TDS proposal is to model mathematical knowledge in terms of *fundamental situations*, defined as games

against a *milieu*, that is, an environment without any didactic intention towards the student and providing feedback to the actions received (Brousseau, 1997). The ATD, which is part of the TDS project of building didactics of mathematics as a science, joins its epistemological background about the need to question mathematical practices. It, however, differs in the models chosen and introduces a clearer institutional perspective by pointing at the existence of different conceptions and knowledge constructions connected through didactic transposition processes (Chevallard & Bosch, 2020).

Second point of contact: dealing with the curriculum constraint. Even if competency-based curriculums are on the agenda, to a large extent, curricular content is still described in terms of labelled pieces of preestablished knowledge. In this conception, inquiry-based proposals are often seen as means or methodologies to acquire the specific labelled content. In contrast to this conception, SRPs do not oppose inquiry and transmission but subordinate the learning of contents to the advance of the inquiry, and the elaboration of a final answer to the question initially addressed. This change in the knowledge conception is particularly challenging in a setting where the two paradigms coexist and meets what we call the *curriculum constraint* (Barquero et al., 2020). The TDS' proposal of modelling knowledge as the answers to problematic *situations* appears as a first move towards the paradigm of questioning the world while remaining in the frontier of the paradigm of visiting works because situations are always supposed to model a given piece of knowledge.

Third point of contact: the evolution of the didactic contract. A third and more evident contact relies on the changes produced by SRPs in the traditional sharing of responsibilities among the teachers and the students, which we interpret as an evolution of the *didactic contract* (Brousseau, 1997) that prevails in the paradigm of visiting works. Implementing an SRP requires students to assume different roles in the inquiry process, such as seeking available answers, validating or rejecting them, raising new questions, deciding which ones to follow or discard, planning the work to do, etc. Teachers also experience essential changes in their tasks: they are no longer the “knowledge holders” nor the sole person bringing new knowledge into the classroom. The coexistence of paradigms in the same school institution – and even in the same course – makes this contract negotiation complicated.

Fourth point of contact: didactic and adidactic situations. The students' assumption of new responsibilities in the inquiry process depends on how they engage in the SRP's generating question and the importance they attribute to developing an answer. In the paradigm of questioning the world, the study of works is essential as far as it helps to progress in elaborating a response to the initial question. The SRP generating question needs to remain at stake during the entire process for this to happen. In the analysis of the implemented SRPs, to identify the episodes where students seemed to be engaged in the inquiry for the sake of the considered generating question, we find it helpful to use the TDS distinction between *didactic* and *adidactic* situations.

The four contact points correspond to assumptions and modelling strategies that are common to both approaches. They need to be complemented with some comments about their differences and specificities. We will now approach them in two blocks by first addressing the question of the modelling of mathematical knowledge (points 1 and 2) and then the role played by the notions of didactic contract and adidactic situations in the ATD analysis (points 3 and 4).

The ATD inherits the epistemological programme initiated by the TDS

The first and second contact points correspond to a solid and fundamental connection between the TDS and the ATD. As explained by Artigue and Trouche (2021), Brousseau's project aimed "to find a genuine experimental epistemology of mathematics based on the construction of situations able to make mathematical knowledge emerge from autonomous students' interactions with their environment in the social context of classrooms" (p.3). Moreover, the TDS experimental epistemology relies on the observation and analysis of classroom implementations. Therefore, the central object here is not the cognitive subject, i.e., the pupil or student, but the situations organising the relationship of such subjects with mathematical knowledge and its *raison d'être*. The ATD participates in this *epistemological research programme* in mathematics education (Gascón, 2003), which locates the description, modelling and reconstruction of the knowledge to be taught at the core of didactics research. An important difference between the TDS and the ATD is in the type of reference epistemological models (Bosch & Gascón, 2006) they elaborate to model mathematics, as knowledge and as a human activity. The TDS models are formulated as *fundamental situations*, defined as games against a *milieu*, with different dialectics or phases (Brousseau, 1997). Brousseau clarifies the scope he gives to this concept:

One of the approaches of didactics of mathematics consists in modelling not only the knowledge to teach or learn, but also the conditions in which it manifests itself. Situations are minimal models that "explain" how such knowledge intervenes in the particular relationships a subject establishes with a milieu to exert a determined influence on it. (Brousseau, 2000, p. 4)

When we look at the types of epistemological models proposed by TDS, we can distinguish between a general model in terms of fundamental situations (or games against a milieu) and specific models in terms of sequences of situations. Without going into detail here, it is worth noting that in most of the work developed in TDS, the division of the modelled knowledge does not follow the classic school cartography of knowledge. It corresponds to rather vast domains of the mathematics to be taught, as shown by the work on numeration, measurement, and decimal numbers, which go far beyond what their name indicates. These models also make it possible to identify elements that do not exist in the dominant school epistemological models, a well-known example being that of "enumeration" (Briand, 1993; Rivière, 2017). The international diffusion of the TDS has been often limited by the confusion between epistemological and didactic models. Fundamental situations indeed play a double role in this respect because they define knowledge not by what it is but by the processes that allow it to be constructed. This fusion between didactic and epistemological proposals – or between didactic and mathematical situations – is at the heart of the TDS (Brousseau, 2007).

As said before, the ATD emerged within the scientific project of the TDS. According to Artigue and Trouche (2021, p.4): "ATD then broadened the perspective by placing at the centre of the analysis the diverse institutions, institutional positions, and institutional relationships with the knowledge at stake, and how they condition and constrain what is taught and, therefore, what students ultimately have the possibility to learn or not. To this end, new concepts were introduced, notably the concept of praxeology and the scale of didactic codeterminacy." The notion of *praxeology* is introduced as the basic unit to analyse human action in general and mathematical knowledge in particular. The

dissemination of praxeologies takes place through what we call didactic systems. A didactic system is a triplet $S(X; Y; \mathcal{P})$ formed by a person or a group of persons Y (the teachers) who do something to help another group of persons X (the students) to learn a given body of knowledge or praxeology \mathcal{P} .

In the paradigm of questioning the world, didactic systems are not formed around a given praxeology to be studied, but rather around a question Q , to which X , with the help of Y , has to provide an answer A^\heartsuit . One tool used to analyse inquiry processes is the Herbartian schema: $[S(X; Y; Q) \rightarrow M] \rightarrow A^\heartsuit$. The schema considers that the study of Q generates an *inquiry* process involving a *didactic milieu* M including questions Q_i derived from the initial one, “ready-made” answers A_j^\diamond one can find in the literature or by consulting works and experts (the *media*), together with empirical data D_k and other material and knowledge works W_l : $[S(X; Y; Q) \rightarrow \{Q_i; A_j^\diamond; D_k; W_l\}] \rightarrow A^\heartsuit$. Obviously, an alteration in the available *media* or aspects related to the *milieu* can lead to completely different constructions of A^\heartsuit . The schema is helpful to design and carry out *a priori analyses* about potential paths to follow when approaching Q , and also *in vivo* and *a posteriori* analyses about the path actually taken and the means used to do so. The Herbartian schema identifies some critical elements of the inquiry, namely $\{Q_i; A_j^\diamond; D_k; W_l\}$, which can be detailed in terms of praxeologies. As we will see below, the inquiry dynamics are described in terms of dialectics, like those of *questions and answers* (Bosch & Winslow, 2014) and *media and milieus* (Kidron et al., 2014).

A first important difference between the TDS and the ATD proposals to model mathematical knowledge and activities is terminological (and, consequently, also conceptual). The use of situations enables one to model knowledge in an implicit way, not by pointing at its elements (for what a specific language is required) but by describing the situations it allows to solve. In the ATD, the notions of praxeology and Herbartian schema do propose descriptions of the knowledge elements and therefore require a specific language, which cannot be neutral. It is sometimes provided by the scholarly knowledge complemented with ad hoc elaborations. A second difference is the role given to the *media* in both approaches, which is often reduced to the teacher in the TDS and accessed through *didactic situations* – that is, situations where the teacher intervenes in the students’ knowledge construction.

Developing TDS notions in the ATD analysis

The generating question(s) of an SRP and the notion of situation

In the implemented SRPs (Barquero et al., 2020), the generating question Q always plays a critical role. It should lead the inquiry and be always prominent, appearing as an end in itself – instead of an excuse to visit some preestablished works. And it should also conduct to a final answer A^\heartsuit that is of a particular value for somebody, not only for X and Y . The experiences carried out in university engineering degrees showed a visible change in the students’ involvement in an SRP depending on the conditions under which Q is formulated and A^\heartsuit has to be received: an imaginary client, a fictitious client or a team of students taking part of a car race from the same university. We find here one of the main *raison d’être* of the TDS notion of *situation*. It includes the question raised but also the conditions under which it arises – the initial *milieu* – and the requirements for its reception and validation – the rules of the game and the winning strategy. In a way, when we talk about a question Q in the ATD, we are also implicitly including the institution and corresponding contract that ensures the reception of Q as such a question, together with the type of acceptable answers A_i and A^\heartsuit . Also,

the importance of Q depends on the destiny of the potential final answer A^\heartsuit . Therefore, in the design of an SRP, it is essential to require not only some general interest for Q , but also a kind of external contract with an instance Z about Q , the type of expected answer A^\heartsuit and how it will be valued or validated.

The notion of milieu

The concept of *milieu* proposed by the TDS corresponds to the environment with which students interact and provides them with objective feedback. It can comprise material objects, informative texts, digital tools, and other collaborating or competing students (Artigue & Trouche, 2021). In an SRP, the *milieu* is built along the inquiry process and can be analysed through the elements previously described in the Herbartian schema: $\{Q_i; A_j^\diamond; D_k; W_l\}$. Its dynamics is then explained through the development of some dialectics. In particular, a central one is the *media-milieu* dialectic. This dialectic includes access to already available answers (or pieces of works found in the media) and their integration into the inquirers' milieu. During this integration, answers need to be validated to become useful for the inquiry progress and appear as potential works to be used by others.

Didactic contract and “adidacticity”

The management of an inquiry process in a “question-driven way” requires a new sharing of responsibilities between teachers and students and can be interpreted as the evolution of the didactic contract (Brousseau, 1997). For instance, students must assume new responsibilities, like planning the work to do, proposing the questions to address and those to discard, deciding what media to consult, validating the answers they find or propose, etc. In a way, in the didactic contract of the paradigm of visiting works, teachers are expected to boost the media-milieu dialectics students will then follow, as if the “question-driven” inquiry was only driven by the teacher, not by the students.

Analysing the changes in the didactic contract led us to use the distinction between *didactic* and *adidactic situations*, two key notions of the TDS. However, in our use of these notions, we do not include its functioning as *fundamental situations*, that is, as reconstructions and epistemological models of the mathematical knowledge to be taught. We only include in this notion the assumption that any question or problem is never raised in the vacuum but always appears to X (and Y) under specific circumstances or conditions and with some available resources (and other unavailable ones), a certain *milieu*. In this context, we use the term *adidacticity* to refer to the very moments when students make decisions primarily considering the generating question Q and the targeted final answer A^\heartsuit , without prioritising the instructional process that supposedly envelops the inquiry, that is, the didactic situation in which the SRP takes place. Looking for these adidactic moments can help measure to what extent an SRP is overcoming the constraints of the paradigm of visiting works.

Open questions in the dialogue between TDS and ATD

This paper presents several contact points between the TDS and the ATD, showing many ways, in which the latter is an heir of the former. But there are also aspects in which they seem to differ sufficiently to make it unavoidable to try to refine the formulation of the ingredients that mark the differences. A work that is still in progress.

The paradigm of questioning the world is the paradigm par excellence in research. Accordingly, the choice of theories when addressing research questions should always be subordinated to the question's sake. In other words, theories are good, helpful, valid, or interesting as far as they help researchers elaborate good, helpful, valid, or interesting answers to the questions addressed. In this context, the bottom-up strategy of networking theories is not a big deal. What specific relevance can we find in the networking experience here presented? First, it illustrates the do-it-yourself character of didactics research, and the freedom researchers should adopt in problematising, experiencing and analysing teaching and learning processes. Second, it provides a new perspective of each considered approach when viewed from the other side, establishing a *dialogue between theories* as proposed by Bosch et al. (2017). In this sense, our paper responds to Brousseau's analysis of ATD notions from the TDS perspective, by proposing some analysis of TDS notions from an ATD perspective. Finally, pointing at their complementary aspects is a way to question the scope of the ATD methodology by pointing at some needs in the proposed analyses, and contributes to its evolution.

We believe that such a bottom-up networking strategy is possible because of the robust commonalities shared by the TDS and the ATD. The most important one is the questioning of the mathematical activities to be taught and the need to elaborate scientific models to reproduce these activities without assuming the prevailing visions of school and scholarly institutions. And we should also add the inclusion of new instructional practices that intent to approach the teaching of mathematics to a questioning the world perspective. These are the critical points of contact that enable researchers to easily switch from one approach to the other without distorting any of them.

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References

- Artigue, M., Bosch, M., Gascón, J., & Lenfant, A. (2010). Research problems emerging from a teaching episode: a dialogue between TDS and ATD. In V. Durand-Guerrier, S. Soury-Lavergne & F. Arzarello (Eds.), *Proceedings of CERME 6* (pp. 1535–1544). INRP.
- Artigue, M. et al. (2019). The French didactics tradition in mathematics. In W. Blum et al. (Eds.), *European Traditions in Didactics of Mathematics* (pp.11–55), Springer. https://doi.org/10.1007/978-3-030-05514-1_2
- Artigue, M., & Trouche, L. (2021). Revisiting the French Didactic Tradition through Technological Lenses. *Mathematics*, 9, 629. <https://doi.org/10.3390/math9060629>
- Barquero, B., Bosch, M., Florensa, I., & Ruiz-Munzon (2020). How to integrate study and research paths into university courses? Teaching formats and ecologies. In T. Hausberger, M. Bosch, & F. Chellougui (Eds.), *Proceedings of the INDRUM 2020 conference*. University of Carthage. <https://hal.archives-ouvertes.fr/hal-03113981>
- Bosch, M., & Gascón, J. (2006). 25 years of the didactic transposition. *ICMI Bulletin*, 58, 51–63.

- Bosch, M., Gascón, J., & Trigueros, M. (2017). Dialogue between theories interpreted as research praxeologies: the case of APOS and the ATD. *Educational Studies in Mathematics*, 95, 39–52. <https://doi.org/10.1007/s10649-016-9734-3>
- Briand, J. (1993). *L'énumération dans le mesurage des collections. Un dysfonctionnement dans la transposition didactique* [Doctoral dissertation, Université de Bordeaux I]. TEL. <https://tel.archives-ouvertes.fr/tel-00494623>
- Brousseau, G. (1997). *Theory of Didactical Situations in Mathematics* (N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, Eds.). Kluwer Academic Publishing.
- Brousseau, G. (2000). Les différents univers de la mesure et leurs situations fondamentales. Un exemple d'utilisation de la théorie des situations pour l'ingénierie. *Quaderni di Ricerca in Didattica*, 9, 125–133. <http://dipmat.math.unipa.it/~grim/mesure.pdf>
- Brousseau, G. (2007). Entre la théorie anthropologique du didactique et la théorie des situations didactiques en mathématiques : questions et perspectives. In L. Ruiz-Higueras, A. Estepa, & F. J. García (Eds.), *Sociedad, escuela y matemáticas. Aportaciones de la Teoría Antropológica de lo Didáctico* (pp. 3–28). Universidad de Jaén.
- Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: a case for an oncoming counter paradigm. In S. J. Cho (Ed.), *The Proceedings of the 12th International Congress on Mathematical Education: Intellectual and attitudinal challenges* (pp. 173–187). Springer.
- Gascón, J. (2003). From the cognitive to the epistemological programme in the didactics of mathematics: Two Incommensurable Scientific Research Programmes? *For the learning of mathematics*, 23(2), 44–55.
- Ghedamsi, I., & Lecorre, T. (2019). Towards an interplay between TDS and ATD in a Design-Based Research project at the entrance to the university. In U. T. Jankvist et al. (Eds.), *Proceedings of the 11th Congress of the European Society for Research in Mathematics Education* (pp. 3038–3015). Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.
- Eleventh Congress of the European Society for Research in Mathematics Education* (N. 16). Freudenthal and Institute; ERME. <https://hal.archives-ouvertes.fr/hal-02422633/document>
- Kidron, I., Artigue, M., & Bosch, M. (2014). Context, milieu, and media-milieus dialectic: a case study on networking of AiC, TDS, and ATD. In A. Bikner-Ahsbabs, & S. Prediger (Eds.), *Networking of Theories as a Research Practice in Mathematics Education* (pp. 153–177). Springer. https://doi.org/10.1007/978-3-319-05389-9_10
- Markulin, K., Bosch, M., & Florensa, I. (2021). Project-based learning in statistics: a critical analysis. *Caminhos da Educação Matemática em Revista*, 11(1), 200–220. https://aplicacoes.ifs.edu.br/periodicos/caminhos_da_educacao_matematica/article/view/755
- Rivière, O. (2017). *Continuité des connaissances d'énumération et conséquences sur les savoirs : mieux comprendre les difficultés des élèves confrontés à des problèmes d'énumération* [Doctoral dissertation, Université Clermont Auvergne]. TEL. <https://tel.archives-ouvertes.fr/tel-01754817>