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FABIO ACERBI^a

Barlaam's Paraphrase of Euclid, *Elements* II 1–10. A Critical Edition*

ABSTRACT: The article presents a critical edition, with a translation and an introduction, of the arithmetical rewriting of Euclid, *Elements* II 1–10, authored by the 14th-century scholar and polemicist Barlaam of Seminara, one of the two leading characters in the Palamite and hesychast controversies. In this way, a further item of Barlaam's scientific writings can now be read in a critical edition. The present edition explains in detail the mathematical background of Barlaam's work, describes all of its manuscript witnesses, reconstructs a stemma codicum that does not require any lost witness, and pays due attention to the diagrams that accompany the text.

KEYWORDS: Barlaam of Seminara, Euclid's *Elements*, rewriting, Byzantine mathematics, geometric algebra

INTRODUCTION

Barlaam of Seminara (†1348), born in Calabria and there made a monk, was a high-brow intellectual in Palaiologan Byzantium. He was actively engaged in political life, serving as an imperial ambassador. He was one of the two leading characters in the Palamite and hesychast controversies, where he championed, opposing the monk Gregorius Palamas, an approach to the doctrinal points at issue in sharp contrast with Palamas' mysticism¹. He was defeated, left Constantinople in 1341 for Latin West, settled by the papal Curia in Avignon, taught Latin to Francesco Petrarca, the founding father of Italian lyric poetry², and eventually converted to Catholicism (he was also reluctantly made bishop of Gerace), serving the Pope as he had served the Byzantine Emperor.

Barlaam was a polemicist, and a prolific writer. Many of his works were collected in volume 151 of Migne's *Patrologia Graeca* (the irony is that the focus of this volume is Palamas' production); most of his output has been published in a critical edition³. He also composed five scientific

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* Online reproductions of most manuscripts mentioned in this article can be found through the website <https://pinakes.irht.cnrs.fr/>, which also provides additional bibliography. A very good reproduction of Dasypodius' edition can be found at <https://doi.org/10.3931/e-rara-38516>. I am grateful to L. Corry for triggering me to undertake this edition and for useful discussions, to B. Vitrac for a critical reading and for sharing the results of his research about the textual tradition of the *Elements*, to C. Giacomelli, S. Martinelli Tempesta, and M. Trizio for their suggestions, and to M. Cronier, S. Di Mambro, and G. Pausillo for their logistic support.

¹ For a first orientation on the Palamite and hesychast controversies, see A. RIGO, Gregorio Palamas e oltre. Studi e documenti sulle controversie teologiche del XIV secolo bizantino (*Orientalia Venetiana* 16). Firenze 2004; A. FYRIGOS, Dalla controversia palamitica alla polemica esicastica (con un'edizione critica delle Epistole greche di Barlaam). Roma 2005, 67–97; N. RUSSELL, The Hesychast Controversy, in: *The Cambridge Intellectual History of Byzantium*, ed. A. Kaldellis – N. Siniosoglou. Cambridge 2017, 494–508. On Barlaam, see S. IMPELLIZZERI, Barlaam, in: *Dizionario Biografico degli Italiani*. VI. Roma 1964, 392–397; *PLP* 2284; A. FYRIGOS, Barlaam Calabro: l'uomo, l'opera, il pensiero. Atti del Convegno internazionale, Reggio Calabria, Seminara, Gerace, 10-11-12 dicembre 1999. Roma 2001; FYRIGOS, Dalla controversia palamitica 161–169. On Barlaam's philosophical stance, see most recently M. TRIZIO, «Una è la verità che pervade ogni cosa». La sapienza profana nelle opere perdute di Barlaam Calabro, in: *Byzantine Theology and its Philosophical Background*, ed. A. Rigo (*Byzantios* 4). Turnhout 2011, 108–140, with bibliography.

² See A. FYRIGOS, Barlaam e Petrarca. *Studi Petrarqueschi* 6 (1989) 179–200.

³ See footnote 5 below for the scientific works. Other editions include C. GIANNELLI, Un progetto di Barlaam Calabro per l'unione delle Chiese, in: *Miscellanea Giovanni Mercati*. III (*StT* 123). Città del Vaticano 1946, 157–208; R. E. SINKEWICZ, The Solutions Addressed by Barlaam the Calabrian to George Lapithes and their Philosophical Context. *MS* 43 (1981)

works⁴: a fully-fledged treatise of number theory, in six books (Λογιστική, henceforth *Logistikē*); two strictly parallel texts, in which he computed the circumstances of the solar eclipses of 1333 and 1337 (*De eclipsi* I and II); a pamphlet on the determination of the date of Easter (*De paschate*); a tract in which he showed that the last three chapters of Ptolemy's *Harmonica* as they are handed down by a part of the manuscript tradition cannot be authentic (*Refutatio*)⁵; a number-theoretical rewriting of the first ten propositions of Book II of Euclid's *Elements* (*Demonstratio*). Barlaam was also a scientific polemicist: *De eclipsi* I and II, *De paschate*, and *Refutatio* have Nikephoros Gregoras as their polemical target⁶; as we shall see, the *Demonstratio* is quite obviously intended to denounce George Pachymeres' approach to the same subject-matter as amateurish.

The present article contains a critical edition of the *Demonstratio*—which until now could be read only in the text of Dasypodius' 1564 edition—accompanied by a translation. The edition is preceded by an overview of Barlaam's scientific production, by an outline of the mathematical background of the *Demonstratio*, by an analysis of the deductive structure and of the style of the treatise, by a detailed description of its manuscript witnesses, and by a discussion of the stemmatic relations between these manuscripts. Supplementary material includes a collation of two new witnesses of a geometric problem ascribed to George Gemistos Pletho (Appendix 1); an edition, with a translation and a commentary, of a series of procedural and geometric texts intended to complete Barlaam's *Logistikē* (Appendix 2); and plates setting out all diagrams of Barlaam's *Demonstratio* in two important manuscript witnesses (Iconographic Complement).

151–217 (this contains a complete list of Barlaam's writings, with references to the editions to that date; see also, more recently, FYRIGOS, *Dalla controversia palamitica* 169–182); T. M. KOLBABA, *Barlaam the Calabrian. Three Treatises on Papal Primacy. Introduction, Edition, and Translation. REB* 53 (1995) 41–115; and the complete edition A. FYRIGOS, *Barlaam Calabro. Opere contro i Latini. Introduzione, storia dei testi, edizione critica, traduzione e indici (StT 347)*. I–II. Città del Vaticano 1998; FYRIGOS, *Dalla controversia palamitica*.

⁴ For plausible dates of composition of these works, deduced from allusions in some of Palamas' writings, see FYRIGOS, *Dalla controversia palamitica* 170–172. Barlaam scientific activity can in this way be limited to *ca.* 1330–37.

⁵ See the following editions: P. CARELOS, *Βαρλαάμ τοῦ Καλαβροῦ, Λογιστική*. Barlaam von Seminara, *Logistikē (Corpus philosophorum Medii Aevi. Philosophi byzantini 8)*. Athens – Paris – Bruxelles 1996 (*Logistikē*); J. MOGENET – A. TIHON – D. DONNET, *Barlaam de Seminara, Traités sur les éclipses de soleil de 1333 et 1337*. Louvain 1977 (*De eclipsi* I and II); A. TIHON, *Barlaam de Seminara. Traité sur la date de Pâques. Byz* 81 (2011) 362–411 (*De paschate*); I. DÜRING, *Die Harmonielehre des Klaudios Ptolemaios*. Göteborg 1930, 112–121 (*Refutatio*); Düring's text is a reprint of J. FRANZ, *De musicis graecis commentatio*. Berolini 1840, 14–23 (based on the manuscript Napoli, Biblioteca Nazionale Centrale Vittorio Emanuele III, III.C.3 [1330–50; *Diktyon* 46279]), with corrections coming from manuscripts that also contain Ptolemy's *Harmonica*, among which our witnesses P¹ and V (a critical edition of the *Refutatio*, in collaboration with S. Di Mambro, is forthcoming). As for the *Demonstratio*, the *princeps* is published in *Euclidis Quindecim Elementorum Geometriae secundum: ex Theonis commentarijs Graecè & Latinè*. Item, *Barlaam monachi Arithmetica demonstratio eorum quae in secundo libro elementorum sunt in lineis & figuris planis demonstrata*. Item, *Octo propositiones Stereometricae, eiusdem cum praecedentibus argumentis*. Per Cunradum Dasypodium scholae Argentinensis Professorem. Argentorati MDLXIII, reprinted in J. L. HEIBERG – H. MENGE, *Euclidis Opera Omnia*. I–VIII. Leipzig 1883–1916 V 725–738, both without the proem, for which see G. VITELLI, *Indice de' codici greci Riccardiani, Magliabechiani e Marucelliani. SIFC* 2 (1894) 471–570: 543–544; and P. CARELOS, *Das unedierte Prooimion des Kommentars Barlaams von Kalabrien zum zweiten Buch der Elemente des Euklid. Hellenika* 49 (1999) 367–369, who apparently did not know of Vitelli's earlier edition and asserts that he transcribed the text from our witness M (this statements is surprising: just check the title Carelos gives for the *Demonstratio*).

⁶ The best assessments of the Gregoras-Barlaam controversy in scientific matters are in MOGENET – TIHON – DONNET, *Barlaam 150–157* and in TIHON, *Barlaam*, which also contains a detailed description of Gregoras' contribution to Easter Computation. Gregoras' computation of the solar eclipse of July 1330 is edited in J. MOGENET – A. TIHON – R. ROYEZ – A. BERG, *Nicéphore Grégoras, Calcul de l'éclipse de soleil du 16 juillet 1330 (Corpus des Astronomes Byzantins 1)*. Amsterdam 1983. In his writings, Barlaam never mentions Gregoras, who wrote three pamphlets against Barlaam: possibly the *Antilogia*, certainly the *Philomathēs* and the *Phlorentios*: see the editions in P. L. M. LEONE, *Nicephori Gregorae «Antilogia» et «Solutiones Quaestionum»*. *Byz* 40 (1970) 471–516; P. L. M. LEONE, *Il Φιλομαθής ἢ Περί ὕβριστων di Niceforo Gregora. RSN* 8–9 (1971–72) 171–201; P. L. M. LEONE, *Niceforo Gregora, Fiorenzo o Intorno alla sapienza (Byzantina et Neo-Hellenica Neapolitana 4)*. Napoli 1975.

BARLAAM'S SCIENTIFIC PRODUCTION OTHER THAN THE *DEMONSTRATIO*

Logistikē. Despite its title⁷, the *Logistikē* in six books is a fully-fledged, highly original, treatise of theoretical arithmetic formulated in an impeccable Euclidean style. The *Logistikē* must be considered the most accomplished product of Byzantine mathematics, with no parallel in ancient Greece. Barlaam's goal is expressly stated in the preface: to provide logistic techniques useful in astronomy with a demonstrative basis⁸. Barlaam also briefly describes the contents of his treatise, according to the following outline. Book I (27 propositions): addition and subtraction of parts (these are our fractions). Book II (39): multiplication and division of degrees and parts (in prop. II 39, the Heronian approximation of a square root is discussed; the proof shows that each iteration gives a better approximation; see Appendix 2 *infra*). Book III (12): multiplication and division of degrees and minutes (that is, this book deals with the sexagesimal system). Book IV (12): multiplication of straight lines by straight lines and division of regions by straight lines (these are lengths with respect to a fixed unit of measurement; emphasis is put on issues of homogeneity). Book V (23): compounded ratios and removal of a ratio from a ratio (see Appendix 2 *infra*). Book VI (21): how, from given magnitudes, what is sought becomes given (the magnitudes are numerical proportions and regions, rectangles and right-angled triangles; here, "given" means "assigned in numbers"). Each book is opened by a set of specific definitions; the propositions alternate theorems and problems; the latter may have a counterpart in a specific elementary operation; in this case, the constructions involved in the problems can be rewritten as algorithmic sequences formulated in the stylistic code of "procedures" (see Appendix 2 below). All propositions are illustrated by diagrams. Numerals are attached to most of these diagrams (see below).

De eclipsi I and II. These two treatises, whose structures are strictly parallel (their prefaces are identical), compute, according to the procedures described in the *Almagest*, the circumstances of the solar eclipses that occurred on May 14, 1333 and on March 3, 1337, respectively. In the preface, Barlaam states that he set out to compute the circumstances of the eclipse(s) because the *Almagest* does not contain any worked-out example of such a computation. Accordingly, what follows the preface of each version of *De eclipsi* is a series of bare computations, which determine all circumstances of the intended eclipse. These are, in the order in which they are calculated: position and time of the mean conjunction; position and time of the true conjunction; position and time of the apparent conjunction; latitudinal parallax; apparent position of the Moon in its own orbit; magnitude of the eclipse; positions of the Moon and of the Sun and times at the beginning and end of the eclipse; duration of the eclipse; "inclinations" at beginning of eclipse, mid-eclipse, and end of eclipse. The two treatises differ as to the detail in which the computations are performed. *De eclipsi* II is much longer than *De eclipsi* I: 275 lines vs. 162 lines in the standard edition.

⁷ Logistic is a branch of arithmetic in which a unit can be divided and that deals with numbers, insofar as they represent concrete entities, and with calculations connected with them. Logistic was extensively developed in late antiquity as a support to mathematical astronomy, and also played the same role in the Byzantine period. The first known treatise of this kind is included in the *Prolegomena to the Almagest*, a primer on Ptolemy's treatise made of (non-redacted) lecture notes of a course held at the end of the 5th century in the school of the Neoplatonic philosopher Ammonios. The most comprehensive introduction to Greek logistic is still K. VOGEL, Beiträge zur griechischen Logistik. Erster Teil (*Sitzungsberichte der Bayerischen Akademie der Wissenschaften, Mathematisch-naturwissenschaftliche Abteilung*). München 1936, 357–472. For Byzantine logistic, see F. ACERBI, Arithmetic and Logistic, Geometry and Metrology, Harmonic Theory, Optics and Mechanics, in: A Companion to Byzantine Science, ed. S. Lazaris (*Brill's Companions to the Byzantine World* 6). Leiden – Boston 2020, 105–159: 116–128; F. ACERBI, A New Logistic Text of Nicholas Rhabdas. *Byz* 92 (2022) 17–45: 20–26.

⁸ Barlaam's statement can be read at CARELOS, Βαρλαάμ 1, 10–26. See also the explicit statement opening a primer on the decimal positional system written in 1252: A. ALLARD, Le premier traité byzantin de calcul indien: classement des manuscrits et édition critique du texte. *Revue d'Histoire des Textes* 7 (1977) 57–107: 80, 2–4, and, in a smoother formulation, Planudes' *Great Calculation*, which is a reworking of the 1252 primer: A. ALLARD, Maxime Planude, Le grand calcul selon les Indiens. Louvain-la-Neuve 1981, 27, 1–5.

De paschate. This treatise is not a real Easter Computus⁹, but an exposition of the principles underlying the determination of the date of Easter, completed by an argument that explains why the discrepancy between schematic and real full Moons should not lead to a correction of the traditional dates of Passover. The *De paschate* does not contain any worked-out technical development, even if unfinished computations and tables, in Barlaam's hand, are attached to the treatise in the ancestor of the manuscript tradition (see below).

Barlaam begins by expounding some basics about the celestial sphere and the solar motion; his goal is to clarify what is the Spring equinox (sects. 1–12). The date and the hour of the equinox varies from year to year because the tropical year, according to Ptolemy's value, falls short of $365\frac{1}{4}$ days by $\frac{1}{300}$ of a day. In the Julian calendar, this difference entails a secular backward drift of 1 day every 300 years for the date of the Spring equinox, after which Easter must be celebrated (sects. 13–15). Barlaam asserts that, according to his own computations, in AM 4156 [= BC 1352] the vernal equinox occurred on March 27; the dates of subsequent occurrences of the vernal equinox, spaced 300 years apart, are set out in a table (sect. 16)¹⁰. Barlaam does not draw any consequences from this computation, whose argument takes a good half of his treatise and which shows that progressively earlier dates in March are available to Passover¹¹. He goes on by providing historical reasons for Easter being celebrated after the Spring equinox, adds other precepts, and finally summarizes all of them in a fourfold rule: the Easter date must follow the Spring equinox; it should not coincide with Passover; it must follow the first full Moon after the Spring equinox; it must occur on the first Sunday after such a full Moon (sects. 17–22). On the basis of these precepts, the Fathers of the Church have elaborated a ready-to-use table. This is the Damascene table, which was originally conceived for the 19-year lunar cycle AM 6233–6251 [= AD 725–743] and which contains the traditional list of Passover dates (sects. 23–24). However, this table faces a problem. Over a 19-year cycle, the Moon is in advance by $0;3,37$ of a day, and hence by about 1 day (nearly $0;57,52$ of a day) over 304 years, that is, over 16 lunar cycles of 19 years. Accordingly, every 304 years there arises a discrepancy of about 1 day between real and schematic full Moons¹². This discrepancy, Barlaam writes, has accumulated to 2 days since the conception of the “table of the Fa-

⁹ On Byzantine Easter Computi and their technical background, see F. ACERBI, Byzantine Easter Computi: An Overview with an Edition of *Anonymus* 892. *JÖB* 71 (2021) 1–62. Barlaam's pamphlet's was partly compiled in Matthew Blastares' *Syntagma*, dated 1335, see G. RHALLES – M. POTLES, *Syntagma tōn theiōn kai hierōn kanonōn kata stoicheion*. VI. Athēnai, 404–425 = *PG* CXLV 65–104.

¹⁰ The mentioned table is absent at this point of the text, see below. The shift of 1 day is already apparent if we compare the vernal equinox observations reported in the *Almagest* and ascribed to Hipparchus with the sole observation made by Ptolemy; these observations lie 285 to 267 years apart. When Barlaam set out to study computational matters, the date of the “real” vernal equinox oscillated between March 12 and 13, while the date assumed in Easter Computi was frozen, on Alexandrian authority, to March 21. The date of the equinox is not readily observed with accuracy; even if it were, being a Byzantine astronomer just meant being able to use the *Almagest* and the *Handy Tables*—or the *Persian Tables* from mid 14th century on—it did not mean being engaged in performing accurate observations. To Byzantine astronomers, the date of the equinox could most easily be extrapolated from the dates given in the *Almagest* and from the secular drift induced by the just-mentioned difference between the tropical year and $365\frac{1}{4}$ days. Probably basing himself on this estimate and on the fact that Ptolemy observed the vernal equinox on March 22, AD 140 [= AM 5648], 1^h after noon (*Almagest* III 1 and III 4), Barlaam, who composed his treatise in AM 6841 [= AD 1333], namely, $300 \times 4 - 7$ years later, set out a table of dates of the Spring equinox on a time interval of 2701 years (AM 4156–6856 [= BC 1353/2–AD 1347/8], that is, $300 \times 4 + 8$ years later than AD 140 at the latest) and by 300-year steps. Barlaam asserts that the date of the vernal equinox was March 27 in AM 4156, and that it was March 18 in AM 6856 [= AD 1348]. However, he does not say how he determined the dates of March 18 and 27. It is more likely that he operated as I suggest, and then modified his outputs to get the dates he gives in the list.

¹¹ Every fourth year, the additional day of the Julian calendar offsets the 6-hour forward shift of the equinox induced by the fact that the tropical year is (about) 6 hours longer than 365 days.

¹² A “schematic” new or full Moon is determined according to the lunar calendar that underlies the Damascene table. This lunar calendar is established once and for all and summarizes the 19-year lunar cycle. As no lunar cycle can be exact, the dates of corresponding real and schematic full Moons need not coincide. As Barlaam explains, there occurs a secular increase of the discrepancy between real and schematic phases of the Moon.

thers” (for $1333 - 725 = 608 = 304 \times 2$, a perplexing agreement indeed); this entails that Easter is sometimes celebrated one week later than the real full Moon that follows the Spring equinox would allow one to do (sects. 25–29). However, Barlaam suggests not to proceed to any modifications of the table of the Fathers, for the following reasons: first, the difficulty of reaching any and every Christian community in the world would entail, over a period that could be unpredictably long, unwelcome local disagreements as to the date of Easter; second, the modified Passover list should be updated again after 304 years; third, since the said discrepancy makes real full Moons precede schematic full Moons, adhering to the traditional dates for Passover entails celebrating Easter farther away from the real Jewish Passover than adhering to “real” full Moons would allow one to do, a fact that nicely fits one of the principles of Easter Computi (“never with the Jews”) (sects. 30–33)¹³. On f. 153r–v of Marc. gr. Z. 332, the ancestor of the manuscript tradition, the *De paschate* is followed by computations and tables (including the one alluded to in the text) related to the treatise; these two pages are entirely in Barlaam’s hand.

Refutatio. This is a “refutation” of chapters 14 and 15 of Book III of Ptolemy’s *Harmonica*, added by Nikephoros Gregoras, and of chapter III 16¹⁴, of a different origin but which is out of place in the manuscripts. This pamphlet is more a piece of textual criticism than of harmonic theory: Barlaam states six conditions that should be fulfilled if the chapters were to be regarded as authentic, and shows that none of these conditions is satisfied in the restored chapters. The six conditions are: first, the chapters should not repeat things expounded in previous chapters; second, they should contain particular, and not general, arguments, for, at the end of chapter III 13, Ptolemy states that this will be exactly what he will do; third, the comparisons set out in the chapters should be in agreement with the observative data available in Ptolemy’s times; fourth, these comparisons should be “fitting” (οἰκεῖος, a qualifier used in the last three conditions), that is, they should not equate similar items to dissimilar items; fifth, the language should be correct and it should fit the subject-matter; sixth, the contents of the chapters should fit their titles¹⁵. The tract includes a clarification of the meaning of “prime number” and a geometric argument about the straight line being the shortest path between two points. Barlaam’s *Refutatio* settled the issue of authenticity of *Harmonica* III 14–16 once and for all: this was already recognized by Johannes Kepler in his annotated translation of *Harmonica* III 3–16¹⁶.

There is no reason to ascribe to Barlaam the iterative procedure for computing an approximate square root that opens a series of mathematical texts (henceforth *varia mathematica*) attached to the *Logistikē* in our manuscript witness A (see below). This series of texts comprises the said procedure, six procedures for removing a ratio from a ratio, five of which are explicitly identified as derived from propositions of the *Logistikē*, a (by and large fallacious) geometric argument by Nicholas Kabasilas, and two texts on inequality of ratios¹⁷.

¹³ The core of Barlaam’s argument will be endorsed (that is, abridged, translated into Latin, and embellished with rhetorical bombast) in a letter *De errore Paschatis* (dated to 1470) addressed by Cardinal Bessarion to Pope Paul II: see L. MOHLER, Kardinal Bessarion als Theologe, Humanist und Staatsmann. I–III. Paderborn 1923–42 III, 546–548.

¹⁴ With the sole exception of P¹, in all witnesses listed below the *Refutatio* is accompanied by *Harmonica* III 14–16. That chapter 14 and 15 were added by Gregoras is borne out by partly autograph scholia in the manuscript Paris, Bibliothèque nationale de France, Coislin 173 (1st half of 14th century; *Diktyon* 49312), in particular ff. 32r and 108r; see DÜRING, Die Harmonielehre LXXVIII–LXXXVIII, for this and other documents and for a discussion of the content of the added chapters. More recently, a thorough discussion of *Harmonica* III 14–16 is found in N. M. SWERDLOW, Ptolemy’s *Harmonics* and the ‘Tones of the Universe’ in the *Canobic Inscription*, in: *Studies in the History of the Exact Sciences in Honour of David Pingree*, ed. Ch. Burnett – J. P. Hogendijk – K. Plofker – M. Yano. Leiden 2004, 137–180: 165–176.

¹⁵ The titles are independently preserved in the *pinakes* of Ptolemy’s treatise.

¹⁶ Ed. Ch. FRISCH, Joannis Kepleri astronomi Opera Omnia. V. Francofurti a. M. et Erlangae 1864, 392 Nota I. Kepler asserts that he owned a transcription of the *Refutatio*, *ex dono J. G. Herwarti*. This manuscript is lost.

¹⁷ An edition of the square root algorithm, on the basis of the *deterior* witness L, is found in CARELOS, Βαρλαάμ 114 and, on the basis of the sole independent witness A, in Appendix 2 below.

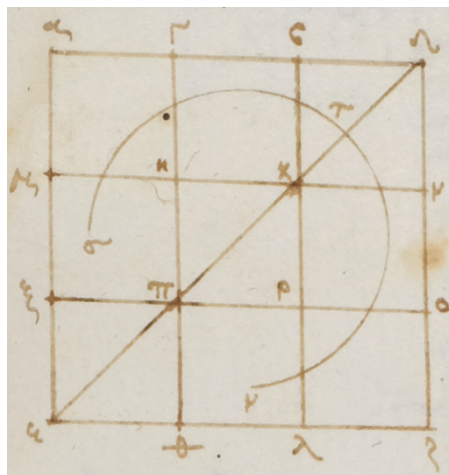
MATHEMATICAL BACKGROUND TO THE *DEMONSTRATIO**Euclid*

Book II of the *Elements* contains cut-and-paste geometric theorems about quadrangles and triangles. These theorems are one of the main resources in the tool-box of Greek mathematics; they are applied—in the *Elements* but also in Apollonius, Pappus, Ptolemy, and Diophantus—in a variety of geometric and number-theoretical configurations, where they operate as “substitution rules”¹⁸. To understand the nature of such theorems, let us read the enunciation of *Elem.* II 8¹⁹:

ἐὰν εὐθεῖα γραμμὴ τμηθῆ ὡς ἔτυχεν, τὸ τετράκις ὑπὸ τῆς ὅλης καὶ ἑνὸς τῶν τμημάτων περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τοῦ λοιποῦ τμήματος τετραγώνου ἴσον ἐστὶ τῷ ἀπὸ τε τῆς ὅλης καὶ τοῦ εἰρημένου τμήματος ὡς ἀπὸ μιᾶς ἀναγραφέντι τετραγώνῳ.

If a straight line be cut at random, four times the rectangle contained by the whole <straight line> and by one of the segments with the square on the remaining segment is equal to the square on the whole <straight line> and on the said segment as if described on one <straight line>.

In the *Elements*, the proof of this statement—as well as the proofs of all theorems in the string II 1–10—runs as follows. First, the whole geometric configuration described in the enunciation is set up, by using standard constructive steps, in the “construction”. The diagram representing the configuration is as the one here reproduced²⁰, where AB is the “whole” straight line, AΓ and ΓB are the two “segments” in which it is divided, any of AK, MP, HΛ, or KZ is “the rectangle contained by the whole <straight line> and by one of the segments”, AΓ is the “remaining” segment and ΞΘ is the square on it, AΔ = AB + BΓ is “the whole <straight line> and the said segment” and AZ is the square on it. Second, the equality stated in the apodosis of the enunciation is proved. The proof consists in identifying all the sub-regions that compose the geometric objects in the constructed configuration: concisely stated, AZ is made of ΞΘ and of AK, MP, HΛ, and KZ—that is, of four times any of them—where one of the two squares overlapping in HP compensates for the square “hole” BN²¹. In this way, the Euclidean proofs of II 1–10 are independent of one another.



To modern eyes, the enunciations of the theorems in *Elem.* II have a strongly “algebraic” connotation; accordingly, modern scholarship since Tannery deems the contents of Book II “geometric

¹⁸ The use of these and other, similar theorems in Greek geometry is studied in F. ACERBI, The Geometry of Burning Mirrors in Greek Antiquity. Analysis, Heuristic, Projections, Lemmatic Fragmentation. *Archive for History of Exact Sciences* 65 (2011) 471–497.

¹⁹ Ed. HEIBERG – MENGE, *Euclidis Opera Omnia* I 138, 2–7.

²⁰ Venezia, Biblioteca Nazionale Marciana, gr. Z. 302 (coll. 730; *Diktyon* 69773), f. 14v, copied by Bessarion. As the figure represents $A\Gamma = \Gamma B$, which is not required, this diagram is “oversymmetrized”. On the diagrammatic practice as it is exhibited by the most ancient witnesses of the *Elements*, see F. ACERBI, Interazioni fra testo, tavole e diagrammi nei manoscritti matematici e astronomici greci, in: La conoscenza scientifica nell’Alto Medioevo. Atti della LXVII Settimana di Studio, Spoleto, 25 aprile – 1 maggio 2019. Spoleto 2020, 585–621: 591–594, 607–614, and 615–618.

²¹ Two lines are enough to explain concisely the proof, but writing it down so as to frame a rigorous argument is another matter: the entire proposition takes 59 lines of Heiberg’s edition.

algebra”²². This approach to Book II is not new: since Greek antiquity, commentators and mathematicians have tried to rewrite the proofs of these theorems, or to adapt them to other mathematical contexts, in particular number-theoretical or algebraic contexts²³. I shall describe what we read in Greek sources; these are Hero of Alexandria's alternative proofs of II 2–10, a few sections of George Pachymeres' *Quadrivium*, and Barlaam' *Demonstratio*.

Hero of Alexandria

To understand the way Hero of Alexandria's alternative proofs of *Elem.* II 2–10 are framed, let us read the first argument of his proof of *Elem.* II 8²⁴:

<Si linea in duas partes dividatur, eique in longum equalis uni dividendum linea iungatur: quod ex ductu totius iam composite in se ipsam fiat, equum erit his que ex ductu prioris lineae in eam que sibi adiecta est quater et quod ex ductu alterius dividensis in se ipsam.>

Ponam lineam ab quam supra punctum g dividam qualitercumque contingat divisio, et adiungam ei lineam bd equalem lineae bg .

(1) Cum ergo resolverimus quadratum lineae ad , resolveretur in probationem figure quarte huius partis. Quod ideo erit quoniam quadratum factum ex linea ad est equale duplo superficiei quam continent due lineae ab bd cum duobus quadratis factis ex duabus lineis ab bd (2) et quia bd posita est equalis sectioni bg , ergo duplum superficiei que continetur a duabus lineis ab bg , cum duobus quadratis factis ex duabus lineis ab bg est equale quadrato facto ex linea ad . (3) Secundum probationem vero figure septime huius partis erunt duo quadrata facta ex duabus lineis ab bg equalia duplo superficiei que continetur a duabus lineis ab bg cum quadrato ag ; (4) cum ergo illud coniungetur, erit quadruplum superficiei que continetur a duabus lineis ab bg cum quadrato ag equale duplo superficiei que continetur a duabus lineis ab bg cum duobus quadratis factis ex lineis ab bd ; (5) sed iam ostendimus quod ista sunt equalia quadrato facto ex linea ad ; ergo quadruplum superficiei que continetur a duabus lineis ab bg , cum quadrato ag est equale quadrato ad . Ergo iam resolutum est hoc in figuram quartam prius, post in figuram septimam. Et illud est quod demonstrare voluimus.

I shall set a line AB that I shall cut at random at point G , and I shall add to it a line BD equal to line BG .

(1) Therefore when we shall resolve the square of line AD , it will be resolved to the proof of the fourth proposition of this part. Then since it will be that the square made from line AD is equal to double the surface that the two lines AB , BD contain with the two squares made from the two lines AB , BD , (2) and since BD turns out to be set equal to segment BG , therefore double the surface that is contained by the two lines AB , BG with the two squares made from the two lines AB , BG is equal

²² The best-balanced critical overview of the paradigm of “geometric algebra”, a thorny issue of modern scholarship, can be read in I. MUELLER, *Philosophy of mathematics and deductive structure in Euclid's Elements*. Cambridge (MA) 1981, 41–53.

²³ For an overview of such avatars in Greek, Arabic, and Mediaeval mathematics (Barlaam included), see L. CORRY, *Geometry and arithmetic in the medieval traditions of Euclid's Elements: a view from Book II*. *Archive for History of Exact Sciences* 67 (2013) 637–705. For a discussion of the Heronian proofs, see F. ACERBI – B. VITRAC, *Héron d'Alexandrie, Metrica (Mathematica Graeca Antiqua 4)*. Pisa – Roma 2014, 36 and 368–371; for their logical import, see F. ACERBI, *The Logical Syntax of Greek Mathematics (Sources and Studies in the History of Mathematics and the Physical Sciences)*. Heidelberg – New York 2021, 65–68.

²⁴ Ed. P. M. J. E. TUMMERS, *Anaritus' Commentary on Euclid. The Latin Translation, I–IV*. Nijmegen 1994, 81, 20–82, 10 (the enunciation is not included in the text; Tummers adds the one found in the so-called Adelard II Arabo-Latin version, which I shall not translate). The entire sequence of Heronian proofs is at pages 73, 25–86, 5 of Tummers' edition. All these Heronian proofs comprise two equivalent arguments, one the inverse of the other; this feature is irrelevant to our purposes. Recall that scraps of Hero's commentary on the *Elements* are preserved, in Arabic and Arabo-Latin translations, only in the analogous commentary of the Persian mathematician an-Nayrīzī (*Anaritus*).

to the square made from line AD . (3) But according to the proof of the seventh proposition of this part, the two squares made from the two lines AB, BG will be equal to double the surface that is contained by the two lines AB, BG with the square of AG ; (4) therefore if we compose that, the quadruple of the surface that is contained by the two lines AB, BG with the square of AG will be equal to double the surface that is contained by the two lines AB, BG with the two squares made from lines AB, BD ; (5) and we have already proved that these are equal to the square made from line AD ; therefore the quadruple of the surface that is contained by the two lines AB, BG , with the square <from> AG is equal to the square <from> AD . Therefore this has now been resolved to the fourth proposition first, then to the seventh proposition. And that is what we wished to prove.

Using symbols, the enunciation can be transcribed as follows: if we cut ab at g and we set $bd = bg$, then $q(ad) = 4r(ab, bg) + q(ag)$ ²⁵. Hero's proof above operates on term $q(ad)$ of this equality and can be schematized as follows:

	equality	justified by
(1)	$q(ad) = 2r(ab, bd) + q(ab) + q(bd)$	II 4
(2)	$q(ad) = 2r(ab, bg) + q(ab) + q(bg)$	(1), $bd = bg$
(3)	$q(ab) + q(bg) = 2r(ab, bg) + q(ag)$	II 7
(4)	$2r(ab, bg) + q(ab) + q(bd) = 4r(ab, bg) + q(ag)$	(3) + $2r(ab, bg)$
(5)	$q(ad) = 4r(ab, bg) + q(ag)$	(2), (4)

If we exclude a mathematical trick as the introduction of segment bd , which is used in the *Elements* to generate the structure of double gnomon from which the factor “four” in the proof naturally arises, and by Hero to generate a linear configuration suited to applying *Elem.* II 4 and II 7, the above argument does not construct any geometric configuration. The proof carries out a twofold reduction. First, the reduction operates on the two geometric configurations that are constructed starting from the objects listed in the enunciation; it is required to prove that the constructed configurations are equal. One of the configurations is assumed as the starting point—this is $q(ad)$ in the proof just seen, $4r(ab, bg) + q(ag)$ in the inverse argument—the other as the end point; the starting point is reduced to the end point by means of theorems in the same sequence *Elem.* II 2–10, where theorem II 1 serves as a “principle”. Second, the declared goal of the proof is to set up a complete list of the theorems in the sequence *Elem.* II 2–10 to which the theorem at issue is reduced. In the case of II 8, such theorems (which are called *figure*) are II 4 and II 7; the items of this micro-list are expressly declared in the course of the proof and at the very end of it: *ergo iam resolutum est hoc in figuram quartam prius, post in figuram septimam*. The presence of such clauses shows that Hero's approach is eminently metalogical: they intimate that the goal of these alternative proofs—and, as seen above, contrary to Euclid's deductive strategy—is to reduce a proposition of the sequence II 2–10 to some other propositions that precede it in the sequence. The Heronian proofs are thus conceived both as a deployment of the deductive structure in its complete form and as a decomposition of the geometric configuration in its ultimate components.

George Pachymeres

In sects. 47–54 of the arithmetical “way” of his *Quadrivium*, George Pachymeres proposed arithmetical versions of *Elem.* II 3–10. All the proofs are carried out by using specific numerical data. Let us read sect. 52, which rewrites *Elem.* II 8²⁶:

²⁵ The sign $q(a)$ denotes the square on straight line a ; the sign $r(a, b)$ denotes the rectangle contained by straight lines a and b .

²⁶ Ed. P. TANNERY, *Quadrivium de Georges Pachymère (StT 94)*. Città del Vaticano 1940, 81, 22–31.

Ἐὰν ἀριθμὸς ὅστισοῦν τμηθῆ ὡς ἔτυχεν, ὁ τετράκις ὑπὸ τε τοῦ ὅλου καὶ ἐνὸς τῶν ἀφαιρέματων προμήκης ἀριθμὸς (δηλονότι οἱ τέσσαρες τοιοῦτοι ἀριθμοί) μετὰ τοῦ ἀπὸ τοῦ λοιποῦ ἀφαιρέματος τετραγώνου ἴσος ἐστὶ τῷ ἀριθμῷ τῷ τετραγώνῳ τῷ συνισταμένῳ ἐκ τοῦ ὅλου ἀριθμοῦ καὶ τοῦ εἰρημένου ἀφαιρέματος ὑφ' ἐν τῶν δύο ἀριθμῶν γεγονότων.

Οἷον ἔστω ὁ ιβ. οὗτος τεμηθήσθω ὡς ἔτυχεν εἰς η καὶ δ· τετράκις γοῦν τὰ ιβ, μη· καὶ ὀκτάκις τὰ η, ξδ· οὗτος ὁ ἀριθμὸς συνάμα τετράκις, ἤγουν ὁ μη τετράκις, ρρβ· οὗτος μετὰ τοῦ ξδ, σνς· οὗτος ὁ ἀριθμὸς ἴσος ἔσται τῷ τετραγώνῳ τῷ γεγονότι ἀπὸ τε τοῦ ὅλου ἀριθμοῦ τοῦ ιβ καὶ τοῦ δηλωθέντος ἀφαιρέματος τοῦ δ ὑφ' ἐν γεγονότων· ις γὰρ ἐπὶ ις, σνς.

If any number as we please be cut at random, four times the oblong number <contained> both by the whole and by one of the removals (clearly, these four numbers) with the square on the remaining removal is equal to the square number constructed from the whole number and the said removal, the two numbers coming to be under one <number>.

For instance, let it be 12. This be cut at random into 8 and 4; then, four times 12, 48; and eight times 8, 64; this number all together four times, that is, 48 four times, 192; this with 64, 256; this number will be equal to the square resulting on both the whole number 12 and on the exhibited removal 4, coming to be under one <number>; for 16 times 12, 256.

The proof takes number 12, divides it into 8 and 4, and checks that $4r(12,4) + q(8) = 4 \times 48 + 64 = 192 + 64 = 256 = q(16) = q(12 + 4)$. Other theorems in the sequence are closed by sentences like “and the same argument for all numbers” (καὶ ἐπὶ πάντων ἀριθμῶν ὁ αὐτὸς λόγος), intended to convey generality to the argument. Pachymeres adopts the algorithmic code to perform his operations²⁷. His lexicon does not follow the standard of Greek mathematics: compare for instance the parenthetical explanation introduced by δηλονότι (“clearly”) and the qualifier “exhibited” (δηλωθείς); the “removals” (ἀφαιρέματα)²⁸ and the “oblong” (προμήκης) number that replace the Euclidean “segments” (τμήματα) and “rectangle” (ὀρθογώνιον), respectively; the adverbs γοῦν (“then”) and συνάμα (“all together”)²⁹; the incoherent designations of a square as a number “constructed from” (συνιστάμενος ἐκ) and “resulting on” (γεγονὼς ἀπὸ) another; the repeated use of the perfect tense of γίγνομαι (“to result”) to designate a number that is the result of an operation.

THE DEDUCTIVE STRUCTURE OF THE *DEMONSTRATIO*

Barlaam's approach is strictly arithmetical, yet different from Pachymeres'. He stresses this difference in the preface, when he writes that “it is possible to prove each of the[se theorems] by induction,³⁰ for every arithmetical problem can be proven by induction once we set out some particular

²⁷ For a description of the three stylistic codes adopted in Greek and Byzantine mathematics, see ACERBI, *The Logical Syntax*, sect. 1, § 1–3.

²⁸ This late coinage finds its first occurrences in Philo of Alexandria (5), in Flavius Josephus (1), and especially in the Septuaginta (40). The occurrences in a later metrological compilation (1) and in Peditasimos' *Geometria* (3) refer to something that is actually removed.

²⁹ The latter adverb was belated by Pachymeres, as a TLG search shows (Pachymeres scores 215 occurrences out of 826 in the entire Greek corpus).

³⁰ The term “induction” (ἐπαγωγή) is canonically applied to any mental act that allows us to recognize the generality of a state of affairs from a particular example; this is exactly Pachymeres' approach. This meaning of ἐπαγωγή is already Aristotelian: see R. MCKIRAHAN, Aristotelian Epagoge in *Prior Analytics* 2. 21 and *Posterior Analytics* 1. 1. *Journal of the History of Philosophy* 21 (1983) 1–13, with bibliography. For Greek mathematical sources, see Eutocius, in *Sph. cyl. II 4* (ed. J. L. HEIBERG, *Archimedis opera omnia cum commentariis Eutocii*. I–III. Lipsiae 1910–15 III 120, 5–11), who dismisses previous expositions of compounded ratios—by Pappus, by Theon, which we read at *in Alm. I 13* (ed. A. ROME, *Commentaires de Pappus et de Théon d'Alexandrie sur l'Almageste [StT 54, 72, 106]*. I–III. Città del Vaticano 1931–43 II 532, 1–535, 9), and by some Arcadius—as “inductive”. At the beginning of his preface, Barlaam asserts that “the mathe-

numbers to which the general argument applies. But since this is quite amateurish and in anyone's wheelhouse, I regarded it as mandatory, by disregarding the proof by induction, to set out a demonstrative theory of them". Accordingly, Barlaam transforms *Elem.* II 1–10 into a sequence of number-theoretical propositions. Let us read the enunciation of his proposition 8:

Ἐὰν ἀριθμὸς εἰς δύο ἀριθμοὺς διαιρεθῆ, ὁ τετράκις ἐκ τοῦ ὅλου καὶ ἐνὸς τῶν μερῶν ἐπίπεδος μετὰ τοῦ ἀπὸ τοῦ λοιποῦ μέρους τετραγώνου ἴσος ἐστὶ τῷ ἀπὸ τοῦ ὅλου καὶ τοῦ προειρημένου μέρους ὡς ἀφ' ἐνὸς τετραγώνῳ.

If a number be divided into two numbers, four times the plane <number> from the whole and one of the parts with the square on the remaining part is equal to the square on the whole and the said part as on one.

The preface of the *Demonstratio* is followed by the definitions of two operations and of three objects. The operations are multiplication, in a wording that slightly modifies the formulation of *Elem.* VII def. 16, and the inverse relation of “measuring”, a notion that is not defined in the *Elements*. The defined objects are the “plane” number, which is assigned a different categorial status with respect to Euclid's terminology³¹; the “square” number, in a formulation different from *Elem.* VII def. 19; and, most importantly, the “part” of a number, in whose definition Barlaam merges *Elem.* VII def. 3 and 4³².

As for the deductive structure proper, Barlaam introduces a harmless but significant innovation³³: he interchanges *Elem.* II 1 and 2, which in the *Demonstratio* become propositions 2 and 1, respectively. As both propositions serve as “principles” in the deductive chain 1–10, the innovation does not change the deductive structure. It may be that Barlaam regarded his proposition 1 more basic because it involves a square. The Euclidean sequence will be restored in a manuscript copy of the *Demonstratio* penned by Bartolomeo Zamberti, a copyist-scholar and editor of Euclid, and in Dasypodius' *editio princeps* of 1564.

Barlaam's proofs hinge on basic arithmetical facts and on previous theorems in the sequence *Elem.* II 1–10 as rewritten by him. The most remarkable feature of such a rewriting is that the propositions, whose original, geometric proofs are independent of one another, are now organized as a connected deductive sequence, very much as in the strictly geometric Heronian rewriting of the same Book II. Propositions 1–3 are so basic that they cannot be linked in this way; from prop. 4 on, the preceding proposition(s) within the sequence that figure in each proof are as follows: 4 (2); 5 (4), 6 (4); 7 (4, 3); 8 (4, 7); 9 (5, 4); 10 (7, 4, 6). From proposition 7 on, no arithmetical results other than those just indicated are used; accordingly, the proofs and the diagrams are radically simplified. Propositions 7 and 8 are the only ones which are grounded on the same previous propositions both in Hero and in Barlaam.

The arithmetical facts that Barlaam uses as axioms are two general results: the distributivity law that a number that measures any of the numbers in a set also measures their sum (this is applied

maticians use in many instances the theorems of the second <book> of the Elements as if they were arithmetical". In ancient Greek sources, this claim is confirmed for instance by Diophantus' *De polygonis numeris*, which is crucially grounded on the number-theoretical counterpart of *Elem.* II 8, and by a standard procedure of extraction of an approximate square root, which applies the number-theoretical counterpart of *Elem.* II 4, see Theon of Alexandria's exposition in ROME, Commentaires II 469, 16–473, 8.

³¹ To Barlaam, a plane number is an ultimate numerical species, whereas in the *Elements* this is used as a name of genre; a number that is the result of the multiplication of two numbers is identified as “the one resulting from” (ὁ γενόμενος ἐκ) them.

³² In the *Elements*, a “part” (only in the singular) of a number is a divisor of it, “parts” (only in the plural) of a number is any number less than it and which is not a part of it in the said sense.

³³ No extant manuscript of the *Elements* interchanges these two propositions (B. Vitrac, *per litteras*).

passim)³⁴; the principle that “the numbers equimultiple of a same number are equal to one another” (props. 1–2, 5)³⁵; this principle is once used in a formulation that refers to the operation of “measuring” (prop. 3). Barlaam also applies the equals-plus-equals axiom (codified as *Elem.* I cn. 2) and replacement of equals with equals as a matter of course. Finally, in proposition 9 he applies a multiplication-rule of multiples³⁶.

As usual in Greek number theory, the generic numbers that figure in the proof are designated by letters and are represented by diagrams that depict them as line segments. The lettering is compositional, a convention whose mathematical import Barlaam employs many times in his proofs: if two numbers are designated by AB and BΓ, their sum is designated by AΓ (so, no need to look at any diagram).

As for the diagrams which we find in the prototype of the whole manuscript tradition and in most of its copies³⁷, they feature not only line segments and denotative letters, but also signs obviously standing for specific numbers intended to exemplify the theorems. These signs are Indian numerals of the Eastern form³⁸. As the prototype of the manuscript tradition was revised by Barlaam himself, we may wonder how this move can be consistent with the aims he so clearly expounds in the preface of the *Demonstratio*. However, Indian numerals also accompany almost all diagrams of the *Logistikē*, and attaching specifying numerals to diagrams of the *Elements* (mainly those of Books II and X) was an activity Byzantine readers of the Euclidean treatise have been engaged in since the 12th century³⁹. We may speculate that Barlaam added this metamathematical touch to his monographs in order to stress their scholarly character.

As all of Barlaam's enunciations are formulated in terms of multiplication, but his axioms use parts and the operation of measuring, his proofs of propositions 1–6 repeatedly use definition 1 to go back and forth from two numbers to their product and vice versa. To apply the compositionality of the lettering system, Barlaam must introduce new lettered designations for the numbers elsewhere designated by definite descriptions. The point is that, in order to apply his definitions, Barlaam needs to regard the same number both as a species of composite number, plane or square, and as the number that results from the multiplication of two numbers. For instance, the square on AB in prop. 1 is designated by Δ when it is regarded as the result one gets when AB is multiplied by itself. Barlaam introduces such one-letter designations in the specific part of a proposition called “construction” (this is the third paragraph in my segmentation of a proposition), but he must “translate back” such designations at the end of each proof of propositions 1–6. All in all, Barlaam proofs are well-thought, and his *Demonstratio* a remarkable piece of sophisticated mathematics.

³⁴ We may write this law $r(a,b) + r(a,c) + r(a,d) \dots = r(a,b + c + d \dots)$.

³⁵ We may write this principle as follows: if $b = c$, then $r(a,b) = r(a,c)$, where the Euclidean syntagm “equimultiples” (ισάκις πολλαπλάσιοι) captures both that the multiplier a is the same and that $r(a,b)$ and $r(a,c)$ are generated as multiples of a .

³⁶ We may write this rule as follows: if $a = k \times m \times c$ and $b = m \times c$, then $a = k \times b$.

³⁷ Zamberti (our siglum K below) “translated” the Indian numerals into Greek numerals and re-oriented the line segments from vertical to horizontal; a manuscript witness (siglum d) eliminated the numerals; two other manuscript witnesses (N and F) squarely eliminated all diagrams; Dasypodius' edition and an apograph of it modified all numerical values and used the numerals that were standard in middle-16th century West. In our notation, Barlaam's numerals are as follows (see the Iconographic Complement at the end of this paper). 1: 3, 5; 64; 24, 40. 2: 3, 4, 5; 6; 72; 18, 24, 30. 3: 3, 5; 40; 15, 25. 4: 3, 4; 49; 9, 12, 16, 12. 5: 5, 2, 3; 25; 21, 4; 9, 6, 6, 4. 6: 4, 4, 3; 49; 33, 16; 9, 12, 12, 16. 7: 3, 4. 8: 5, 3, 3. 9: 6, 3, 3. 10: 3, 3, 2. Dasypodius' corresponding numerals are as follows (note the defect of generality in prop. 2). 2: 4, 2; 36; 24, 12. 1: 2, 2, 2; 4; 24; 8, 8, 8. 3: 4, 2; 12; 8, 4. 4: 6, 2; 64; 36, 12, 4, 12. 5: 4, 2, 2; 16; 12, 4. 6: 3, 3, 4; 49; 40, 9. 7: 5, 3. 8: 6, 2. 9: 5, 3, 2. 10: 3, 3, 2.

³⁸ See Ch. BURNETT, Indian Numerals in the Mediterranean Basin in the Twelfth Century, with Special Reference to the “Eastern Forms”, in: From China to Paris: 2000 Years Transmission of Mathematical Ideas, ed. Y. Dold-Samplonius – J. W. Dauben – M. Folkerts – B. van Dalen (*Boethius* 46). Stuttgart 2000, 237–288.

³⁹ See the overview in F. ACERBI – B. VITRAC, Les mathématiques de Michel d'Éphèse. *REB* 80 (2022) 229–255: 240–241 and n. 26.

Barlaam’s proof of proposition 8 can be symbolically transcribed as follows. If ab is divided into two numbers at g , then $q(ab + bg) = 4r(ab, bg) + q(ag)$. Let bd be set equal to bg .

	equality	justified by
(1)	$q(ad) = q(ab) + q(bd) + 2r(ab, bd)$	prop. 4
(2)	$q(ad) = q(ab) + q(bg) + 2r(ab, bg)$	(1), $bd = bg$
(3)	$q(ab) + q(bg) = 2r(ab, bg) + q(ag)$	prop. 7
(4)	$q(ad) = 4r(ab, bg) + q(ag)$	(2), (3)
(5)	$q(ab + bg) = 4r(ab, bg) + q(ag)$	$ad = ab + bd = ab + bg$

LANGUAGE AND STYLE OF THE *DEMONSTRATIO*

Barlaam adheres strictly to the demonstrative stylistic code of Greek mathematics. He is even faithful to the sectorial idiom adopted in the arithmetic books VII–IX of the *Elements*, one of the traits of which is the presence of aorist and perfect tense of the verb “to make” (ποιέω) in specific formulaic expressions. Barlaam modifies only slightly the form of the enunciations of *Elem.* I.1–10, due allowance being made for the need to replace the lexical range of the verb “to cut” (τέμνω), ill-suited to number-theory, with “to divide” (διαίρω) in the case of verb forms, and with “part” (μέρος), in the case of nouns such as “segment” (τμήμα). The latter is Barlaam’s main lexical and conceptual innovation, as we have seen.

Barlaam’s proofs are extremely rigid as to style. The formulaic character of Greek mathematics is deployed without a single oversight and almost without *variationes*. We find in the *Demonstratio* more inversions of the order subject–object than in any random sequence of the *Elements*; the objects set out in the specific part of a proposition called “setting-out” are non-articular, as they should; the imperfect, the verb “to prove” (ἀποδείκνυμι), and the verb “to be supposed” (ὑπόκειμαι) are used exactly as they must be used; “and” (καί) and “with” (μετά or σύν) and the adjective “both ... together” (συναμφοτέρος) alternate as they should to express addition; apodotic “therefore” (ἄρα) in the consequent of a “paraconditional” is present without exceptions⁴⁰. Had the *Demonstratio* been transmitted anonymously—and very much as we identify a script as imitative because of its stiffness—we might even guess that it is a late scholarly product from such an unfailing adherence to the standard code. This guess is corroborated by several signs, which denounce Barlaam as a Byzantine mathematician: the presence of the uncanonical connectors “now then” (τοίνυν), in propositions 1 and 4, “then” (γούνη) in proposition 5, and “but of course” (ἀλλὰ μήν), in propositions 4 and 6; the elimination of the article in expressions like “the <number> from Γ , AB” (ὁ ἐκ τῶν Γ AB) when these have the syntactic function of nominal complement of the copula, whereas the article should be retained because it is a part of the designation syntagm; the formulation of equalities in the idiom of inequalities in proposition 9, where, for instance, Barlaam writes “the <number> from $A\Delta$, ΔB is less than that on AB by twice that on $\Gamma\Delta$ ” in place of “that on AB is equal to the <number> from $A\Delta$, ΔB and twice that on $\Gamma\Delta$ ”; the nexus “equally measured” (ἰσάκις μετρούμενοι) in proposition 3; the active imperative “let it multiply” (πολλαπλασιασάτω) in the construction of proposition 3⁴¹, for a participle is required; the use of “to be really” (ὑπάρχω) in proposition 4; pronominal “in between” (μεταξύ) in proposition 5; the adverb “unequally”

⁴⁰ See ACERBI, *The Logical Syntax*, sect. 5, § 3, 2. A “paraconditional” is made of a causal subordinate introduced by “since” (ἐπεὶ) and of a principal clause. More generally, see the same book for all features of Greek demonstrative style mentioned in the text.

⁴¹ Barlaam provides the only occurrences of this verb form in the Greek and Byzantine technical corpus; the third occurrence is in *Log.* VI 16 (106, 19 CARELOS).

(ἀνισαχῆ) in the setting-out of proposition 5, which is a *hapax* in the entire Greek and Byzantine corpus.

An interesting feature of the *Demonstratio* is that the general conclusion of a theorem is often, and to various degrees, shortened in the prototype of the manuscript tradition; the other witnesses made a mess out of this⁴².

MANUSCRIPT WITNESSES OF THE *DEMONSTRATIO*

The manuscript witnesses of Barlaam's *Demonstratio* are listed below; they are arranged by increasing *Diktyon* number. I anticipate such family relations among them as can be guessed on the basis of the contents of the manuscripts and on the stemmas established in critical editions of treatises other than Barlaam's⁴³:

F. Firenze, Biblioteca Nazionale Centrale, Fondo Nazionale II.III.428 (Vitelli 1; *olim* Magl. XI.8; *Diktyon* 16954)⁴⁴, middle of 16th century⁴⁵. The *Demonstratio* is on ff. 195r–201r. Other works in the manuscript: ff. 1r–31r Aristoxenus, *Elementa harmonica* I–III; 35r–112r, 123r–125r Ptolemy, *Harmonica* I–III; 125v–126v *scholium in Ptolemaei Harmonica*⁴⁶; 113r–122v, 129r–182v Porphyry, *in Ptolemaei Harmonica* (excerpts)⁴⁷; 185r–186v *excerpta optica e Gemino*; 186v–191r Damianus, *Opticae Hypotheses*⁴⁸; 204r–210v Hippocrates, *excerpta de urinis*; 213r–214v Barlaam, *De eclipsi* I (incomplete)⁴⁹.

The contents of this manuscript already show that most of it must descend from Neap. III.C.2 (our witness N), see below. However, the contents of F and N do not fit exactly: the former also contains Ptolemy's *Harmonica* and Porphyry, the latter Gaudentius, Theon of Smyrna, Cleonides (both versions), Euclid, the *Excerpta Neapolitana*, and two other treatises by Barlaam. It is not said that F drew Ptolemy's *Harmonica* from another source: we must not forget that the ateliers and their copyist first and foremost produced sets of loose quires containing one or more works, and that, as a rule, only parts of these sets were given the stable form of a codex. In his edition of Ptolemy's *Harmonica*, I. Düring postulates two hyparchetypes between F and its most remote ancestor München, Bayerische Staatsbibliothek, Cod. graec. 361a (end of 13th century; *Diktyon*

⁴² As the general conclusion of a theorem is identical to its enunciation apart from the insertion of a liminal particle “therefore” (ἄρα), restoring shortened general conclusions is very easy.

⁴³ For descriptions of the same manuscripts in the editions of Barlaam's scientific treatises other than the *Demonstratio*, a reference to the relevant pages of CARELOS, Βαρλαάμ (*Logistikē*), MOGENET – TIHON – DONNET, Barlaam (*De eclipsi* I and II), TIHON, Barlaam (*De paschate*), and DÜRING, Die Harmonielehre (*Refutatio*) is understood. I use the abbreviations and numberings in SINKEWICZ, The Solutions 185–194 for Barlaam's non-scientific works: Or. = Greek discourses; AL = Antilatin treatises; EG = Greek Letters.

⁴⁴ The two Florence manuscripts are described in VITELLI, Indice de' codici greci 540–541 and 543–544, respectively. The manuscript witnesses of the *Demonstratio* that contain treatises of harmonic theory are also described in J. MATHIESEN, Ancient Greek Music Theory. A Catalogue Raisonné of Manuscripts. München 1988 (F is Mathiesen's n. 157; the siglum of this manuscript in most editions of the treatises of harmonic theory it contains is Fn).

⁴⁵ Two hands are engaged in the copy: 1–186 and 204–214; 195–201. The watermark range is 1535–55. I use here the descriptive file, by D. Speranzi, at https://manus.iccu.sbn.it/opac_SchedaScheda.php?ID=290392.

⁴⁶ The scholium is edited in DÜRING, Die Harmonielehre 60–61. This scholium is characteristic of the stemmatic family mentioned below and of Isaak Argyros' recension of the *Harmonica*, witnessed in the autograph manuscript Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 176 (*Diktyon* 66807), see f. 133r–v. On Argyros, see most recently A. GIOFFREDA, Tra i libri di Isacco Argiro (*Transmissions* 4). Berlin – Boston 2020.

⁴⁷ See DÜRING, Porphyrios Kommentar zur Harmonielehre des Ptolemaios (*Göteborgs Högskolas Årsskrift* 38). Göteborg 1932 XXVI–XXVIII and XXIX (stemma).

⁴⁸ The latter two texts are edited in R. SCHÖNE, Damianos Schrift über Optik. Berlin 1897, 22, 14–30, 11 and 2, 1–22, 9, respectively. In both cases, F belongs to the same family as Neap. III.C.2 (our witness N).

⁴⁹ *De eclipsi* I *des. mut.* MOGENET – TIHON – DONNET, Barlaam 56 line 80 εἰσαγαγών.

72898)⁵⁰; one of these hyparchetypes might well have been a part of the copying campaign that partly got a stabilization in N⁵¹. We also know that some codices of the Magliabechi collection in Florence's Biblioteca Nazionale Centrale are the result of stabilizing a collection of manuscripts originally kept as loose quires in the library of the mathematician and Arabist Giovanni Battista Raimondi (1536–1614)⁵²; among them figures a copy of Theon of Smyrna, now lost, which might well be a further item copied from N.

- R. Firenze, Biblioteca Riccardiana 1192 (*Diktyon* 17111), composite, 1565–67 (watermarks)⁵³. The *Demonstratio* is on ff. 76r–87r; a Latin translation of it, coinciding with Dasypodius', is on ff. 89r–98v. Other Greek works in the manuscript: ff. 72r–75v *scholia in Aristarchum*.

This codex is an assemblage of disparate—and codicologically inhomogeneous—texts written in Greek, Latin, and Italian. According to B. Noack⁵⁴, the Aristarch scholia, copied *ca.* 1515 (watermark), are drawn directly from the manuscript Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 204 (9th century; *Diktyon* 66835), which is the most authoritative witness of Aristarchus' text. The presence of the Latin translation is enough to show that R is a copy—in fact, it is a conformal copy—of Dasypodius' edition.

- K. Kremsmünster, Stiftsbibliothek, Ms. 343 (*Diktyon* 37295), 1505, copyist Bartolomeo Zamberti (entire manuscript)⁵⁵. The *Demonstratio* is on ff. 126v–133r. Other works in the manuscript: ff. 1r–58r Theodosius, *Sphaerica* I–III; 58v–126r Barlaam, *Logistikē* I–VI; 133v–134r George Gemistos Pletho, *Problema geometricum*⁵⁶.

Zamberti also copied, and subscribed on December 12, 1505 (f. 434r), the manuscript Leiden, Universiteitsbibliotheek, B.P.G. 7 (*Diktyon* 42700), which contains *excerpta e Procli commentarium in Elem. I*; Euclid, *Elementa* I–XII 5, *Catoptrica*, *Phaenomena* b, *Optica* B, *Data*; Marinus *in Data*⁵⁷. The two manuscripts have the same layout and identical codicological features: they are the stabilization of a set of independent blocks of quires copied during one and the same copying campaign. Both codices were owned by Gian Francesco d'Asola⁵⁸, whose library is now almost entirely preserved in the Bibliothèque nationale de France. As we shall see, Zamberti used Marc. gr. Z. 302 (our witness M¹) as a model for the texts now included in Krems. 343⁵⁹. However, he employed the Venice manuscript only as a support for the treatises now included in Leid. B.P.G.

⁵⁰ On this manuscript, see F. ACERBI – A. GIOFFREDA, *Manoscritti scientifici della prima età paleologa in scrittura arcaizzante*. *Scripta* 12 (2019) 9–52 *passim*, and further below.

⁵¹ See DÜRING, *Die Harmonielehre LVIII–LIX and LXIX (stemma)*.

⁵² The *membra disiecta* of the library of Raimondi have been identified, on the basis of an inventory penned by Gian Vincenzo Pinelli, in C. GIACOMELLI, *I libri greci di Matteo Macigni*. *Contributo allo studio di una biblioteca umanistica*. *La Parola del Passato* 74 (2019) 361–420: 412–415.

⁵³ The watermarks were identified by D. Speranzi, unpublished descriptive file. See also the description by D. Nardi at https://manus.iccu.sbn.it/opac_SchedaScheda.php?ID=87238.

⁵⁴ See B. NOACK, *Aristarch von Samos. Untersuchungen zur Überlieferungsgeschichte der Schrift Περὶ μεγεθῶν καὶ ἀποστημάτων ἡλίου καὶ σελήνης (Serta Graeca 1)*. Wiesbaden 1992, 112–117.

⁵⁵ A dated subscription is found on ff. 58r and 133r. On this manuscript, see E. GOLLOB, *Verzeichnis der griechischen Handschriften in Österreich außerhalb Wiens (Sitzungsberichte der Kais. Akad. der Wissenschaften in Wien. Phil.-Hist. Klasse CXLVI)*. Wien 1903, 31–33.

⁵⁶ Pletho's text is edited in F. ACERBI – S. MARTINELLI TEMPESTA – B. VITRAC, *Gli interventi autografi di Giorgio Gemisto Pletone nel codice matematico Marc. gr. Z. 301. Segno e Testo* 14 (2016) 411–456. Manuscripts K and Par. gr. 2384 (our P²) are two new witnesses of Pletho's geometric problem. See Appendix 1 for a collation of their texts.

⁵⁷ See K. A. DE MEYER, *Codices Bibliothecae Publicae graeci (Bibliotheca Universitatis Leidensis, Codices Manuscripti VIII)*. Lugduni Batavorum 1965, 10–12.

⁵⁸ See A. CATALDI PALAU, *Gian Francesco d'Asola e la tipografia aldina. La vita, le edizioni, la biblioteca dell'Asolano*. Genova 1998.

⁵⁹ It is likely that Theodosius' treatise was also copied from Marc. gr. Z. 302. The most recent critical editor records this witness, but did not collate it: see C. CZINCZENHEIM, *Édition, traduction et commentaire des Sphériques de Théodose*. PhD Thesis, Université de Paris IV – Sorbonne 2000. This witness of Barlaam's *Logistikē* is unknown to Carelos.

7, for the model for *Phaenomena* **b**, *Data*, Marinus in *Data*, *Optica* **B**, and *Catoptrica*, is Monac. gr. 361a⁶⁰; the model for *Elem.* I–XI is the manuscript Paris, Bibliothèque nationale de France, gr. 2344 (1120–40; *Diktyon* 51976)⁶¹; the lacuna at *Elem.* VIII 25–IX 14 in the Paris manuscript was filled, up to the end of Book IX, by means of Marc. gr. Z. 302; as for the short segment *Elem.* XII 1–5, Zamberti used the manuscript Leiden, Universiteitsbibliotheek, Scal. 36 (14th century; *Diktyon* 37988; it only contains *Elem.* XI–XV), which he also retained, for *Elem.* XII–XIII, as the Greek model of his Latin translation, printed in 1505⁶².

- A. Milano, Biblioteca Ambrosiana, E 76 sup. (gr. 292; *Diktyon* 42700)⁶³, composite, ca. 1340, Barlaam's copyist III⁶⁴. The *Demonstratio* is on ff. 172r–178r. Other works in the manuscript: ff. 1r–107v Ptolemy, *Harmonica* I–III; 108r–110v *varia mathematica*; 111r–172r Barlaam, *Logistikē* I–VI; 178v–190r Barlaam, *Refutatio*; 191r–243r Barlaam, *De Spiritu Sancto adv. Latinos* (AL 1–6); 243r–247v Barlaam, *De principatu Papae* (AL 7); 247v–252r Barlaam, *Legatus seu de Spiritu Sancto* (AL 11); 252r–254r Barlaam, *Confutatio dogmatis Latinorum* (AL 10); 254v–267v Barlaam, *Solutiones ad Georgium Lapitham*; 268r–283v Barlaam, *Oratio de concordia* (Or. 1); 283v–290r Barlaam, *Oratio ad Synodum de unione* (Or. 2); 291r–294v Barlaam, *De eclipsi* I; 294v–300v Barlaam, *De eclipsi* II; 301r–v *scholium de cyclo lunari et computationes astronomicae*; 302r–305v, 290v (*alia manu*) Barlaam, *De paschate*.

Barlaam's works in this manuscript are penned by two of his collaborators. Contrary to what happens in Marc. gr. Z. 332 (our witness M), he did not revise his own texts, which exhibit typical copying mistakes. A is the sole independent witness for the *varia mathematica* it carries on ff. 108r–110v, which I have used for my edition in Appendix 2 below. Düring shows that A is a copy of Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 185 (composite; first half of 14th century; *Diktyon* 66816) as for Ptolemy's *Harmonica*⁶⁵.

- a. Milano, Biblioteca Ambrosiana, P 72 sup. (gr. 626; *Diktyon* 43103), composite, 1563, copyist <Nicaise Hellbaut (Ellebodius)>⁶⁶. The *Demonstratio* is on ff. 54v–59r. Other works in the manuscript: ff. 1r–54v Barlaam, *Logistikē* I–VI; 59r–61v *varia mathematica*; 62r–65r Barlaam, *De eclipsi* I; 65r–70v Barlaam, *De eclipsi* II; 71r–v *scholium de cyclo lunari et computationes astronomicae*;

⁶⁰ For *Data*, *Optica* **B**, and *Phaenomena* **b** see HEIBERG – MENGE, Euclidis VI XXIII, VII XXIV, and VIII XXIX–XXX, respectively.

⁶¹ On this manuscript, see F. ACERBI – A. LAMI, Una pagina di antropologia filosofica nel codice matematico *Par. gr. 2344. Galenos* 8 (2014) 133–148.

⁶² The findings, here summarized, that concern the *Elements* are argued in B. VITRAC, A propos de l'histoire du texte des *Éléments* d'Euclide : Préalables à une nouvelle édition critique. 2022. hal-03328161, sect. 3, § VI "Les modèles de Zamberti".

⁶³ The three Milan manuscripts are described in E. MARTINI – D. BASSI, *Catalogus codicum Graecorum Bibliothecae Ambrosianae*. I–II. Milano 1906, 326–328, 711–712, and 836–839, respectively. On Pinelli's library, see M. GRENDLER, A Greek Collection in Padua: The Library of Gian Vincenzo Pinelli (1535–1601). *Renaissance Quarterly* 33 (1980) 386–416 (407–408 on Manuel Moros, see below) and, more recently, A. M. RAUGEI, Gian Vincenzo Pinelli e la sua biblioteca (*Cahiers d'Humanisme et Renaissance* 151). Genève 2018.

⁶⁴ On this copyist, who penned ff. 111r–190r and 291r–305v, see A. GIOFFREDA, Su scrittura, libri e collaboratori di Barlaam calabro. *Segno e Testo* 14 (2016) 361–378. Ff. 191r–290r of Ambr. E 76 sup. were copied by Barlaam's copyist II, who is also found in Marc. gr. Z. 332 (see below); other hands are engaged in the copy, for instance on ff. 108r–110v. Ff. 2r–v, 8r–10v, 38v–40v are in the hand of Theodoros Rentios (S. MARTINELLI TEMPESTA, Per un repertorio dei copisti greci in Ambrosiana, in: *Miscellanea Graecolatina I*, a cura di F. Gallo. Roma 2013, 101–153: 140; see also *RGK* III 215 [a copy is subscribed in 1557]).

⁶⁵ See DÜRING, Die Harmonielehre LXV and LXIX (stemma).

⁶⁶ Hellbaut copied ff. 1–42, 44–103v, and 120v–127r; a second hand added f. 43; a third hand penned ff. 104r–120r; Lazzaro Bonamico (†1552) copied ff. 128r–134r (the latter is identified in MARTINELLI TEMPESTA, Per un repertorio 139). On Hellbaut (†1577), a distinguished scholar and a close associate of Pinelli, who acquired his library, see S. MARTINELLI TEMPESTA, L'Isocrate di Michele Sofianòs. *Acme* 58 (2005) 301–316: 308–309 and n. 26, with a rich bibliography, to which F. SCHREIBER, Unpublished Renaissance Emendations of Aristophanes. *TAPA* 105 (1975) 313–332 can be added.

72r–75v Barlaam, *De paschate*; 75v–80v Barlaam, *De principatu Papae* (AL 7); 80v Marcus Eugenicos, *De Spiritu Sancto* (excerpt); 81r–127r Barlaam, *De Spiritu Sancto adv. Latinos* (AL 1–6); 128r–134r Theodoros Gaza, *epistulae tres*.

Ambr. E 76 sup. (our witness A) and P 72 sup. are the only witnesses of Barlaam's scientific writings that also contain dogmatic works of his.

- L. Milano, Biblioteca Ambrosiana, R 117 sup. (gr. 724; *Diktyon* 43201), composite, ca. 1565, copyist <Manuel Moros>⁶⁷. The *Demonstratio* is on ff. 127r–131v. Other works in the manuscript: ff. 3r–47r Herennios, in *Aristotelis Metaphysica*; 52r–81v [Georgius Codinos], *De officiis*; 86r–127r Barlaam, *Logistikē* I–VI; 132r–140r Barlaam, *Refutatio*; 140v–143v Barlaam, *De eclipsi* I; 143v–148r Barlaam, *De eclipsi* II; 148r–149r *scholium de cyclo lunari et computationes astronomicae*; 149v–152v Barlaam, *De paschate*; 152v–155r *varia mathematica*; 158r–188r Dionysius of Halicarnassus, *De Thucydide*; 191r–195r, 196r–200r, 203r–207r (triplex) *De comoedia*; 209r–225r, 247r–262v (duplex), 265r–266r Georgius Hamartolos, *Chronicon* (fragm.); 227r–235r Iohannes Scylitzes, *Synopsis Historica* (excerpts) *cum additamenta Theodori Scutariotae*; 237r–239v Theodoros Gaza, *Epistula de origine Turcarum*; 241r–v, 242r–v (duplex) Polyaeus, *Stratagemata* (excerpt); 243r–244v *apographum inscriptionis*; 269r–270v, 273r–274v (duplex) *Vita Aristotelis*; 277r–323v Origenes, *Contra Celsum cum Gennadii Scholarii scholia*.

This codex is a recent assemblage of parts of several manuscripts. The Barlaam block was entirely copied by Manuel Moros, one of Pinelli's favourite calligraphists.

- m. Moskva, Gosudarstvennyj Istoriceskij Musej, Sinod. gr. 315 (Vlad. 441; *Diktyon* 43940), 1590–1600, copyist <Maximus Margounios>⁶⁸. The *Demonstratio* is on ff. 290v–294r. Other works in the manuscript: ff. 1–13v Leo medicus, *Conspectus medicinae*⁶⁹; 16r–129r Arethas, *Opuscula*; 135r–159r Barlaam, *epistulae tres* (EG 1–3); 161r–251v Demetrios Cydones, *De processione Spiritus Sancti ad amicum*; 252r–255r Manuel Chrysoloras, *De processione Spiritus Sancti*⁷⁰; 257r–290v Barlaam, *Logistikē* I–VI; 294r–296v Barlaam, *De paschate*; 297r–303v Barlaam, *Refutatio*; 303v–307v Barlaam, *epistulae quinque* (EG 4–8); 307v–309r Barlaam, *De eclipsi* I; 309v–312v Barlaam, *De eclipsi* II; 313r *scholium de cyclo lunari et computationes astronomicae*; 317r–321v Nikephoros Gregoras, *excerptum e Byzantina Historia X* 8; 321v–327r Nikephoros Gregoras, *In annuntiationem Deiparae*; 327r–328r Nikephoros Gregoras, *excerpta*; 331r–334v Nikephoros Gregoras, *Epistula*

⁶⁷ On Moros, see *RGK* I 252 (which confirms the identification by Martini and Bassi), II 348, III 417; P. GÉHIN, Évagre le Pontique dans un recueil de mélanges grammaticaux du fonds Pinelli, l'Ambr. C 69 sup., in: Nuove ricerche sui manoscritti greci dell'Ambrosiana. Atti del Convegno Milano, 5–6 giugno 2003, a cura di C. M. Mazzucchi – C. Pasini. Milano, 2004, 265–313. The four hands engaged in the copy are distributed as follows: Camillo Zanetti on ff. 3–47, 191–241, 243–323; an anonymous hand supplies f. 242; Maximus Margounios on ff. 52–81 (†1602; *RGK* I 259 [which identifies the copyist], II 356, III 427; G. FEDALTO, Massimo Margunio e il suo commento al “De trinitate” di S. Agostino [1588]. Brescia 1967; F. CICOLELLA, Maximos Margounios and Anacreontic Poetry: An Introductory Study, in: Greeks, Books and Libraries in Renaissance Venice, ed. R. M. Piccione [*Transmission* 1]. Berlin – Boston 2021, 147–160, with recent bibliography), who also copied the entire Mosq. Mus. Hist. gr. 315 (our witness m) and was engaged in the copy of Vat. gr. 1756 (our witness v); Manuel Moros on ff. 86–155 and 171–188. Ff. 158–170 were excised from the Aldine nr. 1559 of Dionysius of Halicarnassus.

⁶⁸ Margounios penned the entire manuscript with the exception of ff. 16r–26v, which contain Arethas' first pamphlet. The best description of this manuscript is found in L. G. WESTERINK, *Arethae archiepiscopi Caesariensis scripta minora*. I–II. Lipsiae 1968–72 I IX–XVIII. This volume of the edition of Arethas' writings exactly comprises all works contained in the Moscow manuscript (57 items), which is the sole witness of most of them.

⁶⁹ Leo medicus is edited in F. Z. ERMERINS, *Anecdota Medica Graeca*. Lugduni Batavorum 1840, 79–275.

⁷⁰ See A. SPOURLACOU, *Einai o Manouel Chrysoloras o suggrapheus tou ergou Kephalaia oti kai ek tou uiou to agion pneuma ekporeuetai*. *Θησαυρία* 2 (1963) 88–117.

XVI⁷¹; 334v–339v Nikephoros Gregoras, *Oratio in Deiparam*⁷²; 341r–351v Nikephoros Gregoras, *Vita Iohannis episcopi Heracleensis*; 360r–366r Maximus Planudes, *De compassione*; 366v–401r Maximus Planudes, *Laudatio Sanct. Petri et Pauli*; 401v–419v Maximus Planudes, *In Sanctum Diomedem*; 420r–440r Maximus Planudes, *Basilikos logos*⁷³; 440r–442v *excerpta e Planudis operis et epistulis*; 442v–443r Maximus Planudes, *Versus politici*.

The copyist Maximus Margounios collaborated with Alvis Lollino in the copy of Vat. gr. 1756 (our witness v). Westerink shows that several works contained in Mosq. Mus. Hist. gr. 315 were copied from manuscripts held in Venice: Barlaam's works from Marc. gr. Z. 332 (our witness M); the two treatises on the procession of the Holy Spirit from the manuscript Venezia, Biblioteca Nazionale Marciana, gr. Z. 157 (coll. 399; 1442; *Diktyon* 69628); Gregoras' and Planudes' works from two manuscripts held in the library of the monastery of St Anton in Venice, destroyed by a fire in 1687.

- N. Napoli, Biblioteca Nazionale Centrale Vittorio Emanuele III, III.C.2 (*Diktyon* 46278), composite, ca. 1493–94 (watermarks), copyist <Iohannes Rhosos>⁷⁴. The *Demonstratio* is on ff. 83r–88v. Other works in the manuscript⁷⁵: ff. 1r–5r Hippocrates, *excerpta de urinis*⁷⁶; 6r–13r Gaudentius, *Introductio harmonica*; 13v–15v Theon of Smyrna, *excerpta musica*; 16r–21r [Pappus], *immo* Cleonides, *Introductio harmonica*; 21r–41v Aristoxenus, *Elementa harmonica* I–III; 41v–45r *Excerpta Neapolitana*; 45v–46v *excerpta optica e Gemino*; 47r–50r Damianus, *Opticae Hypotheses*; 55r–63v Cleonides, *Introductio harmonica*; 63v–68v Euclid, *Sectio canonis*; 73r–80v Barlaam, *Logistikē* I; 89r–92v Barlaam, *De eclipsi* I; 92v–98v Barlaam, *De eclipsi* II.

⁷¹ See the edition in S. BEZDEKI, Nicephori Gregorae epistulae XC. *Ephemeris Dacoromana* 2 (1924) 239–377: 303–311.

⁷² This item is edited in L. G. WESTERINK, Nikephoros Gregoras, Dankrede an die Mutter Gottes. *Helikon* 7 (1967) 259–271.

⁷³ These four Planudean works are edited in *PG* 147, 985–1016 and 1017–1112; L. G. WESTERINK, Trois textes inédits sur Saint Diomède de Nicée. *AnBoll* 84 (1966) 161–227; L. G. WESTERINK, Le *Basilikos* de Maxime Planude. *BSI* 27 (1966) 98–103; 28 (1967) 54–67; 29 (1968) 34–50, respectively.

⁷⁴ This manuscript is described in M. FORMENTIN – F. RICETTI – L. SIBEN, *Catalogus codicum Graecorum Bibliothecae nationalis Neapolitanae. Volumen III*. Roma 2015, 87–89. The codex comprises two different codicological units, penned by two different, albeit contemporary, copyists, on ff. 1–54 and 55–100. On Rhosos, a copyist celebrated for his *mira in transcribendo celeritas* and one of Bessarion's favourite copying-machines, see *RGK* I 178; II 237; III 298; F. MAVROIDI-PLOUMIDI, Eggrapha anapheromena stis erides tōn Ellēnōn tēs Benetias sta telē tou IE' aiōna. *Θησαυρίσματα* 8 (1971) 115–187: 130–138 and pl. 2 (edition and reproduction of two autograph letters, penned in Rhosos' cursive script); C. SCHIANO, Sulla tradizione del *De febribus* dello pseudo-Alessandro di Afrodisia (con appunti sulla lista di Lascaris). *Bollettino dei Classici*, 3rd s., 26 (2005) 39–67: 50–60 (on Rhosos' late career); D. SPERANZI, Omero, i cardinali e gli esuli. Copisti greci di un manoscritto di Stoccarda. Madrid 2016 *passim*; D. SPERANZI, Scritture, libri e uomini all'ombra di Bessarione. I. Appunti sulle lettere del Marc. Gr. Z. 527 (coll. 679). *Rinascimento* 57 (2017) 137–197: 172 n. 125 (identification of the earliest manuscript copied by Rhosos).

⁷⁵ All musical treatises in this manuscript are copied from the coherent set of quires now dismembered in the relevant parts of the manuscript Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 2338 (end of 13th century; *Diktyon* 68969) and of Monac. gr. 361a: F. ACERBI – A. GIOFFREDA, Harmonica Membra Disiecta. *GRBS* 59 (2019) 646–662 (for the Renaissance trajectory of Vat. gr. 2338, see the complementary study L. CALVIÉ, Un manuscrit médiéval d'anciens musicographes grecs : le Vaticano, BAV, gr. 2338. *Script* 74 [2020] 219–250). As for Gaudentius, see K. VON JAN, *Musici Scriptores Graeci*. Lipsiae 1895, LI–LIV (Neap. III.C.2) and 319–356 (edition, but see in particular 326). As for Cleonides, see J. SOLOMON, Cleonides: Εἰσαγωγή ἁρμονική; Critical Edition, Translation, and Commentary. PhD Thesis, University of North Carolina 1980, 60–99, and J. SOLOMON, Vaticanus gr. 2338 and the Εἰσαγωγή ἁρμονική. *Philologus* 127 (1983) 247–253. As for Aristoxenus, see R. DA RIOS, *Aristoxeni Elementa Harmonica*. Roma 1954, LXXV and LXXIX–LXXXI. As for the so-called *Excerpta Neapolitana* (edited in VON JAN, *Musici* 411–420 and 266–271), see F. ACERBI – S. PANTERI, Eratosthenes in the *Excerpta Neapolitana*. *GRBS* 59 (2019) 663–679. As for Damianus, see again ACERBI – GIOFFREDA, *Harmonica Membra* 9–10. As for Euclid, see A. BARBERA, *The Euclidean Division of the Canon. Greek and Latin Sources*. Lincoln (NE) and London 1991, 67–68. In its turn, the Naples manuscript is the model of the manuscripts Firenze, Biblioteca Riccardiana 41 (middle of 16th century; *Diktyon* 17041) and BNCF, Fondo Naz. II.III.428, which is our witness F (see again the just mentioned references). The excerpt from Theon of Smyrna's *Expositio* is E. HILLER, *Theoni Smyrnaei philosophi platonici Expositio rerum mathematicarum ad legendum Platonem utilium*. Leipzig 1878, 46, 20–57, 6.

⁷⁶ See H. DIELS, *Die Handschriften der antiken Ärzte*. I–II. Berlin 1905–6 I 44.

The extracts from Geminus precede Damianus, as in BNCF, Fondo Naz. II.III.428 (our witness F).

P¹. Paris, Bibliothèque nationale de France, gr. 2381 (*Diktyon* 52013), composite, ca. 1371–73 (certainly before 1392: tables on ff. 100r and 101r; later note of the main hand on f. 104v)⁷⁷. The *Demonstratio* is on ff. 30v–32r. Other works in the manuscript: ff. 1r–2v *notae chronologicae et metrologicae*; 3r–12v Maximus Planudes, *Psephophoria secundum Indos*; 13r–30v Barlaam, *Logistikē* I–VI; 32r–35r Barlaam, *Refutatio*; 35v–41v Gregorius Palamas, *Physica, theologica moralia et practica capita* CL⁷⁸; 41v–46v Gregorius Palamas, *Pro Hesychastis Orationes duo*⁷⁹; 46v *excerpta theologica*; 47r–62r Cleomedes, *Caelestia cum scholiis Pediasimi*; 55r marg. [Apollonius], *On finding two mean proportionals*⁸⁰; 56r marg. Anatolius, *De generatione*; 56r marg. *nota astrologica*; 56r marg. [Melampus], *De divinatione ex naevis*⁸¹; 56v marg. *geographica et astronomica varia*; 62r Hermes Trismegistos, *De partibus hominis*; 62r *Oneirocriticon e Danielis psalmis*; 62r *excerptum e Galeni De dignotione ex insomniis*⁸²; 62v Nicholas Rhabdas, *Methodus de arithmetice et geometricis meditationibus, et problemata arithmetica octo*; 63r–v *De Persici astrolabii usu*; 64r–77v Aratea, *astrologica et brontologica varia*⁸³; 78r–79v Anonymous and Demetrius Triclinius, *De lunae schematismis*; 80r *De climatibus*⁸⁴; 80v *excerpta ex Adamantii De ventis*⁸⁵; 81r–85v Iohannes Pediasimos, *Geometria*; 85v–86r *notae et tabulae metrologicae chronologicae astrologicae* (dated to 1371–73); 86r–88v [Aristotle], *De mundo*; 93r–96v Alexander of Aphrodisias medicus, *Quaestiones et solutiones physicae*⁸⁶; 96v–99r [Philo], *De mundo*⁸⁷; 99r–v [Aristotle], *De virtute*; 99v, 102r–v Theophylact Simocatta, *Dialogus de quaestionibus physicis*⁸⁸; 100r–101v *tabulae (partim vacuae) et notae vari-*

⁷⁷ A very detailed description of this manuscript (here completed) is found in *Catalogus Codicum Astrologorum Graecorum*. I–XII. Bruxelles 1898–1953, VIII 3 (P. BOUDREAUX) 43–59. See also P. SCHREINER, *Die byzantinischen Kleinchroniken*. I–III (*CFHB* 12). Wien 1975–79 I 191–192; P. CABALLERO SANCHEZ, *El Comentario de Juan Pediasimo a los «Cuerpos celestes» de Cleomedes* (*Nueva Roma* 48). Madrid 2018, 107–110 (watermarks and identification of the main copyist).

⁷⁸ This text is edited in R. E. SINKEWICZ, *Saint Gregory Palamas, The One Hundred and Fifty Chapters* (*Studies and Texts* 83). Toronto 1988.

⁷⁹ These are parts 2 and 3 of the first Triad of Palamas' treatises edited in J. MEYENDORFF, *Grégoire Palamas. Défense des saints hésychastes* (*Spicilegium Sacrum Lovaniense. Études et documents* 30). Louvain 1973.

⁸⁰ This is a method for finding two mean proportional lines between two given straight lines, witnessed in several sources and variously assigned to Hero of Alexandria or to Apollonius; see W. R. KNORR, *Textual Studies in Ancient and Medieval Geometry*. Boston – Basel – Berlin 1989, 11–28 and 41–61, and earlier, and paying attention to Byzantine authors, V. DE FALCO, *Sul problema delico*. *Rivista Indo-Greco-Italica* 9 (1925) 41–56. The proof in Par. gr. 2381 is an abridged version of Knorr's text PK, transmitted in late witnesses of Philoponus' *in Apo*.

⁸¹ The first of these three texts is edited in *Catalogus Codicum Astrologorum Graecorum* VIII 3, 188, the third is edited in J. FRANZ, *Scriptores physiognomoniae veteres*. Altenburgi 1780, 501–508, and now in S. COSTANZA, *Una versione bizantina e una metafrasi neogreca dello ps. Melampo De Naevis*. *Byz* 83 (2013) 83–102.

⁸² The first of these three texts is edited in H. DIELS, *Beiträge zur Zuckungsliteratur des Okzidents und Orients*. I. Die griechischen Zuckungsbücher (Melampus *Περὶ παλμῶν*), *Abhandlungen der königlich preussischen Akademie der Wissenschaften*. Philosophisch-historische Klasse. Berlin 1907 IV, 41–42; for the third see C. G. KÜHN, *Claudii Galeni opera omnia*. I–XX. Lipsiae 1821–33 VI 832–835.

⁸³ This sequence of extracts is very accurately described, and partly edited, in *Catalogus Codicum Astrologorum Graecorum* VIII 3, 47–53 and 189–191, to which I refer.

⁸⁴ This text is edited in J. A. CRAMER, *Anecdota Graeca e codicibus manuscriptis bibliothecae Regiae Parisiensis*. I. Oxonii 1839, 362, 1–364, 7 μέρεσι.

⁸⁵ These excerpts are edited in V. ROSE, *Anecdota Graeca et Gaecolatina*. I–II. Berlin 1864–70 I 49–52.

⁸⁶ This is a version of the compilation of problems edited in J. L. IDELER, *Physici et medici Graeci minores*. I–II. Berolini 1841–42 I 3–80.

⁸⁷ This work is a compendium of Philo's *De aeternitate mundi*: see F. CUMONT, *Philonis De aeternitate mundi*. Berolini 1891, xxvii; L. COHN – P. WENDLAND, *Philonis Alexandrini opera quae supersunt*. I–VI. Berolini 1896–1915 VI xxxiv–xxxvi. The edition can be read in K. E. RICHTER, *Philonis Judaei opera omnia*. I–VIII. Lipsiae 1828–30 VI 148–174.

⁸⁸ This text is edited in IDELER, *Physici et medici* I 168, 1–177, 16, and L. MASSA POSITANO, *Teofilatto Simocatta. Questioni naturali*. Napoli 1965, 7, 1–26, 15.

*ae*⁸⁹; 103v–r Barlaam, *De paschate*; 104r–105v line 2 Matthew Blastares, *Computus Paschalis*⁹⁰; 105v line 3–107r Michael Psellos, *Opus chronologicum* (excerpts); 107r–v *notae physiognomoniae*; 107v–108r *notae astronomicae*; 108v *pauca theologica*.

This is a high-brow manuscript written for personal use by a distinguished scholar, who possibly added the quire comprising ff. 3–12 (Planudes) to his notebook (my doubts come from the fact that our scholar did not leave traces in the added quire); he also briefly collaborated with another copyist on f. 88v. As first remarked by P. Caballero Sánchez, the main copyist also penned the so-called *textus tripartitus* of Manuel Bryennios' *Harmonica*, whose three *membra disiecta* we read in the manuscripts Paris, Bibliothèque nationale de France, gr. 2549 (*Diktyon* 52181), ff. 43r–46v and 75v–78v, Madrid, Biblioteca nacional de España 4625 (*Diktyon* 40105), ff. 2r, 68r–71v, and 122v–123v, and München, Bayerische Staatsbibliothek, Cod. graec. 487 (*Diktyon* 44935), ff. 272r–289r⁹¹. As is to be expected, the scientific texts contained in Par. gr. 2381, possibly by the intermediation of hyparchetypes, have very important witnesses as ancestors. This is the case for the treatises of Planudes: the ancestor of P¹ is the manuscript Firenze, Biblioteca Medicea Laurenziana, Ashb. 1599 (14th century; *Diktyon* 15767); of Barlaam: the ancestor is the manuscript Marc. gr. Z. 332 (our witness M); and of Cleomedes: the ancestor is the manuscript Edinburgh, National Library of Scotland, Adv. 18.7.15 (*ca.* 1290; copied for the most part by Maximus Planudes; *Diktyon* 13730)⁹². Moreover, Par. gr. 2381 is an independent witness of Triclinius' treatise and of Pediasimos' scholia to Cleomedes; it is the only witness of Rhabdas' short logistic text, which I have published elsewhere⁹³. The presence of Barlaam's *De paschate* in P¹ is not recorded in the standard edition (the folio is bound with recto and verso interchanged), nor is it the fragment from Psellos' chronological treatise⁹⁴.

⁸⁹ The contents of these folios are as follows: f. 100r, table of the yearly mean longitude, mean anomaly, and double elongation of the Moon, years AM 6879–6900 [= AD 1371–90]; table of the mean longitude, mean anomaly, and double elongation of the Moon, for 1 to 10, 20 to 90, 100 to 300 days, 1 year of 365 days and 1 year of 366 days; astrological *thema*; f. 100v, day (Nov. 8 and 23, Dec. 8, Jan. 6, Feb. 5, Mar. 6, Mar. 11, Jun. 2, Dec. 19 AM 6881 [= AD 1372]), hour, longitude, distance from nodes of new and full Moons; definition of the base of the Moon; notes and expense reports for trips to Rhodes and back to Constantinople, and to Cyprus; f. 101r, table of the yearly anomaly and apogee of the Sun, years AM 6879–6900 [= AD 1371–90]; table of the anomaly and apogee of the Sun, for 1 to 10, 20 to 90, 100 to 300 days, 1 year of 365 days and 1 year of 366 days; reason for taking (0)0;59,8 as the value of the mean daily motion in longitude of the Sun, with associated tabular computations; f. 101v, table 2b according to the list in R. LEURQUIN, *La Tribiblos astronomique* de Théodore Méliénote (Vat.gr. 792). *Janus* 72 (1985) 257–282: 270–276, as in the manuscript Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 210 (middle of 14th century; *Diktyon* 66841), f. 50r (only the last two columns).

⁹⁰ See RHALLES – POTLES, Σύνταγμα VI 404–419, 8.

⁹¹ See G. H. JONKER, De textu Bryennii tripartito. *Mnemosyne* 19 (1966) 399–400; G. H. JONKER, Μανουήλ Βρυεννίου Ἀρμονικά. The Harmonics of Manuel Bryennius. Groningen 1970, 36, 37, 40, 46–47; B. MONDRAIN, Les écritures dans les manuscrits byzantins du XIV^e siècle. Quelques problématiques. *RSBN* 44 (2007) 157–196: 194 and n. 70, who identified a fourth limb in the composite manuscript München, Bayerische Staatsbibliothek, Cod. graec. 489 (*Diktyon* 44937), f. 246r–v and provides references to other manuscripts where the hand of the copyist of Par. gr. 2381 can be found.

⁹² See ALLARD, Maxime Planude 12–14 (Planudes); below for Barlaam; R. B. TODD, Cleomedis Caelestia (Metēora). Leipzig 1990 x (Cleomedes).

⁹³ See ACERBI, A New Logistic Text.

⁹⁴ For Pediasimos' scholia, see CABALLERO SÁNCHEZ, El Comentario 139 (stemma) and 165–166. For the Anonymous and Triclinius, see A. WASSERSTEIN, An Unpublished Treatise by Demetrius Triclinius on Lunar Theory. *JÖB* 16 (1967) 153–174 and F. ACERBI, I problemi aritmetici attribuiti a Demetrio Cidone e Isacco Argiro. *Estudios Bizantinos* 5 (2017) 131–206: 136 n. 16 and Testo 2. The excerpts from Psellos' treatise are sects. 1–3 and 21–22 (but other material is added); see G. REDL, La chronologie appliquée de Michel Psellos. *Byz* 4 (1927–28) 197–236 and G. REDL, La chronologie appliquée de Michel Psellos (suite). *Byz* 5 (1929–30) 229–286. Pediasimos' *Geometria* is published in G. FRIEDLEIN, Die Geometrie des Pediasimus. Programm Ansbach 1866.

P². Paris, Bibliothèque nationale de France, gr. 2384 (*Diktyon* 52016) 1539–50, copyist <Iacopus Diasorinos>⁹⁵. The *Demonstratio* is on ff. 55r–60v. Other works in the manuscript: ff. 1r–55r Barlaam, *Logistikē* I–VI; 61r–v George Gemistos Pletho, *Problema geometricum*.

The date and the contents of this manuscript make it a priori almost certain that it derives from our witness K. Moreover, both K and P² “translate” the Indian numerals (Eastern form) that accompany the original diagrams into Greek numerals; moreover, they are two new witnesses of Pletho’s geometric problem (see Appendix 1 for a collation of their texts).

P³. Paris, Bibliothèque nationale de France, suppl. gr. 1 (*Diktyon* 52773), 1547–59, copyist Constantinos Palaiocappas (entire manuscript)⁹⁶. The *Demonstratio* is on ff. 1r–8v. Other works in the manuscript: ff. 9r–76r Barlaam, *Logistikē* I–VI.

K, P², and P³ “translate” the Indian numerals (Eastern form) that accompany the original diagrams into Greek numerals.

H. Vatican City, Biblioteca Apostolica Vaticana, Pal. gr. 62 (*Diktyon* 65795), composite, ca. 1549 (watermark)⁹⁷. The *Demonstratio* is on ff. 112v–115v. Other works in the manuscript: ff. 1r–36v Theodosius, *Sphaerica* I–III; 38r–41r Anonymus (*immo* Nicholas Rhabdas), *Epistula ad Khatzykem*; 41v–47r Maximus Planudes, *Psephophoria secundum Indos* (incomplete)⁹⁸; 49r–57v *tabulae arithmeticae*; 59r–72r Iohannes Pediasimos, *Geometria*; 72v–77v [Hero], *Geodaesia*; 78r–v Isaak Argyros, *Epistula ad Colybam*; 78v–80r *excerpta ex [Heronis] Geodaesia*; 81r–112v Barlaam, *Logistikē* I–VI; 118r–125v Michael Psellos, *in Platonis Psychogoniam*; 126r–185r *Excerpta e Strabonis Geographia a Plethone emendata*; 190r–198v George Gemistos Pletho, *Chorographia Thessaliae*; 200r–213r Agathemerus, *Geographiae hypotyposis*; 213v Dionysius, *Navigatio Bospori*.

H is an apograph of Marc. gr. Z. 302 (our witness M¹) as for Theodosius’ *Sphaerica* and Barlaam’s *Logistikē*; it is a systematic copy of the manuscript Venezia, Biblioteca Nazionale Marciana, gr. Z. 323 (coll. 630; 1370–80 and beginning of 15th century [watermark range 1406–21], copyists Isaak Argyros, one of his collaborators, and “anonymus AG”, see GIOFFREDA, Tra i libri 257–264; *Diktyon* 69794) for all subsequent works up

⁹⁵ Christopher Auer (who is possibly to be identified with one of the revisers) penned the subscription of the *Demonstratio* on f. 60v and Pletho’s problem on f. 61r–v. On Diasorinos, see *RGK* I 143, II 191 (where Omont’s identification is confirmed), III 241; C. GARCÍA BUENO, El copista griego Jacobo Diasorino (s. XVI): estudio paleográfico y codicológico de sus manuscritos. PhD Thesis Madrid, Universidad Complutense 2017 (348–350 for Par. gr. 2384, with the identification of Auer’s hand); C. GARCÍA BUENO, Jacobo Diasorino in Italia. *IIIHH/FONS* 3 (2018) 51–69; C. GARCÍA BUENO, The Evolution of Jacobos Diasorinos’ Handwriting in Context, in: Griechisch-byzantinische Handschriftenforschung, hrsg. von C. Brockmann – D. Deckers – D. Harlfinger – S. Valente. Berlin – Boston 2020, 201–210.

⁹⁶ The subscription is located on f. Cv. A description of this magnificent manuscript is found in Ch. ASTRUC et al. Bibliothèque Nationale. Département des Manuscrits. Catalogue des manuscrits grecs. Supplément grec, numéros 1 à 150. Paris 2003, 17–18. On Palaiocappas, see *RGK* I 225; II 316; III 364; C. GARCÍA BUENO, El copista cretense Constantino Paleocapa: un estado de la cuestión. *Estudios bizantinos* 1 (2013) 198–218. This manuscript was owned by Cardinal Richelieu: Ch. ASTRUC, Les manuscrits grecs de Richelieu. *Scripta* 10 (1952) 3–17. This witness of Barlaam’s *Logistikē* is unknown to Carelos.

⁹⁷ A description of this manuscript is found in H. STEVENSON, Codices manuscripti Palatini graeci Bibliothecae Vaticanae. Romae 1885, 31–32. The watermark Ancre 21 corr. (1549) is identified on ff. 1–117 and 126–189 in D. HARLFINGER – J. HARLFINGER, Wasserzeichen aus griechischen Handschriften. II. Berlin 1980 *ad indicem*. The four hands engaged in the copy are distributed as follows: ff. 1–36; ff. 38–115; ff. 118–125; 126–185; ff. 190–198 and 200–213. I am grateful to C. Giacomelli for an exchange on this manuscript. H is partly a copy of the manuscript Cantab. UL Gg.II.33, penned in tandem by Nicholas Sophianos and Constantinos Mesobotes (see *RGK* I 318 and 224 [identification of both, the former already in A. DILLER, The Vatopedi Manuscript of Ptolemy and Strabo. *American Journal of Philology* 58 (1937) 174–184; 183], II 437 and 315, III 517 and 363, respectively), and it figures in an inventory of the library of Ulrich Fugger dated to 1555 (P. LEHMANN, Eine Geschichte der alten Fuggerbibliotheken [*Studien zur Fuggergeschichte* 12, 15]. I–II. Tübingen 1956–60 II 76). The inventory also lists the *tabulae persicae*—which are contained in Marc. gr. Z. 323—as belonging to H.

⁹⁸ Planudes’ *Psephophoria des. mut.* ALLARD, Maxime Planude 73, 10 ἐτέρους δέ.

to the excerpts from the pseudo-Heronian *Geodaesia*; the excerpts from Strabo are copied from Pletho's autograph Venezia, Biblioteca Nazionale Marciana, gr. Z. 379 (coll. 520; *Diktyon* 69850), ff. 1–108; Agathemerus and Dionysius are copied from the manuscript Cambridge, University Library Gg.II.33 (gr. 1463; ca. 1530; *Diktyon* 12191)⁹⁹.

- V. Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 187 (*Diktyon* 66818), middle of 14th century, one single copyist for the entire manuscript¹⁰⁰. The *Demonstratio* is on ff. 214v–220r. Other works in the manuscript: ff. 2r–71r Ptolemy, *Harmonica* I–III; 71r–81v Barlaam, *Refutatio*; 82r–161r Porphyry, in *Ptolemaei Harmonica I*; 162r–214r Barlaam, *Logistikē* I–VI; 220r–223r Barlaam, *De eclipsi* I; 223r–227v Barlaam, *De eclipsi* II (incomplete); 228r–v Barlaam, *De paschate* (fragm.)¹⁰¹.

Düring shows that V is a copy of Monac. gr. 361a for Ptolemy and an independent witness for Porphyry; the editors of *De eclipsi* I and II give reasons to posit a further witness between M and V¹⁰².

- v. Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 1756 (*Diktyon* 68385), composite, end of 16th – beginning of 17th century, the copyist is a collaborator of Alvise Lollino¹⁰³. The *Demonstratio* is on ff. 195r–204r. Other works in the manuscript: ff. 1r–88v Bessarion, *Opera theologica quinque*; 89r–127v *epistulae variorum, praesertim Bessarionis et Plethonis*; 129r–142v *excerpta ex Aristotelis HA a Plethone collecta*; 145r–146r Theodoros Gaza, *epistula ad fratres Andronicum et Demetrium*; 148r–153r Barlaam, *De eclipsi* II; 156r–167v Barlaam, *Refutatio*; 169r–170v Vicellius, *Seismologium*; 171r–177v Libanius, *Declamatio XXVI*; 179v–189v Barlaam, *Logistikē* I¹⁰⁴; 190r–192v Barlaam, *ex epistulis excerpta* (EG 3, 4, 8, 1, 3)¹⁰⁵; 207r–223r Barlaam *epistulae duo* (EG 2–3); 231r–278v *Operae quinque ad hesychasticam controversiam pertinentia*; 279r–294r Barlaam, *epistula ad Gregorium Palamam* (EG 1); 295r–297r Barlaam, *De paschate* (compendium); 298r–306v Barlaam, *epistulae quinque* (EG 5–8, 4); 307r–346r Demetrios Kydones, *De processione Spiritus Sancti*¹⁰⁶;

⁹⁹ See CZINCZENHEIM, Édition 553–603 (Theodosius); my own collations for Rhabdas, the arithmetical tables (which complete those in Rhabdas' treatise, for whose source see F. ACERBI – D. MANOLOVA – I. PÉREZ MARTÍN, The Source of Nicholas Rhabdas' Letter to Khatzykes: An Anonymous Arithmetical Treatise in Vat. Barb. gr. 4. *JÖB* 68 [2018] 1–37), and Argyros; ALLARD, Maxime Planude 12–14 (Planudes); J. L. HEIBERG – L. NIX – W. SCHMIDT – H. SCHÖNE, Heronis Alexandrini opera quae supersunt omnia. I–V. Lipsiae 1899–1914 V LXVI–LCVIII and XCIV–XCVII (*Geodaesia* and excerpts therefrom, whose edition is found in the intervening pages); A. DILLER, The Autographs of Georgius Gemistus Pletho. *Script* 10 (1956) 27–41: 41 (excerpts from Strabo); A. DILLER, The Tradition of the Minor Greek Geographers. American Philological Association 1952, 15–19 (Agathemerus and Dionysius). Psellos' treatise is PHI 72a in P. MOORE, *Iter Psellianum*. A detailed list of manuscript sources for all works attributed to Michael Psellos, including a comprehensive bibliography (*Subsidia Mediaevalia* 26). Toronto 2005. CARELOS, Βαρλαάμ incorrectly makes Marc. gr. Z. 302 and Vat. Pal. gr. 62 independent copies of Marc. gr. Z. 332 (see below).

¹⁰⁰ A description of this manuscript is found in G. MERCATI – P. FRANCHI de' Cavalieri, *Codices Vaticani graeci. Codices 1–329*. Romae 1923, 217–218.

¹⁰¹ *De eclipsi* II *des. mut.* MOGENET – TIHON – DONNET, Barlaam 76 line 269 ἀπὸ τῆς; *De paschate inc. mut.* TIHON, Barlaam 378 sect. 26 πανσελήνους.

¹⁰² See DÜRING, Die Harmonielehre LIV–LVI and LXIX (stemma) and DÜRING, Porphyrios Kommentar XX–XXI and XXIX (stemma); MOGENET – TIHON – DONNET, Barlaam 40–44.

¹⁰³ A detailed description of this manuscript is found in P. CANART, *Codices Vaticani graeci. Codices 1745–1962. I–II*. In *Bibliotheca Vaticana 1970–73 I* 54–62. Nine hands, Lollino included, are engaged in the copy; one of these copyists is Maximus Margounios, who also penned Mosq. Mus. Hist gr. 315 (our witness m). On Lollino, see P. CANART, Alvise Lollino et ses amis grecs. *Studi Veneziani* 12 (1970) 553–587; P. CANART, *Les Vaticani Graeci 1487–1962. Notes et documents pour l'histoire d'un fonds de manuscrits de la Bibliothèque Vaticane (StT 284)*. Città del Vaticano 1979 *passim* and in particular 41–78.

¹⁰⁴ The proemium is missing, the correct folio order is 180–181, 186, 184–185, 182, 187–189 (no folio is numbered 183), see CARELOS, Βαρλαάμ XLII.

¹⁰⁵ See FYRIGOS, Dalla controversia palamitica 29–32.

¹⁰⁶ See M. RACKL, Die ungedruckte Verteidigungsschrift des Demetrios Kydones für Thomas von Aquin gegen Neilos Kabasilas. *Divus Thomas* 2nd series 7 (1920) 303–317.

347r–348v; Demetrios Cydones, *De processione Spiritus Sancti ad amicum* (incomplete); 350r–352r Hippolytus, *De universo* (fragm.).

Lollino, who copied Barlaam's works that precede the *Demonstratio*, notes on ff. 191v–192r that the model of the excerpted letters also contains, in this order, Barlaam, *Logistikē* I–VI, *Demonstratio*, *De paschate*, *Refutatio*, *De eclipsi* I and II. The only manuscripts that fit this description are Mosq. Mus. Hist. gr. 315 (our witness m, copied by Maximus Margounios, who was also engaged in the copy of Vat. gr. 1756) and Marc. gr. Z. 332 (M); M, m, and v are also the only witnesses that contain Barlaam's letters.

- d. Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 2176 (*Diktyon* 68807), composite, middle of 14th century (before 1361: note on f. 52v)¹⁰⁷. The *Demonstratio* is on ff. 20r–21v; its preface is missing. Other works in the manuscript: ff. 1r–2v Stephanus of Alexandria, in *Ptolemaei Tabulae Manuales* 28–30¹⁰⁸; 3r–19v Barlaam, *Logistikē* I–VI; 21v–22v Barlaam, *De eclipsi* I; 22v–24v Barlaam, *De eclipsi* II; 25r–31v Theon of Alexandria, in *Ptolemaei Tabulae Manuales* (*comm. parvum*); 32v *notae geographicae*; 33r–48v Stephanus of Alexandria, in *Ptolemaei Tabulae Manuales* (incomplete, *des. sect.* 18); 49v *notae astronomicae*¹⁰⁹; 50r–51r enunciations of Euclid, *Elementa* I 1–III 18¹¹⁰; 53r–293r Theodoros Metochites, *Elementatio astronomica* I–II; 293r–294r *notae astronomicae incerti auctoris*.

M¹. Venezia, Biblioteca Nazionale Marciana, gr. Z. 302 (coll. 730; *Diktyon* 69773)¹¹¹, composite, ca. 1430 (watermark range 1408–69), copyist <Bessarion>¹¹². The *Demonstratio* is on ff. 260v–263v.

¹⁰⁷ A detailed description of this manuscript is found in S. LILLA, *Codices Vaticani graeci. Codices 2162–2254*. In *Bibliotheca Vaticana* 1985, 50–57. Ten hands seem to be engaged in the copy. The part containing Metochites' *Elementatio astronomica* carries a short note by one of his sons: I. ŠEVČENKO, *Études sur la polémique entre Théodore Métochite et Nicéphore Choumnos* (*Corpus Bruxellense Historiae Byzantinae. Subsidia* 3). Bruxelles 1962, 283, and 280–284 for the relations of Vat. gr. 2176 with the other manuscript witnesses of the *Elementatio*. See also J. LEMPIRE, *Le commentaire astronomique aux Tables Faciles de Ptolémée attribué à Stéphane d'Alexandrie. Tome I. Histoire du texte. Édition critique, traduction et commentaire* (chapitres 1–16) (*Corpus des Astronomes Byzantins* 11). Louvain-La-Neuve 2016, 31–34, 50–52, 56–58, 61–63, 67–68 (Stephanus of Alexandria: d is a copy of a lost model and, for the final part of the text, of the manuscript Paris, Bibliothèque nationale de France, Coislin 338 [watermark range 1346–65; *Diktyon* 49479]); A. TIHON, *Le "Petit Commentaire" de Théon d'Alexandrie aux Tables Faciles de Ptolémée* (*StT* 282). Città del Vaticano 1978, 74–79 (Theon of Alexandria: d is a copy of the manuscript Vatican City, Biblioteca Apostolica Vaticana, Pal. gr. 137 [watermark range 1336–38; *Diktyon* 65869]); B. BYDÉN, *Theodore Metochites' Stoicheiosis Astronomike and the Study of Natural Philosophy and Mathematics in Early Palaiologan Byzantium* (*Studia Graeca et Latina Gothoburgensia* 66). Göteborg 2003, 385 and 409–411 (Metochites: d is a copy of the manuscript Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 182 + 181 [1316–32; *Diktyon* 65813–12], an authorized copy realized during Metochites' lifetime).

¹⁰⁸ The edition of these chapters of Stephanus' commentary, written by the emperor Heraclius, is still the one in H. USENER, *De Stephano Alexandrino*, in: H. USENER, *Kleine Schriften* III. Leipzig – Berlin 1914, 311–317.

¹⁰⁹ This is text 11 in the list of TIHON, *Le "Petit Commentaire"* 359–369.

¹¹⁰ The numbering goes up to item 20 of Book III, but it includes the alternative proofs of *Elem.* III 9 and 10, a numbering convention that is highly non-canonical and that allows us to identify the stemmatic family to which Vat. gr. 2176 belongs (B. Vitrac, *per litteras*).

¹¹¹ The two Venice manuscripts are described in E. MIONI, *Bibliothecae Divi Marci Venetiarum codices graeci manuscripti. Volumen II. Thesaurus Antiquus. Codices 300–625*. Roma 1985, 7–8 and 60–61, respectively. See also F. ACERBI, *I codici matematici di Bessarione*, in: *I libri di Bessarione. Studi sui manoscritti del Cardinale a Venezia e in Europa*, a cura di A. Rigo, N. Zorzi (*Bibliologia* 59). Turnhout 2021, 95–206 *passim*.

¹¹² Bessarion penned ff. 1–47r line 11, 155r–155v line 1, 157v, 161–164; the rest of the manuscript is by the same hand that copied ff. 24r–326v of the manuscript Venezia, Biblioteca Nazionale Marciana, gr. Z. 301 (coll. 635; watermark range 1401–30; *Diktyon* 69772), which is the model of Marc. gr. Z. 302 as for Marinus' *prolegomena*, Euclid's *Data*, *Phaenomena* b, and *Catoptrica*, Theodosius' *Sphaerica*, and Pletho's problem. As for the *Almagest*, Marc. gr. Z. 302 is a copy of the manuscript Paris, Bibliothèque nationale de France, gr. 2391 (a watermark is dated 1334; copyist G; I. PÉREZ MARTÍN, *El 'estilo Hodegos' y su proyección en las escrituras constantinopolitanas. Segno e Testo* 6 [2008] 389–458, 437–438 and n. 182; *Diktyon* 52023); see J. L. HEIBERG, *Claudii Ptolemaei opera quae exstant omnia. I–II*. Lipsiae 1898–1907 II LXXV–LXXVI and LXXVI (stemma). As for the *Elements*, Marc. gr. Z. 302 derives for the most part from the manuscript El Escorial,

Other works in the manuscript: ff. 1r–156v Euclid, *Elementa* I–XIII; 157v George Gemistos Pletho, *Problema geometricum*; 161r–164r Marinus of Neapolis, *Prolegomena ad Euclidis Data*; 164r–184r Euclid, *Data*; 184v–208v Theodosius, *Sphaerica* I–III; 209r–220r Euclid, *Phaenomena* b; 224r–228v Euclid, *Catoptrica*; 232r–260v Barlaam, *Logistikē* I–VI; 265r–494r Ptolemy, *Almagestum* I–XIII.

Folios 160–494 were copied separately from the *Elements*, as confirmed by the quire signatures (α' – $\mu\alpha'$). This codicological unit begins with a blank folio (f. 160); within it, Barlaam's works (ff. 232r–263v) and the *Almagest* were in their turn copied separately, as it is shown by two additional series of signatures (α' – δ' and α' – $\kappa\theta'$) and by the fact that folio 264 is blank. As Marc. gr. Z. 302 was among the manuscripts which Bessarion showed, during the Ferrara-Firenze Council, to Ambrogio Traversari, who lists them in a letter to Filippo Pieruzzi¹¹³, it is likely that Bessarion himself assembled the codex on that occasion, using sets of quires copied in earlier times.

M. Venezia, Biblioteca Nazionale Marciana, gr. Z. 332 (coll. 643; *Diktyon* 69803), ca. 1340 (watermark range 1335–38), Barlaam's copyist II¹¹⁴. The *Demonstratio* is on ff. 61v–67r. Other works in the manuscript: ff. 1r–61r Barlaam, *Logistikē* I–VI; 67r–71v Barlaam, *De paschate*; 73r–85r Barlaam, *Refutatio*; 85r–140v Barlaam, *epistulae octo* (EG 4–8, 1–3); 142r–145v Barlaam, *De eclipsi* I; 146r–152v Barlaam, *De eclipsi* II; 153r–v *scholium de cyclo lunari et computationes astronomicae*.

As is well known¹¹⁵, this is Barlaam's edition of some of his own writings; he did not copy the manuscript himself, but revised it.

THE RELATIONSHIPS BETWEEN THE MANUSCRIPT WITNESSES OF THE *DEMONSTRATIO*

Establishing the relationships between the manuscript witnesses of the *Demonstratio* is eased by a number of facts: Barlaam's scientific writings were often copied as a corpus; most of them, the *Demonstratio* and the *Refutatio* excepted, have been published in a critical edition; these editions reconstruct one and the same stemma for all witnesses the edited works share; in all these stemmas, Marc. gr. Z. 332, whose text was revised by Barlaam himself, is the prototype of the entire tradition.

With two exceptions, my edition confirms all the family relations between manuscript witnesses established in such critical editions. As we shall see, the tradition of the *Demonstratio* comprises three families, and four copies of Marc. gr. Z. 332 (M) that were not further copied. The four isolated copies are Mosq. Mus. Hist. gr. 315 (m), Par. gr. 2381 (P¹), Vat. gr. 1756 (v), and Vat. gr. 2176 (d). The three families are led by Ambr. E 76 sup. (A), whose independent copies are Ambr.

Real Biblioteca del Monasterio de S. Lorenzo Φ .III.5 (gr. 224; end 13th – beginning 14th century; *Diktyon* 15178); the first half of Book X of the Euclidean treatise belongs to a different textual family, and Marc. gr. Z. 302 is a copy of the manuscript Modena, Biblioteca Estense Universitaria α .U.9.7 (Puntoni 56; 3rd quarter of the 14th century; *Diktyon* 43474) and a stemmatic brother of Marc. gr. Z. 301 (on these relations, see VITRAC, *Préalables*, sect. 4, § X, b). On Marc. gr. Z. 301 and its other stemmatic relations, see most recently ACERBI – MARTINELLI TEMPESTA – VITRAC, *Gli interventi autografi* 413–419, with bibliography. On Bessarion's handwriting, see most recently D. SPERANZI, *Le mani del Cardinale. Note sulla scrittura greca di Bessarione*, in: *I libri di Bessarione. Studi sui manoscritti del Cardinale a Venezia e in Europa*, a cura di A. Rigo, N. Zorzi (*Bibliologia* 59). Turnhout 2021, 13–23.

¹¹³ See L. LABOWSKY, *Bessarion's Library and the Biblioteca Marciana* (*Sussidi Eruditi* 31). Roma 1979, 7, and, for Traversari's letter, G. MERCATI, *Ultimi contributi alla storia degli umanisti* (*StT* 91). Città del Vaticano 1939, 25–26.

¹¹⁴ This copyist is responsible for ff. 1r–140v of Marc. gr. Z. 332 and ff. 191r–290r of Ambr. E 76 sup.; ff. 142r–152v of Marc. gr. Z. 332 were copied by Barlaam's copyist I; f. 153r–v is entirely in Barlaam's hand: GIOFFREDA, *Su scrittura*.

¹¹⁵ See MOGENET – TIHON – DONNET, *Barlaam* 46–49; FYRIGOS, *Dalla controversia palamitica* 23–24, 43; TIHON, *Barlaam* 363, 408–410; GIOFFREDA, *Su scrittura*.

P 72 sup. (a) and Ambr. R 117 sup. (L); by Vat. gr. 187 (V), whose apograph is Neap. III.C.2 (N), in its turn copied in BNCF, Fondo Naz. II.III.428 (F); by Bessarion's Marc. gr. Z. 302 (M¹), whose independent copies are Vat. Pal. gr. 62 (H) and Zamberti's Krems. 343 (K), copies of the latter being Par. gr. 2384 (P²) and Par. suppl. gr. 1 (P3). Locating Dasypodius' 1564 edition, where the text of the *Demonstratio* is heavily reworked, is less obvious; a conformal copy of this edition is found in Ricc. 1192 (R). The variant readings I call "characteristic" or "peculiar" are the *Leitfehler* and are not shared by other (families of) manuscript witnesses. The minor variant readings are listed in reduced font size; they are categorized by the kind of error. With one exception, all variant readings in the general conclusions are omitted. All variant readings are also identified by two digits separated by a dot, as in x.y, where x stands for the textual unit (tit = title; pr = proem; 1, 2, etc. = proposition number), y for the line in this textual unit.

The ancestor of the tradition: Marc. gr. Z. 332 (M)

The copy of the *Demonstratio* in Marc. gr. Z. 332 (M) is almost flawless. The copyist, who must have used a carefully written master copy, corrected a handful of his own mistakes; a further lettering error was corrected by using a different ink; an omitted τοῦ ἀπὸ at the very end of the treatise was restored, in the outer margin after the end of line 7 of f. 67r, by Barlaam himself¹¹⁶. The shortened general conclusions appended to most theorems and the numerals that accompany the lettered diagrams appear to be authorial initiatives.

The family of Ambr. E 76 sup. (A): Ambr. P 72 sup. (a) and Ambr. R 117 sup. (L)

Ambr. E 76 sup. (A) is an almost flawless copy of Marc. gr. Z. 332; its only characteristic reading is the composite mistake at 5.13 τὸν ΛΜ ἐποίησεν] ἐποίησε τὸν ΚΛ ἐποίησεν. Other variant readings are as follows.

Omissions. 5.11 δὲ¹ *Mistakes.* 6.4 (a misinterpretation of a correction in M) and 6.24 τετραγώνου (M abbreviates the word by omitting the termination) *Additions.* 2.19, 3.19, 4.18, and 10.20 δεῖξαι 5.24 and 6.24 ἔδει 8.15 ἔδει δεῖξαι *Variants.* 1.1 πρῶτον *Inversions.* 9.2 καὶ εἰς ἀνίσους διαιρεθῆ *Lettering.* 1.15 ΕΗ²] ΕΚ

If we exclude M, Ambr. E 76 sup. is the earliest manuscript of the *Demonstratio*; therefore, it cannot be a copy of any other witness. The readings listed above and the discussions in the editions of *De eclipsi* I and II and of the *Logistikē* show that Ambr. E 76 sup. is a copy of Marc. gr. Z. 332¹¹⁷.

Ambr. P 72 sup. (a) reproduces the characteristic reading of A at 5.13, but Hellbaut emends the text by deleting the second ἐποίησεν (which he had himself copied), by underlining KM, and by writing in the margin "f(ortasse) ΛΜ". More generally, Hellbaut confirms his reputation as a high-brow scholar by emending both himself whenever he had committed a copying mistake (this hap-

¹¹⁶ Expertise by A. Gioffreda; as Gioffreda confirms, Barlaam also penned the *entire* f. 153r–v, the prototype of the *scholium de cyclo lunari et computationes astronomicae*, which complete Barlaam's *De paschate*.

¹¹⁷ See MOGENET – TIHON – DONNET, Barlaam 22–24; CARELOS, Βαρλαάμ LVI–LVIII. As for the *Demonstratio*, the scarce variant readings of A with respect to M might justify the scenario in which both witnesses are independent copies of a master text, but the basic stemmatic principle of parsimony forces us to make A a copy of M: the existence of a master text (something like the definitive author copy) must always be supposed in such cases, but positing a unique master copy from which two independent witnesses derive amounts to positing an archetype, and hence to insert an additional object in the stemma. A further reason for not positing a master text will be given in the discussion located after the stemma. Note, also, that I adhere to the convention that an "apograph" is a direct copy of its model.

pens two dozens of times), and the received text when he detects a problem (see below)¹¹⁸. The only characteristic reading of Ambr. P 72 sup. is the omission of 3.11 ἐμέτρει δὲ καὶ — μονάδας, which separates Ambr. R 117 sup.¹¹⁹. Other variant readings of A with respect to A are as follows.

Emendations. 1.15 EK] EH 6.4 and 6.24 τετραγώνω *Variants.* 1.1 α^{ov} 6.3 ἡμίσεως

Moros' Ambr. R 117 sup. (L) reproduces the characteristic reading of A at 5.13, while omitting the second ἐποίησεν. L is a careless copy; his characteristic readings, which separate Ambr. P 72 sup., are the frequent misreading of the numerals in the diagrams as Greek letters, the omissions 1.12–13 ἐκάτερον — μονάδας, 3.15 ΓΒ ἐπίπεδος σὺν — ΑΒ ΒΓ followed by the wrong lettering ΑΒ, 8.10–11 τὰ δὲ ἀπὸ — ἐπιπέδω, and 10.16–17 ΔΒ — δὲ ἀπὸ τοῦ¹. Other variant readings of L with respect to A are as follows.

Mistakes. 6.18 τῷ ΝΞ] τῶν Ξ 7.4 μέρους] μέρη (a misreading of an abbreviation in A) 7.18 τετραγώνω 8.7 ὁ] ἡ 9.6 τῶν²] τοῖς *Dittographies.* 6.10–11 ἔστω — ΔΒ ΒΓ *Variants.* 1.1 α^{ov} *Lettering.* 2.11 ΙΚ] ΕΚ 4.8 ΑΓ²] ΓΒ 5.7 ΔΓ] ΔΒ 5.19 ΚΝ] ΚΗ 7.12 ΒΓ] ΓΒ and ΓΒ] ΒΓ

*The family of Marc. gr. Z. 302 (M¹): Vat. Pal. gr. 62 (H), Krems. 343 (K),
Par. gr. 2384 (P²), and Par. suppl. gr. 1 (P³)*

The text of Marc. gr. Z. 302 (M¹) is characterized by the title τοῦ αὐτοῦ Βαρλαάμ μοναχοῦ ἀριθμητικῆ ἀπόδειξις ἐπ' ἀριθμῶν ἀνάλογον ἔχουσα τοῖς γραμμικῶς ἐπὶ εὐθειῶν ἐν τῷ δευτέρῳ τῶν στοιχείων ἀποδεδειγμένοις, by the abridged form ἰδιαίτ^(ov) at pr.8 (the word is placed at the end of a line in M¹), by the mistake 1.14 ἀριθμοῦ] ἀριθμοὶ—which derives from misinterpreting a correction in Marc. gr. Z. 332—and by the omission 3.10–11 πάλιν — ΑΓ μονάδας. Other variant readings are as follows.

Mistakes. 6.4 τετραγώνου (a misinterpretation of a correction in M) *Inversions.* 5.13 τὸν ΑΜ ἐποίησεν] ἐποίησε τὸν ΑΜ *Lettering.* 1.7 ΑΒ] ΑΓ (again a misinterpretation of a correction in M) 1.11 ΕΗ] ΕΚ

Marc. gr. Z. 302 does not share any of the characteristic readings of the other independent copies of Marc. gr. Z. 332. Therefore, it is an independent copy of it; I assume that it is a *direct* copy. Contrary to his habits, Bessarion did not take, with the *Demonstratio*, the liberties he takes with the texts he copies; the only exception is the heavily modified title. I now show that the other witnesses in this family derive from Marc. gr. Z. 302¹²⁰.

Vat. Pal. gr. 62 (H) is an obvious conformal copy of M¹, whose layout and characteristic readings it reproduces, the abridged form ἰδιαίτ^(ov) included. The copyist omitted a number of sequences, which are restored by a corrector. This reviser also corrected the mistakes of lettering and resolved, in the margin, the abridged form ἰδιαίτ^(ov). The variant readings of H with respect to M¹ are as follows.

¹¹⁸ The discussion in FYRIGOS, *Opere contro i latini* 87–91, which makes A and a independent copies of a common ancestor is grounded on the curious belief that the copyist of A could not commit so many mistakes when transcribing such an easily readable manuscript as A.

¹¹⁹ Recall that a *Trennfehler* in witness A with respect to (a set of) witness(es) B “separates” B from A and not A from B.

¹²⁰ As for the *Logistikē*, the relationships between the manuscripts included in this family were incorrectly assessed by Carelos, who makes M¹ and H independent copies of a lost model and places P² under the same hyparchetype as P¹. See CARELOS, Βαρλαάμ LIX–LXI (contrary to what Carelos claims, however, the sequence at CARELOS, Βαρλαάμ 12, 24–25 is not omitted in M¹, where it can be read on f. 235r line 11) and LXIV–LX (contrary to what Carelos claims, P² reads περιάινονται [συν- is added in the margin by a corrector] and ΔΕ at CARELOS, Βαρλαάμ 2, 8 and 63, 30, see f. 1v line 14 and f. 31r line 11, respectively). These (non-existent) variant readings are the sole grounds on which Carelos establishes the said relationships.

Omissions. 1.18 ὄλου (marg. m.2) 3.15 ὁ ἐκ (marg. m.2) 3.15–16 ὁ ἄρα ἐκ τῶν — τετραγώνω (marg. m.2) 5.2 καὶ 5.14–15 ὄλον ἄρα — μονάδας 5.22 καὶ ἔστιν (marg. m.2) 10.16–17 ΔΒ — ἀπὸ τοῦ¹ (marg. m.2) *Mistakes.* tit. γραμματικῶς 1.12 ἐν Ε αὐτῷ 1.13 ὁσαπλασίονα 1.14 τοῦ² τῷ 5.20 τῷ ΝΞ] τῶν Ξ 7.11 ἐκ τῶν²] αὐτῶν 7.12 τετράγωνοι] –νι *Lettering.* 2.7 Γ¹] ΝΓ (N del. m.2) 2.9 Γ²] ΑΓ (A del. m.2) 5.7 ΑΔ] ΑΒ (ΑΔ marg. m.2) 7.5 ΑΓ²] ΔΓ

Zamberti's Krems. 343 (K) is a copy of Marc. gr. Z. 302, whose characteristic readings it shares, the abridged form *ἰδιαίτ^(ov)* excepted. However, en connoisseur of Euclid's text, Zamberti restored the "original" order of propositions 1 and 2 of the *Demonstratio*. Zamberti "translated" all numerals in the diagrams into Greek numerals, and added a diagram to prop. 9 to illustrate the multiplication-rule of multiples there applied. Zamberti also provided the text with standard scholarly paraphernalia: the partial title *προοίμιον*; a partition of the definitions into three units by means of the titles ὄρος α' etc.; the word *θεώρημα* in front of all proposition numbers, the entire heading being displaced from the margin to the main text; short scholia that clarify what theorem is applied in specific deductive steps, like *διὰ τὸ δ^{ov} τοῦ τούτου βιβλίου*. Zamberti also completed the title by applying the epithet *Πυθαγόριος* to Barlaam, and supplied the fully-fledged subscription *τῆς ἀριθμητικῆς ἀποδείξεως εἰς τὸ β^{ov} τοῦ Ἐυκλείδου στοιχείων Βαρλαάμ μοναχοῦ Πυθαγορίου τέλος*. The variant readings of K with respect to M¹ are as follows.

Restorations. 1.7 AB 6.4 τετραγώνου sed –φ s.l. m.2 *Omissions.* 1.14 ἀριθμοὶ¹ 3.3 μέρους 3.16 τε 3.18 τε 6.6 τις 6.8 ἐπίπεδος 8.3 τοῦ¹ 8.3 καὶ 9.2 καὶ *Mistakes.* pr.2 ἐπεδείχθησαν pr.5 μερικὰς def.3 ποιήσει 10.3 προσκειμένω] προκ– *Additions.* def.5 ἐπίπεδον τε 1.13 ἐν τῷ ἑαυτῷ 1.19 ἐκ τε τοῦ 9.3 τετράγωνοι ἀριθμοὶ *Dittographies.* 1.5 καὶ 3.15 τοῦ *Variants.* def.6 λέγω τὸν] καλῶ τὸ 2.7 ἐκ μὲν] ὁ ἐκ 5.3 ἡμίσεως 6.4 ἡμίσεως 9.3 ἡμίσεως *Inversions.* 4.14 τοῖς δυσι 10.9 τετράγωνοὶ εἰσι ἴσοι *Lettering.* 2.12 ΗΚ] ΚΗ 9.5 ΓΒ] ΒΓ

Diassorinos' Par. gr. 2384 (P²) is an obvious conformal copy of K, whose scholarly paraphernalia it also reproduces. Later hands collated P² with Par. gr. 2381, as is shown by a number of corrections (the incorrect deletion of 10.14–15 is diagnostic) and by the addition of the *hexastichos* (see below). The variant readings of P² with respect to K are as follows.

Restorations m.2–3. tit. τοῦ αὐτοῦ ἀριθμητικῆ ἀπόδειξις τῶν γραμμικῶς ἐν τῷ δευτέρῳ τῶν στοιχείων ἀποδειχθέντων pr.2 ἀπεδείχθησαν pr.5 μερικούς pr.6 τῶν τυχόντων (misreading of two abbreviations in Vat. gr. 2381) def.5 del. τε def.6 λέγω 1.5 καὶ (semel m.1) 2.7 ἐκ μὲν 1.13 ἐν ἑαυτῷ 8.3 τοῦ¹ 8.3 καὶ 9.2 καὶ 9.3 del. ἀριθμοὶ 10.3 προσκειμένω (m.1) 10.14–15 del. οἱ ἄρα — ἀπὸ τοῦ ΓΔ (perperam m.3 Vat. gr. 2381 secutus) *Omissions.* 3.12–13 ἐμέτρει — ΑΒ μονάδας (marg. m.3) 5.5–7 ἐπιπέδω — ΑΔ ΔΒ (marg. m.3) 6.11 τῶν¹ (suppl. m.3) 6.20–21 ΒΓ, ὁ δὲ Ε — ἀπὸ τοῦ¹ (suppl. m.3) 8.13 ὁ³ (suppl. m.3) *Mistakes.* 3.16 ἀπὸ τοῦ] ἀποῦ 6.10 τῶν¹] τοῦ 8.4 τετραγώνου (corr. m.3) *Dittographies.* 3.13 ἰσάκις ἄρα ὁ ΓΒ ἐκάτερον τῶν Δ κατὰ τὰς ἐν τῷ ΑΒ μονάδας post μονάδας¹ (del. m.3) 7.10 τοῦ² (del. m.3) 7.14 ΑΓ ἴσος — ἀπὸ τοῦ (del. m.3) 9.14 ἐκ τῶν¹ (del. m.3) *Variants.* 3.15 τοῦ τοῦ] τούτου 6.3 ἡμίσεως *Inversions.* 1.8 ἄρα ὁ (corr. m.2)

Palaiocappas' Par. suppl. gr. 1 (P³) is a magnificently illuminated manuscript, offered as a gift to King Henri II. This witness is an almost flawless conformal copy of K, whose scholarly paraphernalia it also reproduces. P³ restores some of the mistakes of K, but omits the subscription and Pletho's problem, which proves that Par. gr. 2384 (P²) cannot be a copy of P³. Conversely, the title shows that P³ cannot be a copy of P². The variant readings of P³ with respect to K are as follows.

Restorations. pr.5 μερικούς def.5 del. τε 1.5 καὶ semel 1.11 ΕΚ] ΕΗ 6.4 τετραγώνω *Omissions.* 2.2 ὁ 8.11 τοῦ² *Additions.* tit.1 τῶν Εὐκλείδου pr.7 οὕτω καὶ *Variants.* tit.1 β^{ov} 5.3 ἡμίσεως *Inversions.* 4.8 ἄρα ΑΓ prop.5 and 9 marg. βιβλίου τούτου 10.9 εἰσι ἴσοι

*The family of Vat. gr. 187 (V): Neap. III.C.2 (N) and
BNCF Fondo Naz. II.III.428 (F)*

The text of Vat. gr. 187 (V) is characterized by the omission of 2.10 διὰ τὰ αὐτὰ — ΑΔ μονάδας and of the subsequent δὲ, by the anticipation mistakes at 2.13–14 τῷ ΗΚ· καὶ ἔστιν] μένος ἔκ τε τοῦ Γ and at 8.12 τῷ τετράκις ἐκ τῶν ΑΒ] καὶ ἔστιν ὁ ἀπὸ τοῦ + sp. 2 litt. (both to be read in the subsequent lines of Marc. gr. Z. 332), by the omission of 5.21 ὄλω τῷ ΖΘ — ΚΞ, and by the dittography of 9.16–17 ἐκ τῶν — συγκείμενος. The minor variant readings below show that Vat. gr. 187 was penned by a careless copyist.

Omissions. 2.2 εἰς 4.14 δυσι 4.17 τε 5.15 δὲ 6.3 ἀπὸ τοῦ¹ 7.7 τοῦ 8.2 καὶ 9.16 τοῦ⁴ (s.l. m.2) *Mistakes.* pr.1 μαθητικούς pr.8 περιλειφθήσεται def.3 συντιθεῖς 2.4 γινομέν(ας) 3.16 τῷ²] τῶν 4.3 τῷ] τὸ 5.24 τετραγώνω] –ου 6.4 τετραγώνου 7.9 τοῦ¹] τῶν 7.11 κοινῶς 8.6 τετραγώνω] –ου 8.10 δις] δι 9.6 ΔΒ τετράγωνοι] ΓΒ τετραγώνοις *Additions.* 7.11 δις τῶν 8.9 ἔστιν ἄρα] καὶ ἔστιν ἄρα 8.13 ἔστιν] καὶ ἔστιν *Dittographies.* 3.17 εἰς 4.17 καὶ 5.5 τῷ 9.6 τοὺς *Variants.* 1.2¹ and 1.4² δύο] β *Lettering.* 1.9 ΖΗ] ΕΖΗ 1.10 ΓΒ] ΙΓΒ 1.15 ΕΗ²] ΕΚ 2.5 Γ] ΓΒ 2.7 ΑΔ] ΑΔΓ 2.7 ΘΙ] Θ 2.11 ΕΒ] ΒΕ 3.9 ΓΒ¹] Β 4.7 ΑΓ ΓΒ] ΑΓ 5.5 ΓΒ] Γ 5.11 and 5.12 ΒΔ] ΔΒ 5.18 ΔΒ] ΒΔ 5.18 ΒΔ] ΚΔ 5.20 ΝΞ] ΝΖ 5.22 Ε] Ο 6.6 ΑΔ ΔΒ] ΑΔΒ] 6.17 ΔΒ] ΒΔ 6.18 ΝΞ] Ξ 6.19 ΖΘ¹] ΖΑ 6.20 ΒΓ] ΚΓ 7.5 ΓΒ] Γ 7.6 ΒΑ] ΑΒ 9.20 ΑΓ ΔΓ] ΑΓ

Vat. gr. 187 does not share any of the characteristic readings of the other independent copies of Marc. gr. Z. 332. Therefore, it is an independent copy of it; ~~I assume that it is a direct copy.~~

Iohannes Rhosos shows what it is to be a professional copyist. In Neap. III.C.2 (N), apart from eliminating all diagrams, he reproduced almost exactly the text of Vat. gr. 187, including the two anticipation mistakes, the dittographies with the sole exception of the one at 3.17, the absurd reading at 2.13–14, the variants β for δύο. He commits a few mistakes of his own, some of which are consequences of the unclear script of Vat. gr. 187: pr.5 μυρικούς, 5.13 κουάδας, 5.20 τῷ ΝΖ] τῶν Ζ, 10.11 τῷ ΓΒ] τῶν Β. The variant readings of N with respect to V are as follows.

Restorations. 7.9 τῶν] τοῦ *Omissions.* def.2 πολλαπλασιάζειν — ἐν τῷ (marg. m.2) 4.15 ἀπὸ τῶν 4.16 ὁ² *Mistakes.* tit. γραμματικῶς pr.5 ἐπιδή 1.12 τῶν] τὸν 3.13 τῶν] τὸν 6.3 τῷ] τῶν 9.13 αὐτῶν] αὐτῷ 10.19 τοῦ³] τῷ *Additions.* def.6 τέλος τοῦ προοιμίου 10.18 μετὰ τὰ 10.20 τέλος τῆς ἀριθμητικῆς ἀποδείξεως τῶν γραμματικῶς ἐν τῷ δευτέρῳ τῶν στοιχείων ἀποδειχθέντων *Variants.* 1.11 ἑαυτῷ] αὐτῷ 9.3 and 9.15 ἡμίσεως *Lettering.* 5.18 ΒΔ] ΔΒ 6.11 ΝΞ] ΚΞ 7.11 ΓΑ¹] ΑΓ 7.14 ΓΒ] ΒΓ

A later hand lightly revised the text and corrected a part of the lettering.

BNCF, Fondo Naz. II.III.428 (F) is a revised copy of Neap. III.C.2; the text incorporates almost all corrections introduced in N by the later hand just mentioned. The copyist of F may have used a collation manuscript because he corrected several mistakes that Neap. III.C.2 inherited from Vat. gr. 187; however, he retained several of Rhosos' peculiar readings: tit. γραμματικῶς, pr.5 ἐπιδή, 1.11 ἑαυτῷ] αὐτῷ, 1.12 τῶν] τὸν, and the subscription at 10.20. Particularly striking is 10.14 οἶ] ὡς, which is a misreading of οἶ, traced by Rhosos as an inclusion (f. 88v line 12). I thus suspect that the copyist of F corrected the text himself. The other variant readings of F with respect to N are as follows.

Restorations. pr.1 μαθηματικούς pr.5 μερικούς def.6 om. τέλος τοῦ προοιμίου 1.9 ΕΖΗ] ΖΗ 1.10 ΙΓΒ] ΓΒ 2.2 εἰς 2.4 γινομέν(ας)] –μένοις 2.5 ΓΒ] Γ 2.7 ΑΔΓ] ΑΔ 2.7 Θ] ΙΘ 2.9 τὸν ΗΘ κατὰ τὰς ΑΔ 2.13–14 ὁ Ζ μένος ἔκ τε τοῦ Γ ὁ μὲν Ζ ὁ ἐκ τῶν Γ ΑΒ ἐπίπεδος, ὁ δὲ] ὁ μὲν Ζ ὁ ἐκ τῶν Γ ΑΒ ἐπίπεδος, καὶ ὁ 3.9 Β] ΓΒ 4.3 τὸ] τῷ 4.17 καὶ semel 5.5 τῷ semel 5.13 μονάδας 6.3 ἀπὸ τοῦ¹ marg. 7.11 κοινός 8.2 καὶ 8.10 δις 9.6 τετράγωνοι 9.13 αὐτῶν *Omissions.* 1.10 ἐν τῷ² 4.10 τὸν¹ 6.12 ὁ¹ 8.11–12 ὁ ἀπὸ τοῦ ΑΔ τετράγωνος ἴσος — καὶ ἔστιν 8.13–14 ὁ γὰρ ΒΔ ἴσος ἐστὶ τῷ ΒΓ – καὶ ἔστιν ἄρα ὁ ἀπὸ τοῦ ΑΒ ΒΓ ὡς ἀφ' ἑνός *Mistakes.* def.3 ποιήσει 4.16 τοῦ¹] τῆς *Additions.* 9.2 δὲ καὶ *Dittographies.* 5.14–17 ὄλον ἄρα τὸν ΚΜ — ΑΓ μονάδας *Variants.* def.2 λέγεται 1.12 and 1.13 ἑαυτῷ]

αὐτῷ 5.10 γοῦν ἀπὸ μὲν] μὲν οὖν ἀπὸ 6.4 ἡμίσεως 9.17 ἡμίσεως 10.4 ἡμίσεως (bis) 10.17 and 10.18 ἡμίσεως
Inversions. 2.7 ὁ Ζ ἐκ μὲν τῶν Γ ΑΒ *Lettering.* 1.16 and 1.17 ΒΑ] ΑΒ

Mosq. Mus. Hist. gr. 315 (m)

The text of *Mosq. Mus. Hist. gr. 315 (m)* is almost flawless: Margounios strove for accuracy; he corrected himself a handful of times, always in matters of lettering¹²¹. Minor variant readings are as follows.

Mistakes. 6.24 τετραγώνου (M abbreviates the word by omitting the termination) *Dittographies.* 3.8 ἐποίησε τὸν Δ *Variants.* tit. β⁹ pr.1 β⁰⁰

Mosq. Mus. Hist. gr. 315 does not share any of the characteristic readings of the other independent copies of *Marc. gr. Z. 332*. Therefore, it is an independent copy of it; I assume that it is a *direct copy*¹²².

Par. gr. 2381 (P¹)

A characteristic feature of the text of *Par. gr. 2381 (P¹)* is the addition of a dodecasyllable *hexastichos* at the end of the *Logistikē*: Λογιστικῆς ἐνταῦθα Βαρλαάμ πέρας | τῷ τερματοῦντι τοὺς λόγους δόξα λόγῳ || ὁ συντεταχῶς τουτοί τὸ βιβλίον | τὸ μὲν γένος πέφυκε λατῖνος ξένε || πατρίδα δ' ἔσχε Τάραντα πόλιν ὁ δὲ | κλήσι δ' ἀκούει Βαρλαάμ φιλόσοφος. A characteristic reading is the omission of 10.14–15 οἱ ἄρα — ἀπὸ τοῦ ΓΔ. Apart from this omission and from a handful of distractions in the lettering, the copy is remarkably correct; the copyist had a firm grasp of the meaning of the text.

Omissions. 6.18 καὶ *Mistakes.* 9.13 αὐτοῦ] ἐτέρου *Dittographies.* 1.15 ἐκ δύο *Variants.* def.2 πολλαπλασιάζειν 3.6 πολλαπλασιασάτω καὶ] πολλαπλασιάσας 6.4 τετραγών^ν *Inversions.* 5.22 ἴσος ἐστὶ τῷ Ε 10.2 διαιρεθῆ δίχα *Lettering.* 1.9 ΖΗ] ΕΖΗ 4.7 ΑΓ ΓΒ] ΑΓ 6.18 ΝΞ] Ξ 9.6 ΔΒ²] ΓΒ

Par. gr. 2381 does not share any of the characteristic readings of the other independent copies of *Marc. gr. Z. 332*. Therefore, it is an independent copy of it; I assume that it is a *direct copy*¹²³.

Vat. gr. 1756 (v)

With *Mosq. Mus. Hist. gr. 315 (m)*, *Vat. gr. 1756 (v)* is the most recent witness of the *Demonstratio*; no extant witness other than m can be a copy of it. The text of *Vat. gr. 1756* is characterized by the omission of 1.9–10 πάλιν — ΑΓ μονάδας and by the expunction of the subsequent sequence 1.10–11 ὅλον ἄρα — ἐαυτῷ μονάδας. Other peculiar variants are def.3 μετρεῖται, def.4 πολλαπλασιάζοντι, and 6.18 τῷ ΚΝ] κατὰ τὸν ΚΝ. Minor variant readings are as follows.

Omissions. 5.22 ἔστιν *Mistakes.* 2.15 τῶν²] τοῦ 3.5 τῶ²] τοῦ 4.16 τῶν] τοῦ 6.4 τετραγώνου 6.18 τῷ ΝΞ] τῶν Ξ *Additions.* 2.7 Γ ὁ ΑΒ *Dittographies.* 2.4 γινομένοις μένοις

The fact that *Vat. gr. 1756* does not share any of the characteristic readings of the other independent copies of *Marc. gr. Z. 332*, Lollino's description of the model, and the discussions in the editions of *De eclipsi* I and II and, most notably, of Barlaam's letters make it certain that the Vati-

¹²¹ This confirms the remarks in WESTERINK, *Arethae XVI–XVII and XXII*. I had access to a microfilm copy in which some lines of props. 5 and 6 are unreadable.

¹²² See also MOGENET – TIHON – DONNET, *Barlaam 45–46*; FYRIGOS, *Dalla controversia palamitica 37–64*. Fyrigos' discussion allows us to exclude the possibility that *Vat. gr. 1756* is a copy of m.

¹²³ As seen above, CARELOS, *Βαρλαάμ LXIV–LX* incorrectly postulates a hyparchetype between *Marc. gr. Z. 332* and *Par. gr. 2381*.

can manuscript is an apograph of Marc. gr. Z. 332¹²⁴. This copy is independent of the contemporary copy in Mosq. Mus. Hist. gr. 315.

The copyist of Vat. gr. 1756 was negligent. A reviser (Canart's copyist *e* of the same manuscript) fixed the lettering, supplied corrections, integrations *supra lineam*, and marginal integrations of sequences omitted by the copyist: 1.13 ἄρα ἐστὶν — τοσαυταπλασίων 2.11 ὄλον ἄρα — ἐν τῷ AB 3.6–7 Δ, ὁ δὲ ΑΓ τὸν ΓΒ πολλαπλασιασάτω καὶ ποιείτω τὸν 3.7–8 ποιείτω τὸν ΖΗ — ΓΒ πολλαπλασιάσας 4.5 ΓΒ² 4.6–8 ΗΘ, ἐκ δὲ τῶν — ἐπεὶ τοῖνον 5.4–5 δίχα μὲν — εἰς τοὺς ΑΔ ΔΒ. λέγω 6.15 τὰς¹ 7.11 δις 7.14 ΑΓ² (corr.) 8.5 ΓΒ 8.10 post ἐπιπέδω del. τὰ δὲ ἀπὸ τῶν 8.9–12 most of the text 9.14 ὁ τετραπλάσιος διπλάσιός 9.14–16 extensive corrections of a messy sequence, which however do not restore a meaningful text 10.3 τοῦ³ 10.9 εἰς τοὺς 10.11 τοῖς corr. *e* τῶν 10.14 τῶν ΑΔ corr. *e* τοῦ ΑΔ 10.15 εἰσι² 10.17 ΑΓ corr. *e* ΓΒ.

Vat. gr. 2176 (d)

Vat. gr. 2176 was copied shortly after the composition of Barlaam's work. Its text is characterized by the omission of the preface, by the short omission 7.11 ΒΑ ΑΓ ἴσος ἐστὶ τῷ δις ἐκ τῶν, by the dittography of 9.15 μετὰ δύο — ἐκ τῶν ΑΔ ΔΒ, and, as in the *Logistikē*, by the absence of the numerical values and of most of the lettering in the diagrams. No other manuscript of the *Demonstratio* shares these accidents: no extant witness can be a copy of Vat. gr. 2176. On the possible relations with Dasypodius' edition see just below.

Omissions. 5.4 δίχα 7.8 ἀπὸ τοῦ³ 7.13 ἀπὸ τοῦ¹ *Mistakes.* 4.4 ἀπὸ] μὲν 7.9 τοῦ¹] τῶν 8.4 and 8.6 τετραγώνω] – ου *Additions.* 4.2 τοῖς τε 4.10 καὶ ὄλον *Variants.* 1.19 γενομένοις 3.3 εἰρημένου 5.10 γοῦν] οὔν *Inversions.* 5.2 διαμεθῆ] δίχα *Lettering.* 2.5 Γ] ΒΓ 2.7 ΘΙ] ΘΕ 2.8 ΙΚ] ΕΚ 3.4 ΑΒ ΒΓ] ΑΓ ΓΒ 4.9 ΑΓ] ΓΑ 5.14 ΔΓ] ΓΔ 5.18 ΔΒ] ΒΔ 6.11 ΔΒ²] ΒΔ 6.18 ΘΗ] ΗΘ 6.19 ΖΘ²] ΞΘ

The fact that Vat. gr. 2176 does not share any of the characteristic readings of the other independent copies of Marc. gr. Z. 332 and the discussion in the edition of *De eclipsi* I and II prove that the Vatican manuscript is an independent copy of Marc. gr. Z. 332; I assume that it is a *direct* copy¹²⁵. Carelos' arguments that make three manuscripts of the *Logistikē* (among which Vat. gr. 2176) witnesses of three authorial layers of the work prior to the version in Marc. gr. Z. 332 are irrelevant to our purposes since the other two manuscripts involved do not contain the *Demonstratio*¹²⁶. No sign of authorial intervention can be detected in the text of the *Demonstratio* carried by Vat. gr. 2176, which looks very much like an ordinary, moderately negligent copy (but the omission of the preface is deliberate).

¹²⁴ See MOGENET – TIHON – DONNET, Barlaam 44–45; FYRIGOS, Dalla controversia palamitica 37–64. TIHON, Barlaam 367 shows that Vat. gr. 1756 contains only an abridged version of *De paschate*.

¹²⁵ MOGENET – TIHON – DONNET, Barlaam 24–34 and 36–37.

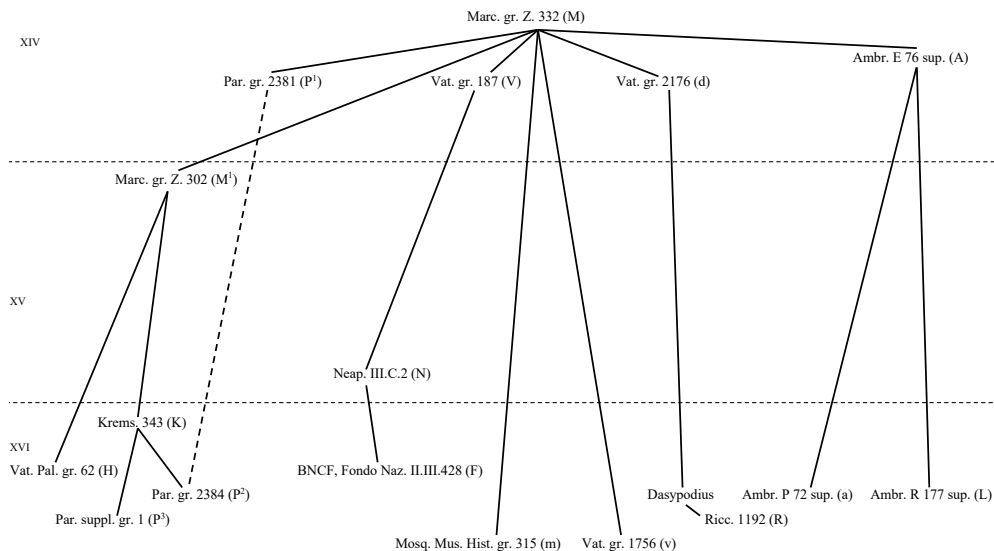
¹²⁶ See CARELOS, Βαρλαάμ LXXII–LXXXIII. The other two manuscripts involved are Athēna, Bibliothēkē tēs Boulēs 5 (*ca.* 1330, with Barlaam's autograph corrections; siglum *c*; *Diktyon* 1101) and Firenze, Biblioteca Medicea Laurenziana, Plut. 89 sup. 48 (composite, here *ca.* 1350 [watermark]; siglum *D*; *Diktyon* 16849), for which see GIOFFREDA, Tra i libri 185–189. However, it is more likely that *D* is a copy of Vat. gr. 2176, assisted by another witness in filling the lacuna that involves *Logistikē* V 8–12; this is the filiation in the case of *De eclipsi* II: MOGENET – TIHON – DONNET, Barlaam 24–34.

Dasypodius' edition and its apograph Ricc. 1192 (R)

In his 1564 edition, while possibly not finding the preface in his manuscript source for the *Demonstratio*, Dasypodius [Konrad Rauchfuß] heavily reworks the text¹²⁷. He introduces, in each proposition, the names of its specific parts (πρότασις “enunciation”, ἔκθεσις “setting-out”, κατασκευή “construction”, ἀπόδειξις “proof”, συμπέρασμα “conclusion”); he restores the general conclusion of all propositions; he frequently modifies the order of the denotative letters in the designations, he (or the printer) commits some peculiar mistakes¹²⁸; he changes the numerical values and the form of all numerals; he adds computations to the diagrams of propositions 7–10 (these computations are in Latin but the diagrams are inserted in the pages that contain the Greek text). As the text is revised and Dasypodius was a good scholar, it is very difficult to identify the *Druckvorlage* of his edition. The absence of the preface points to (a copy of) Vat. gr. 2176; the absence of the numerical values in the diagrams of this manuscript, and in fact the absence of most of the lettering, might have induced Dasypodius to assign them afresh. As for the accidents not shared with Vat. gr. 2176, the short omission that characterizes the text of prop. 7 can easily be restored; the dittography in prop. 9 gives a meaningless text and can easily be detected. On the other hand, Dasypodius might simply have decided to omit Barlaam’s preface because it was irrelevant to his purposes. The fact that both Dasypodius and Zamberti restored the original order of propositions 1 and 2 cannot count as a conjunctive reading because both of them were familiar with the text of the *Elements*.

An obvious apograph of Dasypodius’ edition, copied shortly after 1564 as the watermarks show, is Ricc. 1192 (R)¹²⁹. The copyist exactly reproduced all features of Dasypodius’ text and layout: the long dittography in prop. 2 (Barlaam’s prop. 1); the titles of the specific parts; the internal arrangement of the diagrams; the values and the form of the numerals; the computations associated with some diagrams, etc. He also copied the Latin translation that in the edition faces the Greek text.

The stemma of the manuscript tradition of the *Demonstratio* is depicted below. Accordingly, my edition reproduces the flawless text of Marc. gr. Z. 332.



¹²⁷ Dasypodius’ text was collated with Vat. gr. 2176 in J. L. HEIBERG, *Om Scholierne til Euklids Elementer (Det Kongelige Danske Videnskabernes Selskabs Skrifter, 6te Række, historisk og filosofisk, Afd. II,3)*. Kjøbenhavn 1888, 229–304: 293–295.

¹²⁸ For instance, Dasypodius’s text reads ποιήτω instead of ποιείτω (frequently); duplicates (with two mistakes) the long sequence πάλιν ἐπεὶ — μονάδας at 2.9–11 (Barlaam’s 1.9–11); reads ὁσαπλάσιον and τοσαυταπλάσιον at 2.13 (Barlaam’s 1.13); omits μετρεῖ δὲ — μονάδας at 5.18–19; prints Z for Ξ in the proof of prop. 6; omits ἄρτιος in the enunciation of prop. 9. All these mistakes are recorded in the apparatus of Heiberg’s reprint.

¹²⁹ Ricc. 1192 omits e.g. 2.15–16 ὄ ἄρα — ἐπιπέδοις; therefore, it cannot be the *Druckvorlage* of Dasypodius’ edition.

This stemma and the previous discussion allow me to make a number of methodological points.

(a) A part of the text has no critical significance. This is the general conclusion of every theorem, for it is identical to the enunciation of the same theorem. No copyist can be assumed to be unaware of this general feature of Greek mathematical texts, or, more simply, no copyist can be assumed to have overlooked this specific feature of the text he is copying. Barlaam abridged some general conclusions, but any portion of any of them can be immediately restored by looking at the corresponding enunciation.

(b) Par. gr. 2384 offers a beautiful example of contamination *in statu nascendi*.

(c) It is a commonplace of textual criticism that the *sauts du même au même* have a partial critical significance: insofar as they are (possibly long) omissions, they count as separative errors¹³⁰; insofar as they are potentially polygenetic, they cannot count as conjunctive errors¹³¹. This argument does not apply to Greek mathematical texts such as Barlaam's. For the determiner "polygenetic" formulates an evaluation of the likelihood of an innovation. The reader will immediately realize that, in a highly formulaic text as Barlaam's, it is *very unlikely* that no copyist commits an omission by *homoioteleuton* within an assigned, even reasonably short, stretch of text, but it is also *very unlikely* that two or more copyists omit the same textual sequence by *homoioteleuton*, as there are many sequences conducive to such an omission in any assigned stretch of text, and all of them are equiprobable.

(d) Most *eliminationes codicum descriptorum* in my discussion are straightforward and rest upon extra-textual features such as the presence and the form of the diagrams, the form of the numerals, misunderstood abbreviations, quirks in the script (like Rhosos' inclusion), paratexts of any kind (like the "scholarly paraphernalia"), or even the feeling of *déjà vu* one gets when collating a witness that is a conformal copy of another one¹³².

(e) As our text is short, the independent copies of Marc. gr. Z. 332 are identified as such by a handful of *Leitfehler* each, possibly by just one. But even one *Leitfehler* is enough¹³³.

(f) Decimation has not simplified the part of the stemma nearest to the prototype¹³⁴.

¹³⁰ Only competent revisers are able to detect and to fill a gap of this kind, but normally they are unable exactly to restore the original text, for Greek mathematical style is formulaic but not formalized.

¹³¹ See in the first place P. MAAS, *Leitfehler und stemmatische Typen*. *BZ* 37 (1937) 289–294: 292 n. 1, thereafter in P. MAAS, *Textkritik*. 4. Auflage. Leipzig 1960, 28 (but note the qualification "Trennfehler ohne bindende Kraft sind viele der sog. Homoioteleuta").

¹³² I have applied these criteria to the tradition of Nicomachus' *Introductio arithmetica* in F. ACERBI, *Eliminazioni diagrammatiche*. *Scripta* 13 (2020) 9–37. Much earlier, masterly eliminations grounded on extratextual features were carried out systematically in E. L. DE STEFANI, *I manoscritti della 'Historia Animalium' di Eliano*, *SIFC* 10 (1902) 175–222. Methodological *loci classici* for eliminations based on "mechanical" accidents are G. PASQUALI, *Storia della tradizione e critica del testo*. Firenze 1952, ch. III; J. IRIGOIN, *Accidents matériels et critique des textes*. *Revue d'Histoire des Textes* 16 (1986) 1–36, featuring several examples that explain how to determine the codicological features of the lost model of an extant apograph, if the former was damaged; M. D. REEVE, *Eliminatio codicum descriptorum: A Methodological Problem*, in: *Editing Greek and Latin Texts*, ed. J. N. Grant. New York 1989, 1–35: 9–23, repr. ID., *Manuscripts and Methods. Essays on Editing and Transmission (Storia e Letteratura 270)*. Roma 2011, 145–174: 151–164. On the other hand, G. ORLANDI, *Apografi e pseudoapografi nella «Navigatio sancti Brendani» e altrove*. *Filologia Mediolatina* 1 (1994) 1–35, repr. ID., *Scritti di filologia mediolatina*. Firenze 2008, 63–94, shows that "mechanical" accidents may not be enough to prove that a manuscript is a direct copy of a damaged model: the point is that there may exist, as they do exist in some infamous cases, copies that reproduce their exemplar line by line. As we shall see in a moment, however, twin manuscripts are stemmatically irrelevant, for they can exist only in the "real tree".

¹³³ A single *Leitfehler* is enough to warrant an elimination. However, indifferent variant readings may also acquire a critical significance; see O. PRIMAVESI, *Aristotle, Metaphysics A. A New Critical Edition with Introduction*, in: *Aristotle's Metaphysics Alpha. Symposium Aristotelicum*. ed. C. Steel. Oxford 2012, 385–516: 395–396, and most recently F. ACERBI, *La tradition manuscrite de la « Recension IV » du commentaire à l'Introductio Arithmetica de Nicomaque*. *Revue d'Histoire des Textes* 18 (2023) 35–96 Annexe 1.

¹³⁴ On decimation as the explanation of the predominance of bipartite stemmas see V. GUIDI – P. TROVATO, *Sugli stemmi bipartiti. Decimazione, asimmetria e calcolo delle probabilità*. *Filologia Italiana* 1 (2004) 9–48. This explanation is un-

(g) The genealogical reconstruction of the tradition does not require any lost witness. The verb “require” in the previous sentence means that the principle of parsimony is applied in building the stemma, the latter being conceived as a graph with the minimal number of vertices¹³⁵. In a stemma built according to this principle, branchings are sets of conjunctive innovations, so a branching need not correspond to any lost witness; interposed witnesses (= one-edge branchings) are forbidden unless special conditions apply¹³⁶. All paradoxes of stemmatics evaporate if the principle of parsimony is applied, and if care is taken to keep the stemma and the “real tree” separated¹³⁷.

(h) This distinction being made, I surmise that, as far as the “real tree” is concerned and unless contrary evidence is adduced, the ~~seven~~ independent copies of Marc. gr. Z. 332 are apographs—that is, direct copies—of it. This statement is meaningless if applied to the stemma.

(i) Were Marc. gr. Z. 332 be lost, there would not be a Maasian archetype because the seven independent copies of the Venice manuscript do not share innovations.

(j) Were Marc. gr. Z. 332 be lost, and as there is no Maasian archetype, the seven independent copies of the Venice manuscript could only be traced to an innovation-free exemplar, which, by definition, is the original (this is what I have cautiously called “master text” above). Yet, this original cannot figure in the stemma, for it cannot be reconstructed by means of genealogical methods, which are grounded on innovations.

(k) The tradition of the *Demonstratio* shows that Paul Maas’ rule “[...] ein Zeuge wertlos ist (d.h. als Zeuge wertlos), wenn er ausschließlich von einer erhaltenen oder einer ohne seine Hilfe rekonstruierbaren Vorlage abhängt” contains a methodological mistake (namely, the underlined clause)¹³⁸, for according to the rule and to the nature of the variant readings *any* of the seven independent copies of Marc. gr. Z. 332 should be eliminated as “wertlos”. But if *any* should, nevertheless *all* could not—and this is paradoxical if Maas’ rules are to be regarded as a *method*.

PRELIMINARIES TO THE EDITION

Edition. I have retained the original accents of proclitics and enclitics; otherwise, the accents are normalized to the conventions presently in use. I have punctuated the text anew, following the rules I use in editing Greek and Byzantine mathematical texts¹³⁹. In particular, such rules prescribe that consecutive deductive steps of a proof are separated by an upper point; that a deductive hiatus—which normally precedes a “paraconditional” clause embedded in the proof—is marked by a full stop; that commas separate the antecedent and the consequent of conditional and paraconditional clauses; that N-dashes single out postposed explanations. Short-range correlative μέν ... δέ (*men ... de*) are not separated by a comma; conjunctive δέ (*de*) is normally not preceded by a comma (an exception is in the enunciation of prop. 5). In the Greek text, I have marked the beginning of the pages of Heiberg’s edition and of Marc. gr. Z. 332. The proposition numbers were originally placed in the margins; the title, the subtitle ὅροι (“terms”), and the letters capitalized in my

determined by the fact that it applies to the “real tree” insofar as it is generated root-first, not to the stemma, which is an abstract tree reconstructed from its own leaves.

¹³⁵ That is, the principle of parsimony has been applied tacitly and in a systematic way, not only in assigning the stemmatic position of Ambr. E 76 sup.

¹³⁶ One-leg branchings usually formalize the presence of a recension. See F. ACERBI, Byzantine Recensions of Greek Mathematical and Astronomical Texts: A Survey. *Estudios bizantinos* 4 (2016) 133–213.

¹³⁷ One of these paradoxes has been recently expounded—but not explained away—in P. CHIESA, Tripartito = indimostrabile? *Filologia Mediolatina* 27 (2020) 1–42, who elaborates on J. FOURQUET, Fautes communes ou innovations communes? *Romania* 70 (1948) 85–95: 86–89.

¹³⁸ See MAAS, Textkritik § 4. This lapse was first exposed as such in M. D. REEVE, *Eliminatio* 148–149 of the reprint.

¹³⁹ These rules are not rooted in the punctuation practice of any specific language; they are expounded in ACERBI, The Logical Syntax, sect. 1, § 4.

critical text are rubricated. The critical apparatus is located after each proposition; it is keyed to the text by means of superscript Latin letters. The diagrams are those of Heiberg's edition of 1888; the diagrams of Marc. gr. Z. 332 and of Krems. 343 are reproduced as the plates set out in the Iconographic Complement at the end of this paper.

Translation. Different Greek terms are translated with different English terms; the translations adopted are those used in ACERBI, *The Logical Syntax*. I always translate connective δέ by “and”. I normally translate the two formulae τὸ ὑπὸ “the <plane number resulting> from” and τὸ ἀπὸ “the <square described> on” with “that from” and “that on”, respectively. Words supplied in translation are put within angular brackets <...>. The translation of the proof is punctuated as follows: a semi-colon separates steps in which the deductive chain is not interrupted; a full stop indicates a deductive hiatus; M-dashes single out postposed explanations. The translation contains references to the numbers of the propositions of the *Demonstratio* used in a given proposition.

CRITICAL EDITION OF THE *DEMONSTRATIO*

|_{61v} 725 Βαρλαάμ μοναχοῦ ἀριθμητικὴ ἀπόδειξις τῶν γραμμικῶς ἐν τῷ δευτέρῳ τῶν στοιχείων ἀποδειχθέντων

Ἐπεὶ εἶδον πολλαχοῦ τοὺς μαθηματικοὺς χρωμένους τοῖς τοῦ δευτέρου τῶν στοιχείων θεωρήμασιν ὡς ἀριθμητικοῖς, ἀπεδείχθησαν δὲ τῷ στοιχειωτῇ γεωμετρικῶς, ἔδοξέ μοι μεταβαλόντι αὐτὰ εἰς ἀριθμοὺς ἀριθμητικὴν τὴν τούτων ἐκθέσθαι ἀπόδειξιν. ἔστι μὲν οὖν ἕκαστον αὐτῶν καὶ δι' ἐπαγωγῆς δειχθῆναι, ἐπεὶ καὶ πᾶν ἀριθμητικὸν πρόβλημα καὶ δι' ἐπαγωγῆς δείκνυται ἐκτιθεμένων ἡμῶν μερικοὺς τινὰς ἀριθμοὺς οἷς ὁ καθόλου λόγος ἐφαρμόζεται. ἀλλ' ἐπειδὴ ἄτεχνον τοῦτο καὶ
5 τοῦ τυχόντος δεῖν ῥήθην, ὀλιγορήσας τῆς δι' ἐπαγωγῆς [...]]δείξεως, ἀποδεικτικὴν τὴν τούτων ἐκθέσθαι θεωρίαν, ἐκ τῶν καθολικωτέρων δεικνὺς τὰ μερικώτερα· οὕτω γὰρ ὑπὸ τῆς γνώσεως τὸ γνωστὸν ἅπαν περιληφθήσεται, ὅπερ ἐπιστήμης ἐστὶν ἰδιαίτατον.

ὅροι

<1> Ἀριθμὸν ἀριθμὸν πολλαπλασιάζειν λέγω, ὅταν, ὅσαι εἰσὶν ἐν τῷ πολλαπλασιάζοντι μονάδες, τοσαντάκις συντεθεῖς ὁ πολλαπλασιαζόμενος ποιῆσῃ τινά, ὃν καὶ μετρεῖ κατὰ τὰς ἐν τῷ πολλαπλασιάσαντι μονάδας.

5 <2> καλῶ δὲ αὐτόν, ἦτοι τὸν ἐκ τούτων γενόμενον, ἐπίπεδον.

<3> τετράγωνον δὲ ἀριθμὸν λέγω τὸν γινόμενον ἀπὸ τινος ἑαυτὸν πολλαπλασιάσαντος.

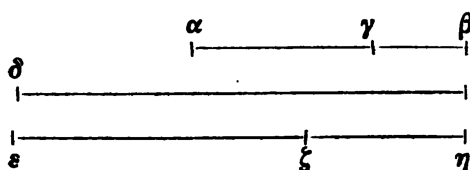
<4> Ἀριθμὸν ἀριθμοῦ μέρος λέγω τὸν ἐλάττονα τοῦ μείζονος, ἂν τε μετρηῖ ἂν τε μὴ μετρηῖ τὸν μείζονα.

α'

Ἐὰν ἀριθμὸς εἰς δύο ἀριθμοὺς διαιρεθῆ, δύο ἐπίπεδοι ἀριθμοὶ οἱ γινόμενοι ἐκ τε τοῦ ὅλου καὶ ἑκατέρου τῶν μερῶν συναμφοτέροι ἴσοι εἰσὶ τῷ ἀπὸ τοῦ ὅλου τετραγώνῳ.

5 ἀριθμὸς γὰρ ὁ AB διηρήσθω|_{62r} εἰς δύο ἀριθμοὺς τοὺς ΑΓ ΓΒ. λέγω ὅτι δύο ἐπίπεδοι ἀριθμοὶ ὅ τε ἐκ τῶν ΑΒ ΑΓ καὶ ὁ ἐκ τῶν ΑΒ ΒΓ συντεθέντες ἴσοι εἰσὶ τῷ ἀπὸ τοῦ ΑΒ τετραγώνῳ.

ὁ γὰρ ΑΒ ἑαυτὸν πολλαπλασιάσας ποιεῖτω τὸν Δ, ὁ δὲ ΑΓ τὸν ΑΒ πολλαπλασιάσας ποιεῖτω τὸν ΕΖ, τὸν δ' αὐτὸν ΑΒ^a καὶ ὁ ΓΒ πολλαπλασιάσας ποιεῖτω τὸν ΖΗ.



ἐπεὶ τοίνυν ὁ ΑΓ τὸν ΑΒ πολλαπλασιάσας ἐποίησε τὸν ΕΖ, ὁ ἄρα ΑΒ μετρεῖ τὸν ΕΖ κατὰ τὰς ἐν τῷ ΑΓ μονάδας. πάλιν ἐπεὶ ὁ ΓΒ τὸν ΑΒ πολλαπλασιάσας ἐποίησε τὸν ΖΗ, ὁ ἄρα ΑΒ μετρεῖ τὸν ΖΗ κατὰ τὰς ἐν τῷ ΓΒ μονάδας· ἐμέτρει δὲ καὶ τὸν ΕΖ κατὰ τὰς ἐν τῷ ΑΓ μονάδας· ὅλον ἄρα τὸν ΕΗ μετρεῖ ὁ ΑΒ κατὰ τὰς ἐν ἑαυτῷ μονάδας. πάλιν ἐπεὶ ὁ ΑΒ ἑαυτὸν πολλαπλασιάσας ἐποίησε τὸν Δ, μετρεῖ ἄρα καὶ τὸν Δ κατὰ τὰς ἐν ἑαυτῷ μονάδας· ἐκάτερον ἄρα τῶν Δ ΕΗ μετρεῖ ὁ ΑΒ κατὰ τὰς ἐν ἑαυτῷ⁷²⁸ μονάδας· ὁσαπλασίων ἄρα ἐστὶν ὁ Δ τοῦ ΑΒ, τοσαυταπλασίων ἐστὶ καὶ ὁ ΕΗ τοῦ ΑΒ· οἱ δὲ τοῦ αὐτοῦ ἀριθμοῦ^b ἰσάκις πολλαπλάσιοι ἀριθμοὶ ἴσοι ἀλλήλοις εἰσὶν· ἴσος ἄρα ἐστὶν ὁ Δ τῷ ΕΗ· καὶ ἔστιν ὁ μὲν Δ ὁ ἀπὸ τοῦ ΑΒ τετράγωνος, ὁ δὲ ΕΗ ὁ^c συντεθειὸς ἐκ δύο ἐπιπέδων ἀριθμῶν τῶν ἐκ τῶν ΑΒ ΒΓ, ΒΑ ΑΓ· ὁ ἄρα ἀπὸ τοῦ ΑΒ τετράγωνος ἴσος ἐστὶ τῷ συγκειμένῳ ἐκ δύο ἐπιπέδων τῶν ἐκ τῶν ΑΒ ΒΓ, ΒΑ ΑΓ.

ἐὰν ἄρα ἀριθμὸς διαιρεθῆ εἰς δύο ἀριθμούς, ὁ ἀπὸ τοῦ ὅλου τετράγωνος ἴσος ἐστὶ δυσὶν ἐπιπέδοις τοῖς ἐκ τοῦ ὅλου καὶ ἐκατέρου τῶν μερῶν γινομένοις, ὅπερ ἔδει.

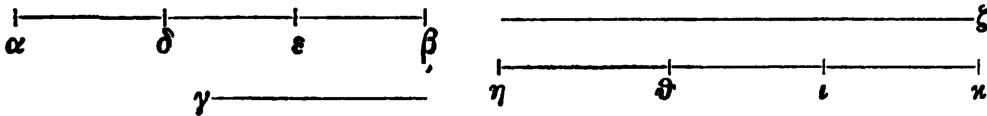
^a ΑΒ corr. ex ΑΓ ^b ἀριθμοῦ corr. ex ἀριθμοὶ ^c ὁ s.l.

β'

Ἐὰν δύο ἀριθμῶν ὄντων διαιρεθῆ ὁ ἕτερος^{62v} αὐτῶν εἰς ὅσουσδηποτοῦν ἀριθμούς, ὁ ἐκ τῶν ἐξ ἀρχῆς δύο⁷²⁶ ἀριθμῶν ἐπίπεδος ἀριθμὸς ἴσος ἐστὶ τοῖς ἐκ τε τοῦ ἀδιαιρέτου καὶ ἐκάστου τῶν μερῶν τοῦ διαιρεθέντος γινομένοις ἐπιπέδοις.

ἔστωσαν δύο ἀριθμοὶ οἱ ΑΒ Γ, καὶ διηρήσθω ὁ ΑΒ εἰς ὅσουσδηποτοῦν ἀριθμούς τοὺς ΑΔ ΔΕ ΕΒ. λέγω ὅτι ὁ ἐκ τῶν Γ ΑΒ ἐπίπεδος ἴσος ἐστὶ τοῖς ἐκ τῶν Γ ΑΔ, Γ ΔΕ, Γ ΕΒ ἐπιπέδοις.

ἔστω γὰρ ἐκ μὲν τῶν Γ ΑΒ ὁ Ζ, ἐκ δὲ τῶν Γ ΑΔ ὁ ΗΘ, ἐκ δὲ τῶν Γ ΔΕ ὁ ΘΙ, ἐκ δὲ τῶν Γ ΕΒ ὁ ΙΚ.



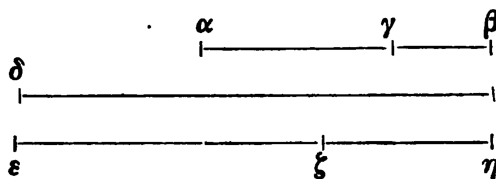
καὶ ἐπεὶ ὁ ΑΒ τὸν Γ πολλαπλασιάσας ἐποίησε τὸν Ζ, ὁ ἄρα Γ μετρεῖ τὸν Ζ κατὰ τὰς ἐν τῷ ΑΒ μονάδας. διὰ τὰ αὐτὰ δὴ καὶ τὸν μὲν ΗΘ μετρεῖ κατὰ τὰς ἐν τῷ ΑΔ μονάδας, τὸν δὲ ΘΙ κατὰ τὰς ἐν τῷ ΔΕ, τὸν δὲ ΙΚ κατὰ τὰς ἐν τῷ ΕΒ· ὅλον ἄρα τὸν ΗΚ μετρεῖ ὁ Γ κατὰ τὰς ἐν τῷ ΑΒ μονάδας· ἐμέτρει δὲ καὶ τὸν Ζ κατὰ τὰς ἐν τῷ ΑΒ μονάδας· ἐκάτερος ἄρα τῶν Ζ ΗΚ ἰσάκις ἐστὶ πολλαπλάσιος τοῦ Γ· οἱ δὲ τοῦ αὐτοῦ ἰσάκις πολλαπλάσιοι ἴσοι ἀλλήλοις εἰσὶν· ἴσος ἄρα ἐστὶν ὁ Ζ τῷ ΗΚ· καὶ ἔστιν ὁ μὲν Ζ ὁ ἐκ τῶν Γ ΑΒ ἐπίπεδος, ὁ δὲ ΗΚ ὁ συγκείμενος ἐκ τε τοῦ Γ καὶ ἐκάστου τῶν ΑΔ ΔΕ ΕΒ ἐπιπέδων· ὁ ἄρα ἐκ τῶν Γ ΑΒ ἐπίπεδος ἴσος ἐστὶ τοῖς ἐκ τε τοῦ Γ καὶ ἐκάστου τῶν ΑΔ ΔΕ ΕΒ ἐπιπέδοις.

ἐὰν ἄρα δύο ἀριθμῶν ὄντων διαιρεθῆ ὁ ἕτερος αὐτῶν εἰς ὅσουσδηποτοῦν ἀριθμούς, ὁ ἐκ τῶν ἐξ ἀρχῆς⁷²⁷ δύο ἀριθμῶν ἐπίπεδος ἴσος ἐστὶ τοῖς ἐκ τε τοῦ ἀδιαιρέτου καὶ ἐκάστου τῶν μερῶν τοῦ διαιρεθέντος ἐπιπέδοις, ὅπερ ἔδει.

γ'

Ἐὰν ἀριθμὸς διαιρεθῆ εἰς δύο ἀριθμούς, ὁ ἐκ τοῦ ὅλου καὶ ἐνὸς τῶν μερῶν ἐπίπεδος ἴσος ἐστὶ^{63r} τῷ ἐκ τῶν μερῶν ἐπιπέδῳ σὺν τῷ ἀπὸ τοῦ προειρημένου μέρους τετραγώνῳ.

ἀριθμὸς γὰρ ὁ ΑΒ διηρήσθω εἰς δύο ἀριθμούς τοὺς ΑΓ ΒΒ. λέγω ὅτι ὁ ἐκ τῶν ΑΒ ΒΒ ἐπίπεδος ἴσος ἐστὶ τῷ τε ἐκ τῶν ΑΓ ΒΒ ἐπιπέδῳ καὶ τῷ ἀπὸ τοῦ ΒΒ τετραγώνῳ.



ὁ γὰρ AB πολλαπλασιασάτω τὸν ΓΒ καὶ ποιείτω τὸν Δ, ὁ δὲ ΑΓ τὸν ΓΒ πολλαπλασιασάτω καὶ ποιείτω τὸν ΕΖ, ὁ δὲ ΓΒ ἑαυτὸν πολλαπλασιάσας ποιείτω τὸν ΖΗ.

καὶ ἐπεὶ ὁ ΑΒ τὸν ΓΒ πολλαπλασιάσας ἐποίησε τὸν Δ, ὁ ἄρα ΓΒ μετρεῖ τὸν Δ κατὰ τὰς ἐν τῷ ΑΒ μονάδας. πάλιν ἐπεὶ ὁ ΑΓ τὸν ΓΒ πολλαπλασιάσας ἐποίησε τὸν ΕΖ,⁷²⁹ ὁ ἄρα ΓΒ μετρεῖ τὸν ΕΖ κατὰ τὰς ἐν τῷ ΑΓ μονάδας. πάλιν ἐπεὶ ὁ ΓΒ ἑαυτὸν πολλαπλασιάσας ἐποίησε τὸν ΖΗ, μετρεῖ ἄρα ὁ ΓΒ τὸν ΖΗ κατὰ τὰς ἐν ἑαυτῷ μονάδας· ἐμέτρει δὲ καὶ τὸν ΕΖ κατὰ τὰς ἐν τῷ ΑΓ μονάδας· ὅλον ἄρα τὸν ΕΗ μετρεῖ ὁ ΓΒ κατὰ τὰς ἐν τῷ ΑΒ μονάδας· ἐμέτρει δὲ καὶ τὸν Δ κατὰ τὰς ἐν τῷ ΑΒ μονάδας· ἰσάκις ἄρα ὁ ΓΒ ἐκάτερον τῶν Δ ΕΗ μετρεῖ· οἱ δὲ ὑπὸ τοῦ αὐτοῦ ἰσάκις μετρούμενοι ἴσοι ἀλλήλοις εἰσίν· ἴσος ἄρα ἐστὶν ὁ Δ τῷ ΕΗ· καὶ ἔστιν ὁ μὲν Δ ὁ ἐκ τῶν ΑΒ ΒΓ ἐπίπεδος, ὁ δὲ ΕΗ ὁ ἐκ τῶν ΑΓ ΓΒ ἐπίπεδος σὺν τῷ ἀπὸ τοῦ ΓΒ τετραγώνῳ· ὁ ἄρα ἐκ τῶν ΑΒ ΒΓ ἐπίπεδος ἴσος ἐστὶ τῷ τε ἐκ τῶν ΑΓ ΓΒ ἐπιπέδῳ καὶ τῷ ἀπὸ τοῦ ΓΒ τετραγώνῳ.

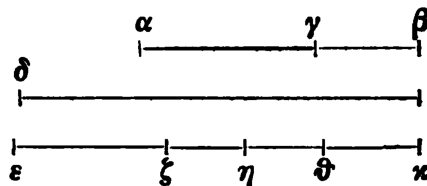
ἐὰν ἄρα ἀριθμὸς εἰς δύο τυχόντας ἀριθμοὺς διαιρεθῆ, ὁ ἐκ τοῦ ὅλου καὶ τοῦ ἐτέρου τῶν μερῶν ἐπίπεδος ἴσος ἐστὶ τῷ τε ἐκ τῶν μερῶν ἐπιπέδῳ καὶ τῷ ἀπὸ τοῦ προειρημένου μέρους τετραγώνῳ, ὅπερ ἔδει.

δ'

Ἐὰν ἀριθμὸς διαιρεθῆ εἰς δύο ἀριθμούς, ὁ ἀπὸ τοῦ ὅλου τετράγωνος ἴσος ἐστὶ τοῖς^{63v} ἀπὸ τῶν μερῶν τετραγώνοις καὶ τῷ δις ἐκ τῶν μερῶν^a ἐπιπέδῳ.

ἀριθμὸς γὰρ ὁ ΑΒ διηρήσθω εἰς δύο ἀριθμοὺς τοὺς ΑΓ ΓΒ. λέγω ὅτι ὁ ἀπὸ τοῦ ΑΒ τετράγωνος ἴσος ἐστὶ τοῖς τε ἀπὸ τῶν ΑΓ ΓΒ τετραγώνοις καὶ τῷ δις ἐκ τῶν ΑΓ ΓΒ ἐπιπέδῳ.

⁷³⁰ ἔστω ἀπὸ μὲν τοῦ ΑΒ τετράγωνος ὁ Δ, ἀπὸ δὲ τοῦ ΑΓ ὁ ΕΖ, ἀπὸ δὲ τοῦ ΓΒ ὁ ΗΘ, ἐκ δὲ τῶν ΑΓ ΓΒ ἐκάτερος τῶν ΖΗ ΘΚ.



ἐπεὶ τοίνυν ὁ ΑΓ ἑαυτὸν πολλαπλασιάσας ἐποίησε τὸν ΕΖ^b, ὁ ἄρα ΑΓ μετρεῖ τὸν ΕΖ κατὰ τὰς ἐν ἑαυτῷ μονάδας. πάλιν ἐπεὶ ὁ ΓΒ τὸν ΑΓ πολλαπλασιάσας ἐποίησε τὸν ΖΗ, μετρεῖ ἄρα τὸν ΖΗ ὁ ΑΓ κατὰ τὰς ἐν τῷ ΓΒ μονάδας· ἐμέτρει δὲ καὶ τὸν ΕΖ κατὰ τὰς ἐν ἑαυτῷ μονάδας· ὅλον ἄρα τὸν ΕΗ μετρεῖ ὁ ΑΓ κατὰ τὰς ἐν τῷ ΑΒ μονάδας· ὁ ἄρα ΑΒ πολλαπλασιάσας τὸν ΑΓ ἐποίησε τὸν ΕΗ· ὁ ΕΗ ἄρα ἐπίπεδος ἐστὶν ὁ ἐκ τῶν ΒΑ ΑΓ. ὁμοίως δὲ δείξομεν ὅτι καὶ ὁ ΗΚ ἐπίπεδος ἐστὶν ὁ ἐκ τῶν ΑΒ ΒΓ· καὶ ἔστιν ὁ ἀπὸ τοῦ ΑΒ τετράγωνος ὁ Δ· ἐὰν δὲ ἀριθμὸς διαιρεθῆ εἰς δύο ἀριθμούς, ὁ ἀπὸ τοῦ ὅλου τετράγωνος ἴσος ἐστὶ δυοῖς τοῖς ἐκ τοῦ ὅλου καὶ ἐκατέρου τῶν μερῶν ἐπιπέδοις· ἴσος ἄρα ὁ Δ τῷ ΕΚ· ἀλλὰ μὴν ὁ ΕΚ συγκείμενός ἐστιν ἕκ τε τῶν ἀπὸ τῶν ΑΓ ΓΒ τετραγώνων καὶ τοῦ δις ἐκ τῶν ΑΓ ΓΒ ἐπιπέδου· ὁ δὲ Δ ὑπάρχει ὁ ἀπὸ τοῦ ΑΒ τετράγωνος· ὁ ἄρα ἀπὸ τοῦ ΑΒ τετράγωνος ἴσος ἐστὶ τοῖς τε ἀπὸ τῶν ΑΓ ΓΒ τετραγώνοις καὶ τῷ δις ἐκ τῶν ΑΓ ΓΒ ἐπιπέδῳ.

ἐὰν ἄρα ἀριθμὸς διαιρεθῆ εἰς δύο ἀριθμούς, καὶ τὰ ἐξῆς, ὅπερ ἔδει.

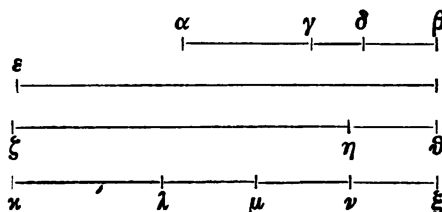
^a μερῶν in ras. m.1 ^b ΕΖ corr. e. Ζ

ε'

Ἐὰν ἄρτιος ἀριθμὸς δίχα διαιρεθῆ, διαιρεθῆ δὲ καὶ εἰς ἀνίσους ἀριθμούς, ὁ ἐκ τῶν^{64r} ἀνίσων μερῶν⁷³¹ ἐπίπεδος μετὰ τοῦ ἀπὸ τοῦ μεταξύ τετραγώνου ἴσος ἐστὶ τῷ ἀπὸ τοῦ ἡμίσεος τετραγώνῳ.

ἔστω γὰρ ἄρτιος ἀριθμὸς ὁ ΑΒ, καὶ διηρήσθω δίχα μὲν εἰς τοὺς ΑΓ ΓΒ, ἀνισαχῆ δὲ εἰς τοὺς ΑΔ ΔΒ. λέγω ὅτι ὁ ἀπὸ τοῦ ΓΒ τετράγωνος ἴσος ἐστὶ τῷ ἐκ τῶν ΑΔ ΔΒ ἐπιπέδῳ μετὰ τοῦ ἀπὸ τοῦ ΓΔ τετραγώνου.

ἔστω γὰρ ἀπὸ μὲν τοῦ ΓΒ τετράγωνος ὁ Ε, ἐκ δὲ τῶν ΑΔ ΔΒ ἐπίπεδος ὁ ΖΗ, ἀπὸ δὲ τοῦ ΔΓ τετράγωνος ὁ ΗΘ.



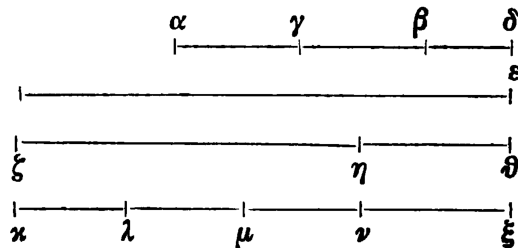
- καὶ ἐπεὶ ὁ ΒΓ ἀριθμὸς διήρηται εἰς τοὺς ΒΔ ΔΓ, ἔστιν ἄρα ὁ ἀπὸ τοῦ ΒΓ τετράγωνος, τουτέστιν
 10 ὁ Ε, ἴσος τοῖς ἀπὸ τῶν ΒΔ ΔΓ τετραγώνοις μετὰ τοῦ δις ἐκ τῶν ΒΔ ΔΓ. ἔστω γοῦν ἀπὸ μὲν τοῦ ΒΔ
 τετράγωνος ὁ ΚΛ, ἀπὸ δὲ τοῦ ΔΓ ὁ ΝΞ, ἐκ δὲ τῶν ΒΔ ΔΓ ἑκάτερος τῶν ΛΜ ΜΝ· ὅλος ἄρα ὁ ΚΞ
 ἴσος ἐστὶ τῷ Ε. καὶ ἐπεὶ ὁ ΒΔ ἑαυτὸν πολλαπλασιάσας ἐποίησε τὸν ΚΛ, μετρεῖ ἄρα αὐτὸν κατὰ τὰς
 ἐν ἑαυτῷ μονάδας. πάλιν ἐπεὶ ὁ ΓΔ τὸν ΔΒ πολλαπλασιάσας τὸν ΛΜ ἐποίησεν, ὁ ἄρα ΔΒ μετρεῖ
 15 τὸν ΛΜ κατὰ τὰς ἐν τῷ ΔΓ μονάδας· ἐμέτρει δὲ καὶ τὸν ΚΛ κατὰ τὰς ἐν ἑαυτῷ μονάδας· ὅλον ἄρα
 τὸν ΚΜ μετρεῖ ὁ ΔΒ κατὰ τὰς ἐν τῷ ΓΒ μονάδας· ἴσος δὲ ὁ ΓΒ τῷ ΓΑ· ὁ ἄρα ΔΒ μετρεῖ τὸν ΚΜ
 κατὰ τὰς ἐν τῷ ΑΓ μονάδας. πάλιν ἐπεὶ ὁ ΓΔ πολλαπλασιάσας τὸν ΔΒ ἐποίησε τὸν ΜΝ, ὁ ἄρα ΔΒ
 μετρεῖ τὸν ΜΝ κατὰ τὰς ἐν τῷ ΔΓ μονάδας· ἐμέτρει δὲ καὶ τὸν ΚΜ κατὰ τὰς ἐν τῷ ΑΓ μονάδας·
 ὅλον ἄρα τὸν ΚΝ μετρεῖ ὁ ΔΒ κατὰ τὰς ἐν τῷ ΑΔ μονάδας· μετρεῖ δὲ καὶ τὸν ΖΗ ὁ ΒΔ κατὰ τὰς
 ἐν⁷³² τῷ ΑΔ μονάδας – ὑπόκειται γάρ – ἴσος ἄρα ἐστὶν ὁ ΖΗ τῷ ΚΝ – οἱ γὰρ τοῦ αὐτοῦ ἰσάκις^{64v}
 20 πολλαπλασίοι ἴσοι ἀλλήλοις εἰσὶν – ἔστι δὲ καὶ ὁ ΗΘ τῷ ΝΞ ἴσος – ἑκάτερος γὰρ ὑπόκειται ἀπὸ
 τοῦ ΓΔ τετράγωνος – ὅλος ἄρα ὁ ΚΞ ὅλῳ τῷ ΖΘ ἴσος ἐστίν· ἔστι δὲ καὶ τῷ Ε ὁ ΚΞ ἴσος· καὶ ὁ ΖΘ
 ἄρα τῷ Ε ἴσος ἐστὶ· καὶ ἔστιν ὁ μὲν ΖΘ ὁ ἐκ τῶν ΑΔ ΔΒ ἐπίπεδος μετὰ τοῦ ἀπὸ τοῦ ΔΓ
 τετραγώνου, ὁ δὲ Ε ὁ ἀπὸ τοῦ ΓΒ τετράγωνος· ὁ ἄρα ἐκ τῶν ΑΔ ΔΒ ἐπίπεδος μετὰ τοῦ ἀπὸ τοῦ ΔΓ
 τετραγώνου ἴσος ἐστὶ τῷ ἀπὸ τοῦ ΓΒ τετραγώνῳ.
 25 ἐὰν ἄρα ἄρτιος ἀριθμὸς διαιρεθῇ δίχα, καὶ τὰ ἐξῆς, ὅπερ.

ζ'

Ἐὰν ἄρτιος ἀριθμὸς διαιρεθῇ δίχα προστεθῇ δὲ τις αὐτῷ, ὁ ἐκ τοῦ ὅλου σὺν τῷ προσκειμένῳ καὶ
 τοῦ προσκειμένου ἐπίπεδος μετὰ τοῦ ἀπὸ τοῦ ἡμίσεος τετραγώνου ἴσος ἐστὶ τῷ ἀπὸ τοῦ συγκει-
 μένου ἕκ τε τοῦ ἡμίσεος καὶ τοῦ προσκειμένου τετραγώνῳ^a.

- 5 ἄρτιος γὰρ ἀριθμὸς ὁ ΑΒ διηρήσθω δίχα εἰς τοὺς ΑΓ ΓΒ ἀριθμούς, καὶ προσκείσθω αὐτῷ ἕτε-
 ρός τις^b ἀριθμὸς ὁ ΒΔ. λέγω ὅτι ὁ ἐκ τῶν ΑΔ ΔΒ ἐπίπεδος μετὰ τοῦ ἀπὸ τοῦ ΓΒ τετραγώνου ἴσος
 ἐστὶ τῷ ἀπὸ τοῦ ΓΔ τετραγώνῳ.

ἔστω γὰρ ἀπὸ μὲν τοῦ ΓΔ τετράγωνος ὁ Ε, ἐκ δὲ τῶν ΑΔ ΔΒ ἐπίπεδος ὁ ΖΗ, ἀπὸ δὲ τοῦ ΓΒ
 τετράγωνος ὁ ΗΘ.



- 10 καὶ ἐπεὶ ὁ ἀπὸ τοῦ ΓΔ ἴσος ἐστὶ τοῖς ἀπὸ τῶν ΔΒ ΒΓ μετὰ τοῦ δις ἐκ τῶν ΔΒ ΒΓ, ἔστω ἀπὸ
 μὲν⁷³³ τοῦ ΔΒ ὁ ΚΛ, ἐκ δὲ τῶν ΔΒ ΒΓ ἑκάτερος τῶν ΛΜ ΜΝ, ἀπὸ δὲ τοῦ ΒΓ ὁ ΝΞ· ὅλος ἄρα ὁ
 ΚΞ ἴσος ἐστὶ τῷ ἀπὸ τοῦ ΓΔ τετραγώνῳ· καὶ ἔστιν ὁ ἀπὸ τοῦ ΓΔ τετράγωνος ὁ Ε· ὁ ἄρα ΚΞ ἴσος
 ἐστὶ τῷ Ε. καὶ ἐπεὶ ὁ ΒΔ ἑαυτὸν πολλαπλασιάσας ἐποίησε τὸν ΚΛ, ὁ ἄρα ΒΔ μετρεῖ τὸν ΚΛ κατὰ
 τὰς ἐν ἑαυτῷ μονάδας· μετρεῖ δὲ καὶ τὸν ΛΜ κατὰ τὰς ἐν τῷ ΓΒ μονάδας· ὅλον^{65r} ἄρα τὸν ΚΜ
 15 μετρεῖ ὁ ΔΒ κατὰ τὰς ἐν τῷ ΓΔ μονάδας. καὶ ἐπεὶ ὁ ΔΒ μετρεῖ καὶ τὸν ΜΝ κατὰ τὰς ἐν τῷ ΓΒ
 μονάδας ἴσος δὲ ὁ ΓΒ τῷ ΓΑ – ὑπόκειται γάρ –, ὅλον ἄρα τὸν ΚΝ μετρεῖ ὁ ΔΒ κατὰ τὰς ἐν τῷ ΑΔ
 μονάδας· ἀλλὰ μὴν καὶ τὸν ΖΗ μετρεῖ ὁ ΔΒ κατὰ τὰς ἐν τῷ ΑΔ μονάδας – ὑπόκειται γὰρ ὁ ΖΗ ἐκ
 τῶν ΑΔ ΔΒ – ἴσος ἄρα ὁ ΖΗ τῷ ΚΝ· ἔστι δὲ καὶ ὁ ΘΗ τῷ ΝΞ ἴσος – ἑκάτερος γὰρ ἐστὶν ἀπὸ τοῦ
 ΓΒ τετράγωνος – ὅλος ἄρα ὁ ΖΘ τῷ ΚΞ ἴσος ἐστίν· ὁ δὲ ΚΞ ἀπεδείχθη τῷ Ε ἴσος· καὶ ὁ ΖΘ ἄρα
 20 τῷ Ε ἴσος ἐστίν· καὶ ἔστιν ὁ μὲν ΖΘ ὁ ἐκ τῶν ΑΔ ΔΒ μετὰ τοῦ ἀπὸ τοῦ ΒΓ, ὁ δὲ Ε ἀπὸ τοῦ ΓΔ· ὁ
 ἄρα ἐκ τῶν ΑΔ ΔΒ μετὰ τοῦ ἀπὸ τοῦ ΓΒ ἴσος ἐστὶ τῷ ἀπὸ τοῦ ΓΔ.

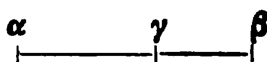
ἐὰν ἄρα ἄρτιος ἀριθμὸς διαιρεθῇ δίχα προστεθῇ δὲ τις αὐτῷ, ὁ ἐκ τοῦ ὅλου σὺν τῷ προσκει-
 μένῳ καὶ τοῦ προσκειμένου ἐπίπεδος μετὰ τοῦ ἀπὸ τοῦ ἡμίσεος τετραγώνου ἴσος ἐστὶ τῷ ἀπὸ τοῦ
 συγκειμένου ἕκ τε τοῦ ἡμίσεος καὶ τοῦ προσκειμένου τετραγώνῳ, ὅπερ.

^a τετραγώνω corr. ex –ου ^b τις s.l.

|⁷³⁴ ζ'

Ἐὰν ἀριθμὸς διαιρεθῆ εἰς δύο ἀριθμούς, ὁ ἀπὸ τοῦ ὅλου τετράγωνος μετὰ τοῦ ἀφ' ἑνὸς τῶν μερῶν τετραγώνου ἴσος ἐστὶ τῷ δις ἐκ τοῦ ὅλου καὶ τοῦ εἰρημένου μέρους ἐπιπέδω μετὰ τοῦ ἀπὸ τοῦ λοιποῦ μέρους τετραγώνου.

- 5 ἀριθμὸς γὰρ ὁ AB διηρήσθω εἰς τοὺς ΑΓ ΓΒ ἀριθμούς. λέγω ὅτι οἱ ἀπὸ τῶν ΒΑ ΑΓ τετράγωνοι ἴσοι εἰσὶ τῷ δις ἐκ τῶν ΒΑ ΑΓ ἐπιπέδω μετὰ τοῦ ἀπὸ τοῦ ΒΓ τετραγώνου.



- ἐπεὶ γὰρ ὁ ἀπὸ τοῦ AB τετράγωνος ἴσος ἐστὶ τοῖς ἀπὸ τῶν ΒΓ ΓΑ καὶ τῷ δις ἐκ τῶν ΒΓ ΓΑ, κοινὸς προσκείσθω ὁ ἀπὸ τοῦ ΑΓ τετράγωνος· ὁ ἄρα ἀπὸ τοῦ ΒΑ μετὰ τοῦ ἀπὸ τοῦ ΑΓ ἴσος ἐστὶ δυοῖς τοῖς ἀπὸ τοῦ ΑΓ τετραγώνοις καὶ ἐνὶ τῷ ἀπὸ τοῦ ΓΒ μετὰ τοῦ δις ἐκ τῶν ΒΓ ΓΑ. καὶ ἐπεὶ ὁ ἅπαξ ἐκ τῶν ΒΑ ΑΓ|^{65v} ἴσος ἐστὶ τῷ ἅπαξ ἐκ τῶν ΒΓ ΓΑ μετὰ τοῦ ἀπὸ τοῦ ΓΑ τετραγώνου, ὁ ἄρα δις^a ἐκ τῶν ΒΑ ΑΓ ἴσος ἐστὶ τῷ δις ἐκ τῶν ΒΓ ΓΑ μετὰ δύο τῶν ἀπὸ τοῦ ΓΑ τετραγώνων· κοινὸς προσκείσθω ὁ ἀπὸ τοῦ ΒΓ τετράγωνος· δύο ἄρα τετράγωνοι ἀπὸ τοῦ ΑΓ καὶ εἷς ἀπὸ τοῦ ΓΒ μετὰ τοῦ δις ἐκ τῶν ΒΓ ΓΑ ἴσοι εἰσὶ τῷ δις ἐκ τῶν ΒΑ ΑΓ μετὰ τοῦ ἀπὸ τοῦ ΓΒ· ὁ ἄρα ἀπὸ τοῦ AB τετράγωνος μετὰ τοῦ ἀπὸ τοῦ ΑΓ ἴσος ἐστὶ τῷ δις ἐκ τῶν ΒΑ ΑΓ μετὰ τοῦ ἀπὸ τοῦ ΓΒ τετραγώνου.

- 15 ἐὰν ἄρα ἀριθμὸς διαιρεθῆ εἰς δύο ἀριθμούς, ὁ ἀπὸ τοῦ ὅλου τετράγωνος μετὰ τοῦ ἀφ' ἑνὸς τῶν μερῶν τετραγώνου ἴσος ἐστὶ τῷ δις ἐκ τοῦ ὅλου καὶ τοῦ εἰρημένου μέρους ἐπιπέδω μετὰ τοῦ ἀπὸ τοῦ λοιποῦ μέρους τετραγώνου^b, ὅπερ ἔδει.

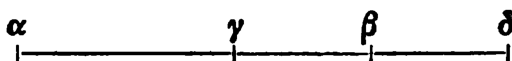
^a δ– e corr. ^b τετραγώνου corr ex –ω

| η'

Ἐὰν ἀριθμὸς εἰς δύο ἀριθμούς διαιρεθῆ, ὁ τετράκις ἐκ τοῦ ὅλου καὶ ἑνὸς τῶν μερῶν ἐπίπεδος μετὰ τοῦ ἀπὸ τοῦ λοιποῦ μέρους τετραγώνου ἴσος ἐστὶ τῷ ἀπὸ τοῦ ὅλου καὶ τοῦ προειρημένου μέρους ὡς ἀφ' ἑνὸς τετραγώνω.

- 5 ἀριθμὸς γὰρ ὁ AB διηρήσθω εἰς δύο ἀριθμούς τοὺς ΑΓ ΓΒ. λέγω ὅτι ὁ τετράκις ἐκ τῶν AB ΒΓ μετὰ τοῦ ἀπὸ τοῦ ΑΓ τετραγώνου ἴσος ἐστὶ τῷ ἀπὸ τοῦ AB ΒΓ ὡς ἀφ' ἑνὸς τετραγώνω.

κεῖσθω γὰρ τῷ ΒΓ ἀριθμῷ ἴσος ὁ ΒΔ.



- καὶ ἐπεὶ ὁ ἀπὸ τοῦ ΑΔ^a ἴσος ἐστὶ τοῖς ἀπὸ τῶν AB ΒΔ τετραγώνοις καὶ τῷ δις ἐκ τῶν AB^b ΒΔ ἐπιπέδω, καὶ ἔστιν ὁ ΒΔ ἴσος τῷ ΒΓ, ἔστιν ἄρα ὁ ἀπὸ τοῦ ΑΔ τετράγωνος ἴσος τοῖς ἀπὸ τῶν AB ΒΓ τετραγώνοις καὶ τῷ δις ἐκ τῶν AB ΒΓ ἐπιπέδω· τὰ δὲ ἀπὸ τῶν AB ΒΓ τετράγωνα ἴσα ἐστὶ τῷ δις ἐκ τῶν AB ΒΓ ἐπιπέδω καὶ τῷ ἀπὸ τοῦ ΑΓ τετραγώνω· ἔστιν ἄρα ὁ ἀπὸ τοῦ ΑΔ τετράγωνος ἴσος τῷ τετράκις ἐκ τῶν AB ΒΓ ἐπιπέδω καὶ τῷ ἀπὸ τοῦ ΑΓ τετραγώνω· καὶ ἔστιν ὁ ἀπὸ τοῦ ΑΔ τετράγωνος ὁ ἀπὸ τοῦ AB ΒΓ ὡς ἀφ' ἑνὸς – ὁ γὰρ ΒΔ ἴσος ἐστὶ τῷ ΒΓ|^{66r} –· ἔστιν ἄρα ὁ ἀπὸ τοῦ AB ΒΓ ὡς ἀφ' ἑνὸς τετράγωνος ἴσος τῷ τετράκις ἐκ τῶν AB ΒΓ καὶ τῷ ἀπὸ τοῦ ΑΓ.

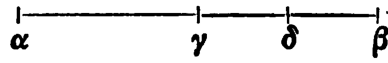
- 15 ἐὰν ἄρα ἀριθμὸς διαιρεθῆ εἰς δύο ἀριθμούς, καὶ τὰ ἐξῆς, ὅπερ.

^a ΑΔ corr. ex AB ^b AB corr. ex ΑΔ m.2 ^c τοῦ corr. e τῶν

| θ'

Ἐὰν ἄρτιος ἀριθμὸς διαιρεθῆ δίχα ἔτι δὲ διαιρεθῆ καὶ εἰς ἀνίσους ἀριθμούς, οἱ ἀπὸ τῶν ἀνίσων ἀριθμῶν τετράγωνοι διπλάσιοι εἰσὶ τοῦ ἀπὸ τοῦ ἡμίσεος τετραγώνου μετὰ τοῦ ἀπὸ τοῦ μεταξύ τετραγώνου.

- 5 ἄρτιος γὰρ ἀριθμὸς ὁ AB δίχα μὲν διηρήσθω εἰς τοὺς AG GB ἀριθμούς, εἰς ἀνίσους δὲ διηρήσθω τοὺς AD DB . λέγω ὅτι οἱ ἀπὸ τῶν AD DB τετράγωνοι διπλάσιοί εἰσι τῶν ἀπὸ τῶν AG GD τετραγώνων.



- 10 ἐπεὶ γὰρ ἄρτιος ἀριθμὸς ὁ AB εἰς ἴσους μὲν διήρηται τοὺς AG GB εἰς ἀνίσους δὲ τοὺς AD DB , ὁ ἄρα ἐκ τῶν AD DB μετὰ τοῦ ἀπὸ τοῦ AG ἴσος ἐστὶ τῶ ἀπὸ τοῦ AG τετραγώνου· ὁ δὲ ἐκ τῶν AD DB μετὰ δύο τῶν ἀπὸ τοῦ GD τετραγώνων διπλάσιός ἐστι τοῦ ἀπὸ τοῦ AG τετραγώνου. καὶ ἐπεὶ ὁ AB δίχα διήρηται εἰς τοὺς AG GB , ὁ ἄρα ἀπὸ τοῦ AB τετράγωνος τετραπλάσιός ἐστι τοῦ ἀπὸ τοῦ AG τετραγώνου. καὶ ἐπεὶ ὁ δὲ ἐκ τῶν AD DB μετὰ δύο τῶν ἀπὸ τοῦ GD διπλάσιός ἐστι τοῦ ἀπὸ τοῦ GA , ἐὰν δὲ ὅσι δύο ἀριθμοὶ καὶ ἡ ὁ μὲν ἕτερος αὐτῶν τοῦ αὐτοῦ τετραπλάσιος ὁ δὲ ἕτερος διπλάσιος, ὁ τετραπλάσιος διπλάσιός ἐστι τοῦ διπλασίου, ὁ ἄρα ἀπὸ τοῦ AB διπλάσιός ἐστι τοῦ δὲ ἐκ τῶν AD DB μετὰ δύο τῶν ἀπὸ τοῦ GD . ἔστιν ἄρα ὁ δὲ ἐκ τῶν AD DB ἐλάττων ἡμίσεος τοῦ ἀπὸ τοῦ AB τῶ δὲ ἀπὸ τοῦ GD .⁷³⁷ καὶ ἐπεὶ ὁ δὲ ἐκ τῶν AD DB μετὰ τοῦ συγκεκριμένου ἐκ τῶν ἀπὸ τῶν AD DB ἴσος ἐστὶ τῶ ἀπὸ τοῦ AB , ὁ ἄρα συγκεκριμένος ἐκ τῶν ἀπὸ τῶν AD DB μείζων ἐστὶν ἡμίσεος τοῦ ἀπὸ τοῦ AB τῶ δὲ ἀπὸ τοῦ GD .^{66v} καὶ ἔστιν ὁ ἀπὸ τοῦ AB τοῦ ἀπὸ τοῦ AG τετραπλάσιος· ὁ ἄρα συγκεκριμένος ἐκ τῶν ἀπὸ τῶν AD DB μείζων ἐστὶ διπλασίου τοῦ ἀπὸ τοῦ AG τῶ δὲ ἀπὸ τοῦ GD . διπλάσιος ἄρα ἐστὶ τῶν ἀπὸ τῶν AG GD .

ἐὰν ἄρα ἄρτιος ἀριθμὸς, καὶ τὰ ἐξῆς, ὅπερ ἔδει δεῖξαι.

^a e corr. Δ

ι'

- Ἐὰν ἄρτιος ἀριθμὸς δίχα διαιρεθῆ προστεθῆ δὲ τις αὐτῶ ἕτερος ἀριθμὸς, ὁ ἀπὸ τοῦ ὅλου σὺν τῶ προσκειμένῳ καὶ ἀπὸ τοῦ προσκειμένου οἱ συναμφοτέροι τετράγωνοι διπλάσιοί εἰσι τοῦ ἀπὸ τοῦ ἡμίσεος τετραγώνου καὶ τοῦ ἀπὸ τοῦ συγκεκριμένου ἕκ τε τοῦ ἡμίσεος καὶ τοῦ προσκειμένου ὡς ἀφ' ἑνὸς τετραγώνου.

- 5 ἔστω γὰρ ἄρτιος ἀριθμὸς ὁ AB , καὶ δίχα διηρήσθω εἰς τοὺς AG GB , καὶ προσκείσθω αὐτῶ ἕτερός τις ἀριθμὸς ὁ BD . λέγω ὅτι οἱ ἀπὸ τῶν AD DB τετράγωνοι διπλάσιοί εἰσι τῶν ἀπὸ τῶν AG GD τετραγώνων.



- 10 ἐπεὶ γὰρ ἀριθμὸς ὁ AD διήρηται εἰς τοὺς AB BD , οἱ ἄρα ἀπὸ τῶν AD DB τετράγωνοι ἴσοι εἰσι τῶ δὲ ἐκ τῶν AD DB ἐπιπέδῳ μετὰ τοῦ ἀπὸ τοῦ AB τετραγώνου· ὁ δὲ ἀπὸ τοῦ AB τετράγωνος ἴσος ἐστὶ τέσ⁷³⁸σαρσι τοῖς^a ἀπὸ τῶν AG GB τετραγώνων – ἴσος γάρ ἐστιν ὁ AG τῶ GB –· οἱ ἄρα ἀπὸ τῶν AD DB τετράγωνοι ἴσοι εἰσι τῶ τε δὲ ἐκ τῶν AD DB καὶ τέσσαρσι τοῖς^b ἀπὸ τῶν BG GA . καὶ ἐπεὶ ὁ ἐκ τῶν AD DB μετὰ τοῦ ἀπὸ τοῦ GB ἴσος ἐστὶ τῶ ἀπὸ τοῦ GD , ὁ ἄρα δὲ ἐκ τῶν AD DB μετὰ δύο τῶν ἀπὸ τοῦ GB ἴσος ἐστὶ δυσι τοῖς ἀπὸ τοῦ GD . οἱ ἄρα ἀπὸ τῶν AD DB τετράγωνοι ἴσοι εἰσι δυσι τοῖς ἀπὸ τοῦ GD καὶ δυσι τοῖς ἀπὸ τοῦ AG . διπλάσιοι ἄρα εἰσι τῶν ἀπὸ τῶν AG GD .^{67r} καὶ ἔστιν ὁ μὲν ἀπὸ τοῦ AD τετράγωνος ὁ ἀπὸ τοῦ ὅλου καὶ τοῦ προσκειμένου, ὁ δὲ ἀπὸ τοῦ DB ὁ ἀπὸ τοῦ προσκειμένου, ὁ δὲ ἀπὸ τοῦ AG ὁ ἀπὸ τοῦ ἡμίσεος, ὁ δὲ ἀπὸ τοῦ GD ὁ ἀπὸ τοῦ συγκεκριμένου ἕκ τε τοῦ ἡμίσεος καὶ τοῦ προσκειμένου· ὁ ἄρα ἀπὸ τοῦ ὅλου σὺν τῶ προσκειμένῳ τετράγωνος μετὰ τοῦ ἀπὸ τοῦ προσκειμένου διπλάσιός ἐστι τοῦ ἀπὸ τοῦ ἡμίσεος μετὰ τοῦ ἀπὸ^c τοῦ συγκεκριμένου ἕκ τε τοῦ ἡμίσεος καὶ τοῦ προσκειμένου, ὅπερ ἔδει.

^a τοῖς corr. e τῶν ^b τοῖς corr. e τῶν ^c τοῦ ἀπὸ marg. m.2

TRANSLATION OF THE *DEMONSTRATIO*

Arithmetical proof, by the monk Barlaam, of what has been proved in a rigorous way in the second <book> of the Elements

Since I realized that the mathematicians use in many instances the theorems of the second <book> of the Elements as if they were arithmetical, whereas they were proven geometrically by the author of the Elements, I deemed it fit to set out an arithmetical proof of them by translating them into numbers. Granted, it is possible that each of them be proven by induction, for every arithmetical problem can also be proven by induction once we set out some particular numbers to which the general argument applies. But since this is quite amateurish and in anyone's wheelhouse, I regarded it as mandatory to set out, disregarding the proof by induction, a demonstrative overview of them, thereby proving what is more particular from what is more general, for in this way every object of knowledge will be encompassed by knowledge itself, which is really the most salient feature of science.

Terms

<1> I say that a number multiplies a number whenever, how many units there are in the multiplier, the <number> multiplied, so many times compounded, makes some <number>, which it also measures according to the units in the multiplier.

<2> I call it, namely, the one resulting from these, plane <number>.

<3> I call it square number the one resulting from some <number> multiplying itself.

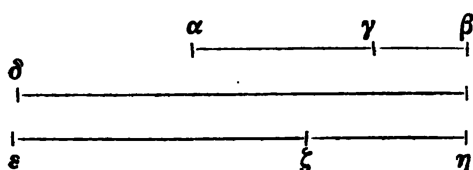
<4> I call a lesser number a part of a greater number, whether it measures the greater or it does not measure it.

1

If a number be divided into two numbers, two plane numbers, namely, those resulting both from the whole and from each of the parts, both <numbers> together, are equal to the square on the whole.

In fact, let a number, AB, be divided into two numbers AΓ, ΓB. I claim that two plane numbers, both that from AB, AΓ and that from AB, BΓ, once compounded, are equal to the square on AB.

In fact, let AB multiplying itself make Δ, and let AΓ multiplying AB make EZ, and let ΓB too multiplying the same AB also ZH.



Now then, since AΓ multiplying AB made EZ, therefore AB measures EZ according to the units in AΓ. Again, since ΓB multiplying AB made ZH, therefore AB measures ZH according to the units in ΓB; and it also measured EZ according to the units in AΓ; therefore AB measures EH as a whole according to the units in itself. Again, since AB multiplying itself made Δ, therefore it also measures Δ according to the units in itself; therefore AB measures each of Δ, EH according to the units in itself; therefore whichever multiple is Δ of AB, such a multiple is also EH of AB; and the numbers equimultiple of a same number are equal to one another; therefore Δ is equal to EH; and Δ is the square on AB, and EH is the <number> compounded of two plane numbers, AB BΓ, BA AΓ; therefore the square on AB is equal to the <number> compounded of two plane <numbers>, AB BΓ, BA AΓ.

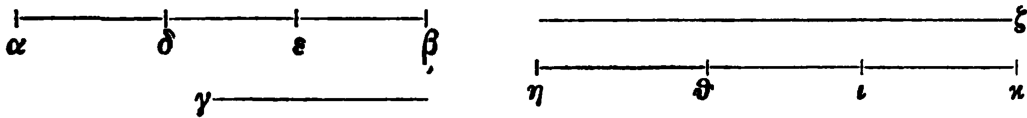
Therefore if a number be divided into two numbers, the square on the whole is equal to two plane numbers, namely, those resulting from the whole and from each of the parts, which it was really required.

2

If, there being two numbers, one of them be divided into as many numbers as we please, the plane number from the original two numbers is equal to the plane <numbers> resulting from both the undivided <number> and from each of the parts of the divided one.

Let there be two numbers, AB, Γ , and let AB be divided into as many numbers as we please, A Δ , ΔE , EB. I claim that the plane <number> from Γ , AB is equal to the plane <numbers> from Γ , A Δ , Γ , ΔE , Γ , EB.

In fact, let Z be the <number> from Γ , AB, and H Θ that from Γ , A Δ , and ΘI that from Γ , ΔE , and IK that from Γ , EB.



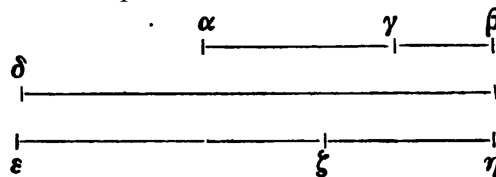
And since AB multiplying Γ made Z, therefore Γ measures Z according to the units in AB. For the very same <reasons> it also measures H Θ according to the units in A Δ , and ΘI according to those in ΔE , and IK according to those in EB; therefore Γ measures HK as a whole according to the units in AB; and it also measured Z according to the units in AB; therefore each of Z, HK is equimultiple of Γ ; and the <numbers> equimultiple of a same <number> are equal to one another; therefore Z is equal to HK; and Z is the plane <number> from Γ , AB, and HK the one compounded both of Γ and of each of the plane <numbers> A Δ , ΔE , EB; therefore the plane <number> from Γ , AB is equal to the plane <numbers> both from Γ and from each of A Δ , ΔE , EB.

Therefore if, there being two numbers, one of them be divided into as many numbers as we please, the plane <number> from the original two numbers is equal to the plane <numbers> resulting both from the undivided <number> and from each of the parts of the divided one, which it was really required.

3

If a number be divided into two numbers, the plane <number> from the whole and one of the parts is equal to the plane <number> from the parts plus the square on the part said above.

In fact, let a number, AB, be divided into two numbers, A Γ , ΓB . I claim that the plane <number> from AB, B Γ is equal both to the plane <number> from A Γ , ΓB and to the square on ΓB .



In fact, let AB multiply ΓB and let it make Δ , and let A Γ multiply ΓB and let it make EZ, and let ΓB multiplying itself make ZH.

And since AB multiplying ΓB made Δ , therefore ΓB measures Δ according to the units in AB. Again, since A Γ multiplying ΓB made EZ, therefore ΓB measures EZ according to the units in A Γ . Again, since ΓB multiplying itself made ZH, therefore ΓB measures ZH according to the units in itself; and it also measured EZ according to the units in A Γ ; therefore ΓB measures EH as a whole according to the units in AB; and it also measured Δ according to the units in AB; therefore ΓB equally measures each of Δ , EH; and the <numbers> equally measured by a same <number> are equal to one another; therefore Δ is equal to EH; and Δ is the plane <number> from AB, B Γ , and

EH the plane <number> from $A\Gamma$, ΓB plus the square on ΓB ; therefore the plane <number> from AB , $B\Gamma$ is equal both to the plane <number> from $A\Gamma$, ΓB and to the square on ΓB .

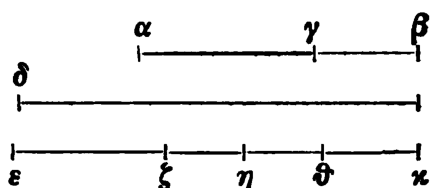
Therefore if a number be divided into two random numbers, the plane <number> from the whole and one of the parts is equal both to the plane <number> from the parts and to the square on the part said above, which it was really required.

4

If a number be divided into two numbers, the square on the whole is equal to the squares on the parts and to twice the plane <number> from the parts.

In fact, let a number, AB , be divided into two numbers, $A\Gamma$, ΓB . I claim that the square on AB is equal both to the squares on $A\Gamma$, ΓB and to twice the plane <number> from $A\Gamma$, ΓB .

In fact, let Δ be the square on AB , and EZ that on $A\Gamma$, and $H\Theta$ that on ΓB , and each of ZH , ΘK the <number> from $A\Gamma$, ΓB .



Now then, since $A\Gamma$ multiplying itself made EZ , therefore $A\Gamma$ measures EZ according to the units in itself. Again, since ΓB multiplying $A\Gamma$ made ZH , therefore $A\Gamma$ measures ZH according to the units in ΓB ; and it also measured EZ according to the units in itself; therefore $A\Gamma$ measures EH as a whole according to the units in AB ; therefore AB multiplying $A\Gamma$ made EH ; therefore EH is the plane <number> from BA , $A\Gamma$. Very similarly we shall prove that HK is the plane <number> from AB , $B\Gamma$; and Δ is the square on AB ; and if a number be divided into two numbers, the square on the whole is equal to the two plane <numbers> from the whole and each of the parts (2); therefore Δ is equal to EK ; but of course EK is compounded both of the squares on $A\Gamma$, ΓB and of twice the plane <number> from $A\Gamma$, ΓB ; and Δ is really the square on AB ; therefore the square on AB is equal both to the squares on $A\Gamma$, ΓB and to twice the plane <number> from $A\Gamma$, ΓB .

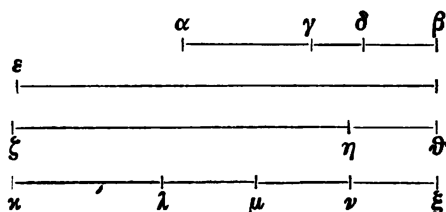
Therefore if a number be divided into two numbers, and what follows, which it was really required.

5

If an even number be divided into two equals and be also divided into unequal numbers, the plane <number> from the unequal parts with the square on what is in between is equal to the square on the half.

In fact, let there be an even number, AB , and let it be divided into two equals, $A\Gamma$, ΓB , and unequally into $A\Delta$, ΔB . I claim that the square on ΓB is equal to the plane <number> from $A\Delta$, ΔB with the square on $\Gamma\Delta$.

In fact, let E be the square on ΓB , and ZH the plane <number> from $A\Delta$, ΔB , and $H\Theta$ the square on $\Gamma\Delta$.



And since number $B\Gamma$ turns out to be divided into $B\Delta$, $\Gamma\Delta$, therefore the square on $B\Gamma$, that is E , is equal to the squares on $B\Delta$, $\Gamma\Delta$ with twice the <number> from $B\Delta$, $\Gamma\Delta$ (4). Then let $K\Lambda$ be the

square on $B\Delta$, and $N\Xi$ the <square> on $\Gamma\Delta$, and each of ΛM , MN that from $B\Delta$, $\Gamma\Delta$; therefore $K\Xi$ as a whole is equal to E . And since $B\Delta$ multiplying itself made $K\Lambda$, therefore it measures it according to the units in itself. Again, since $\Gamma\Delta$ multiplying ΔB made ΛM , therefore ΔB measures ΛM according to the units in $\Gamma\Delta$; and it also measured $K\Lambda$ according to the units in itself; therefore ΔB measures KM as a whole according to the units in ΓB ; and ΓB is equal to ΓA ; therefore ΔB measures KM according to the units in $A\Gamma$. Again, since $\Gamma\Delta$ multiplying ΔB made MN , therefore ΔB measures MN according to the units in $\Gamma\Delta$; and it also measured KM according to the units in $A\Gamma$; therefore ΔB measures KN as a whole according to the units in $A\Delta$; and $B\Delta$ also measures ZH according to the units in $A\Delta$ —for this has been supposed—; therefore ZH is equal to KN —for the <numbers> equimultiple of a same <number> are equal to one another —; and $H\Theta$ is also equal to $N\Xi$ —for each of them has been supposed the square on $\Gamma\Delta$ —; therefore $K\Xi$ as whole is equal to $Z\Theta$; and $K\Xi$ is also equal to E ; therefore $Z\Theta$ is also equal to E ; and $Z\Theta$ is the plane <number> from $A\Delta$, ΔB with the square on $\Gamma\Delta$, and E the square on ΓB ; therefore the plane <number> from $A\Delta$, ΔB with the square on $\Gamma\Delta$ is equal to the square on ΓB .

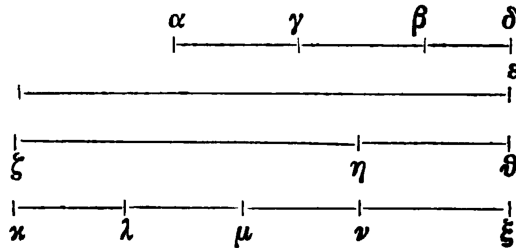
Therefore if an even number be divided into two equals, and what follows, which really.

6

If an even number be divided into two equals and some <number> be added to it, the plane <number> from the whole plus the added one and the added one with the square on the half is equal to the square on the <number> compounded both of the half and of the added one.

In fact, let an even number, AB , be divided into two equals into numbers, $A\Gamma$, ΓB , and some other number be added to it, $B\Delta$. I claim that the plane <number> from $A\Delta$, ΔB with the square on ΓB is equal to the square on $\Gamma\Delta$.

In fact, let E be the square on $\Gamma\Delta$, and ZH the plane <number> from $A\Delta$, ΔB , and $H\Theta$ the square on ΓB .



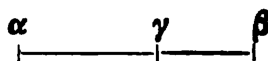
And since that on $\Gamma\Delta$ is equal to those on ΔB , $B\Gamma$ with twice that from ΔB , $B\Gamma$ (4), let $K\Lambda$ be the <square> on $B\Delta$, and each of ΛM , MN the <number> from ΔB , $B\Gamma$, and $N\Xi$ the <square> on $B\Gamma$; therefore $K\Xi$ as a whole is equal to the square on $\Gamma\Delta$; and E is the square on $\Gamma\Delta$; therefore $K\Xi$ is equal to E . And since $B\Delta$ multiplying itself turns out to make $K\Lambda$, therefore $B\Delta$ measures $K\Lambda$ according to the units in itself; and it also measures ΛM according to the units in $B\Gamma$; therefore ΔB measures KM as a whole according to the units in $\Gamma\Delta$. And since ΔB also measures MN according to the units in ΓB and ΓB is equal to ΓA —for it has been supposed—, therefore ΔB measures KN as a whole according to the units in $A\Delta$; but of course ΔB also measures ZH according to the units in $A\Delta$ —for ZH has been supposed that from $A\Delta$, ΔB —; therefore ZH is equal to KN ; and ΘH is also equal to $N\Xi$ —for each of them is the square on $B\Gamma$ —; therefore $Z\Theta$ as a whole is equal to $K\Xi$; and $K\Xi$ was proved equal to E ; therefore $Z\Theta$ is also equal to E ; and $Z\Theta$ is that from $A\Delta$, ΔB with that on ΓB , and E that on $\Gamma\Delta$; therefore that from $A\Delta$, ΔB with that on ΓB is equal to that on $\Gamma\Delta$.

Therefore if an even number be divided into two equals and some <number> be added to it, the plane <number> from the whole plus the added one and the added one with the square on the half is equal to the square on the <number> compounded both of the half and of the added one, which really.

7

If a number be divided into two numbers, the square on the whole with the square on one of the parts is equal to twice the plane <number> from the whole and the said part with the square on the remaining part.

In fact, let a number, AB, be divided into numbers, AΓ, ΓB. I claim that the squares on BA, AΓ are equal to twice the plane <number> from BA, AΓ with the square on BΓ.



In fact, since the square on AB is equal to those on BΓ, ΓA and twice that from BΓ, ΓA (4), let the square on AΓ be added as common; therefore that on BA with that on AΓ is equal to two squares on AΓ and to one on ΓB with twice the <number> from BΓ, ΓA. And since once that from BA, AΓ is equal to once that from BΓ, ΓA with the square on ΓA (3), therefore twice that from BA, AΓ is equal to twice that from BΓ, ΓA with two squares on ΓA; let the square on BΓ be added as common; therefore two squares on AΓ and one on ΓB with twice that from BΓ, ΓA are equal to twice that from BA, AΓ with that on ΓB; therefore the square on AB with that on AΓ is equal to twice that from BA, AΓ with the square on ΓB.

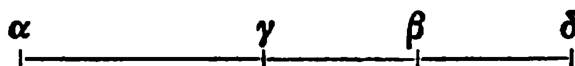
Therefore if a a number be divided into two numbers, the square on the whole with the square on one of the parts is equal to twice the plane <number> from the whole and the said part with the square on the remaining part, which it was really required.

8

If a number be divided into two numbers, four times the plane <number> from the whole and one of the parts with the square on the remaining part is equal to the square on the whole and the part said above as on one.

In fact, let a number, AB, be divided into two numbers, AΓ, ΓB. I claim that four times the <number> from AB, BΓ with the square on AΓ is equal to the square on AB, BΓ as on one.

In fact, let a <number> BΔ be set equal to number BΓ.



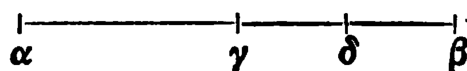
And since that on AΔ is equal to the squares on AB, BΔ and to twice the plane <number> from AB, BΔ (4), and BΔ is equal to BΓ, therefore the square on AΔ is equal to the squares on AB, BΓ and to twice the plane <number> from AB, BΓ; and the squares on AB, BΓ are equal to twice the plane <number> from AB, BΓ and to the square on AΓ (7); therefore the square on AΔ is equal to four times the plane <number> from AB, BΓ and to the square on AΓ; and the square on AΔ is the square on AB, BΓ as on one—for BΔ is equal to BΓ—; therefore the square on AB, BΓ as on one is equal to four times that from AB, BΓ and to that on AΓ.

Therefore if a number be divided into two numbers, and what follows, which really.

9

If an even number be divided into two equals and be further also divided into unequal numbers, the squares on the unequal numbers are double of the square on the half with the square on that in between.

In fact, let an even number, AB, be divided into two equals into numbers, AΓ, ΓB, and let it be divided into unequal <numbers>, AΔ, ΔB. I claim that the squares on AΔ, ΔB are double of the squares on AΓ, ΓΔ.



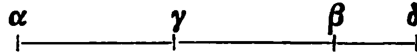
In fact, since an even number, AB, turns out to be divided into equal <numbers>, AΓ, ΓB, and into unequal <numbers>, AΔ, ΔB, therefore the <number> from AΔ, ΔB with that on ΓΔ is equal to the square on AΓ (5); therefore twice that from AΔ, ΔB with two squares on ΓΔ are double of the square on AΓ. And since AB turns out to be divided into two equals into AΓ, ΓB, therefore the square on AB is quadruple of the square on AΓ (4). And since twice that from AΔ, ΔB with two on ΓΔ are double of that on ΓA, and if there be two numbers and one of them be quadruple and the other be double of a same <number>, the quadruple is double of the double, therefore that on AB is double of twice that from AΔ, ΔB with two on ΓΔ; therefore twice that from AΔ, ΔB is less than that on AB by twice that on ΓΔ. And since twice that from AΔ, ΔB with the <number> compounded of those on AΔ, ΔB is equal to that on AB (4), therefore the <number> compounded of those on AΔ, ΔB is greater than half of that on AB by twice that on ΓΔ; and that on AB is quadruple of that on AΓ; therefore the <number> compounded of those on AΔ, ΔB is greater than double of that on AΓ by twice that on ΓΔ; therefore it is double of those on AΓ, ΓΔ.

Therefore if an even number, and what follows, which it was really required to prove.

10

If an even number be divided into two equals and some other number is added to it, the one on the whole plus the added one and that on the added one, the two squares together, are double of the square on the half and of the square on the <number> compounded both of the half and of the added one as on one.

In fact, let there be an even number, AB, and let it be divided into two equals, AΓ, ΓB, and let some other number, BΔ, be added to it. I claim that the squares on AΔ, ΔB are double of the squares on AΓ, ΓΔ.



In fact, since a number, AΔ, turns out to be divided into <numbers>, AB, BΔ, therefore the squares on AΔ, ΔB are equal to twice the plane <number> from AΔ, ΔB with the square on AB (7); and the square on AB is equal to four squares on AΓ, ΓB (4)—for AΓ is equal to ΓB—; therefore the squares on AΔ, ΔB are equal both to twice the <number> from AΔ, ΔB and to four squares on BΓ, ΓA. And since that from AΔ, ΔB with that on ΓB is equal to that on ΓΔ (6), therefore twice that from AΔ, ΔB with two on ΓB are equal to two on ΓΔ; therefore the squares on AΔ, ΔB are equal to two on ΓΔ and to two on AΓ; therefore they are double of those on AΓ, ΓΔ; and the square on AΔ is that on the whole and the added one, and that on ΔB is that on the added one, and that on AΓ is that on the half, and that on ΓΔ is that on the <number> compounded both of the half and of the added one; therefore the square on the whole plus the added one with that on the added one are double of that on the half with that on the <number> compounded both of the half and of the added one, which it was really required.

APPENDIX 1

The following critical apparatus of Pletho's geometric problem updates the one in ACERBI – MARTINELLI TEMPESTA – VITRAC, Gli interventi autografi 438 with a collation of Krems. 343 (K) and of Par. gr. 2384 (P²). As is clear, these witnesses belong to the family of Marc. gr. Z. 302 (N); they also exhibit several conjunctive readings.

1 ἢ τούτου — ἀραβική ἐστὶν atramento rubro habent MVB : om. buKNP²S 4 τρίγωνον] compendium S 5 εὐθεῖαν] om. uKNP²S 6 ἔτυχε] ἔτυχε uKNP²S | οὐδετέρας — 7 ἀπολαμβάνουσα] om. uKNP²S | διὰ τὸ λα^{ov} τοῦ α^{ov} τῶν στοιχείων marg. KP² 8 I] I^a bBV 9 ὀρθὰς] ὀρθὴν P² | διὰ τὸ α^{ov} τοῦ αὐτοῦ marg. KP² 10 ΔΖΚ] ΔΖΗ S | ΔΗ ἄρα KP² 12 τε om. KP² 13 οὕτως ὥστε καὶ] post καὶ inc. spatium 9×2/3 uersuum ubi figuram delineauit m. 1 M : post οὐ- inc. spatium 10×3/5 uersuum ubi figuram delineauit m. 1 B : post ὧσ- inc. spatium 10×3/5 uersuum ubi figuram delineauit m. 1 V | τὸ¹] om. b | post Δ inc. spatium 8×3/5 uersuum ubi figuram delineauit m. 1 b 15 περιηγέσθω] περιαγαγέσθω bBV | καὶ om. uKNP²S | ΔΗ] ΘΗ

uKNP²S 16 ΘΓΙ] ΘΓ bBV | post εὐθεΐαν scr. καὶ τῆ εὐθεΐαν P² sed del. m.2 18 ΔΗ] ΘΗ uKNP²S | ΖΚ] ΖΗ b | ΘΓΙ] ΘΓ bBV sed corr. marg. m. 2 V | ΝΓΞ] ΝΓΖ b 19 ἢ δὲ ΝΓΞ τῆ ΛΜ marg. P² 20 καὶ om. KP² 21 Ν om. buKNP² : Γ S 22 ἔστιν ἄρα] καὶ ἔστιν uKNP²S | δέ] δὴ uKP² | ἢ ΝΔ πρὸς] in rasura M 23 πρὸς τὴν ΔΕ om. b | καὶ ἔστιν] ἔστι δὲ uKNP²S 26 post textum figuram posuerunt NS et sine litteris u : in marg. KP²

APPENDIX 2

This Appendix contains an edition and a translation of the *varia mathematica* in Ambr. E 76 sup., ff. 108r–110v. As said, this sequence of texts comprises an iterative procedure for computing an approximate square root that reformulates the construction of *Logistikē* II 39; six procedures for removing a ratio from a ratio, five of which refer to *Logistikē* V 21, 23, 22, 20, 23, respectively, and the sixth to Theon's commentary on the *Almagest*; a series of geometric proofs by Nicholas Kabasilas, followed by two simple proofs on inequality of ratios possibly to be ascribed to him. It seems clear that these texts were conceived as a supplement to the *Logistikē*. The apparent aim of the first two sets is reformulating some of Barlaam's constructions and proofs as procedures. Kabasilas' geometric proofs appear to have *Logistikē* VI 16–21 as their polemical target; they were certainly inspired by his painstaking study of the *Almagest*, for they treat specific solutions of triangles, a mathematical technique frequently found in Book III of Ptolemy's treatise. On this book Kabasilas wrote a commentary, as Theon's commentary thereon was missing in all manuscripts available in Byzantium¹⁴⁰.

The critical apparatus is inserted in the text between double braces; a commentary is provided after the translation (removal of a ratio from a ratio) or in the notes to it (square root and Kabasilas' arguments). The reader will easily supply the diagrams.

Text

περὶ τετραγωνικῆς πλευρᾶς

προτίθει τὸν ζητούμενον ἀριθμὸν μὴ ὄντα τετράγωνον, οὗ ζητεῖς ἔγγιστα τὴν τετραγωνικὴν πλευρὰν, εἶτα τίθει τὸν ἐγγύς τετράγωνον μείζονα, εἶτα τὴν πλευρὰν αὐτοῦ, εἶτα μέρισον τὸν ζητούμενον παρὰ τὴν τοῦ τετραγώνου πλευρὰν, καὶ ἀπόγραψον τὸν ἀπὸ τοῦ μερισμοῦ, καὶ καλείσθω πρῶτος ἀπὸ μερισμοῦ. ἐνώσας δὲ τὴν πλευρὰν τοῦ τετραγώνου καὶ τὸν ἀπὸ τοῦ μερισμοῦ λάβε τούτων τὸ \surd , καὶ κάλει τοῦτο πρῶτον \surd , καὶ πολλαπλασίασον τοῦτο πρὸς ἑαυτό, καὶ τὸ ἀποτελούμενον πρῶτον ὠνόμασον. τοῦτο δὴ μείζον μὲν ἔσται ὁμολογουμένως τοῦ ζητουμένου ἔλαττον δὲ τοῦ τετραγώνου.

πάλιν δὴ ὡς ἀπ' ἄλλης ἀρχῆς, μέρισον τὸν ζητούμενον ἀριθμὸν παρὰ τὸ ληφθὲν πρῶτον ἥμισυ ἕκ τε τῆς τοῦ τετραγώνου πλευρᾶς καὶ τοῦ ἀπὸ τοῦ α^{ov} μερισμοῦ, καὶ τὸ ἀπὸ τοῦ μερισμοῦ σημείωσον. εἶτα ἔνωσον τοῦτο μετὰ τοῦ ὀλίγου πρότερον ἡμίσεος ῥηθέντος, καὶ λάβε τούτων τὸ \surd , ὃ καὶ δεύτερον ἥμισυ κάλει, καὶ πολλαπλασίασον τοῦτο πρὸς ἑαυτό, καὶ τὸ ἀποτελούμενον δεύτερον ὠνόμασον. τοῦτο δὴ μείζον μὲν ἔσται ὁμολογουμένως τοῦ ζητουμένου ἔλαττον δὲ τοῦ α^{ov} .

πάλιν δὴ ὡς ἀπ' ἄλλης ἀρχῆς, μέρισον τὸ ζητούμενον παρὰ τὸ β^{ov} ἥμισυ, καὶ τὸ ἀπὸ τοῦ μερισμοῦ γινόμενον τάξον. εἶτα ἔνωσον τοῦτο μετὰ τοῦ β^{ov} ἡμίσεος καὶ τὸ ἀποτελεσθὲν προτίθει, καὶ τούτου λαβὼν τὸ \surd , ὃ καὶ τρίτον \surd ὀνομάζεται, πολλαπλασίασον ἐφ' ἑαυτό, καὶ τὸ ἀποτελούμενον μείζον μὲν ἔσται ὁμολογουμένως τοῦ ζητουμένου ἔλαττον δὲ τοῦ β^{ov} , ὃ καὶ τρίτον κάλει.

καὶ τοῦτο ποιεῖ ἕως ἂν ἴδῃς τὸ τελευταίως ἀποτελούμενον ἔγγιστα ἴσα $\{\{lege\} \text{ ἴσον}\}$ ὃν τῷ ζητουμένῳ, καὶ τὴν τούτου πλευρὰν λέγε τὴν αὐτὴν ἔγγιστα εἶναι καὶ τοῦ ζητουμένου. τὴν δὲ τούτου ἀπόδειξιν ζητεῖ ἐν τῇ λογιστικῇ.

marg. Βαρλαάμ α^{ov}

εἰ βούλει λόγον ἐκ λόγου ἀφελεῖν ἔχοντα ὑπόλογον τὸν τοῦ ἡγουμένου λόγου πρόλογον ὥστε τὸν καταλειπόμενον καταλείπεσθαι πρὸς ὄρω δεδομένῳ προλόγῳ ὄντι, ὃς καὶ πρόλογος ἔσται τοῦ ἡγουμένου, τούτων

¹⁴⁰ On Kabasilas' scientific output, see F. ACERBI – I. PÉREZ MARTÍN, Les études géométriques et astronomiques à Thessalonique d'après le témoignage des manuscrits : de Jean Pédiasimos à Démétrios Kydônès. *Byz* 89 (2019) 1–35: 13–17.

υποκειμένων ποίει οὕτως. ὡς τὸν πρόλογον τοῦ ἡγουμένου πρὸς τὸν πρόλογον τοῦ ἐπομένου οὕτω τὸν ὑπόλογον τοῦ ἡγουμένου πρὸς ἄλλον τινά, οἷον τὸν ν, καὶ οὕτως *{{lege οὕτως}}* ἂν εἶη ὑπόλογος |_{108v} τοῦ καταλειπομένου, ὃς καὶ δέδοται διὰ τῆς τοιαύτης μεθόδου. τὴν δὲ τοῦ τοιούτου ἀπόδειξιν ζητεῖ ἐν τῷ κ^α τῆς ε^ο βιβλίου τῆς λογιστικῆς.

marg. β^ο Θέωνος

εἰ βούλει λόγον ἐκ λόγου ἀφελεῖν ἐν ἰδίῳ ὄροις ὥστε τὸν καταλειπόμενον καταλείπεσθαι πρὸς ὄρω δεδομένῳ ὑπολόγῳ ὄντι, ὃς οὐδενὶ τῶν ἐξ ἀρχῆς ὁ αὐτὸς ἔσται, ποίει οὕτως. ὡς τὸν ὑπόλογον τοῦ ἡγουμένου πρὸς τὸν πρόλογον αὐτοῦ οὕτω τὸν ὑπόλογον τοῦ ἐπομένου πρὸς ἄλλον τινά, οἷον τὸν ν, ὃν μείζον τάξας τὸν πρόλογον τοῦ ἐπομένου ποίει πάλιν ὡς τὸν πρόλογον τοῦ ἐπομένου πρὸς τὸν ν οὕτω τὸν δεδομένον ὑπόλογον τοῦ λοιποῦ πρὸς ἄλλον τινά, καὶ οὕτως ἂν εἶη ὁ ζητούμενος, ὃς καὶ πρόλογός ἐστι τοῦ λοιποῦ δοθεὶς διὰ τῆς τοιαύτης μεθόδου.

marg. Βαρλαάμ εἰς τὸ αὐτό

εἰ βούλει λόγον ἐκ λόγου ἀφελεῖν ἐν ἰδίῳ ὄροις ὥστε τὸν καταλειπόμενον καταλείπεσθαι πρὸς ὄρω δεδομένῳ ὑπολόγῳ ὄντι, ὃς οὐδενὶ τῶν ἐξ ἀρχῆς ὁ αὐτὸς ἔσται, ποίει οὕτως. πρῶτον μὲν ὡς τὸν πρόλογον τοῦ ἡγουμένου πρὸς τὸν ὑπόλογον αὐτοῦ οὕτω τὸν πρόλογον τοῦ ἐπομένου πρὸς ἄλλον τινά, οἷον τυχὸν τὸν ν. πάλιν δὲ ὡς αὐτὸν τὸν ν πρὸς τὸν ὑπόλογον τοῦ ἐπομένου οὕτως αὐτὸν τὸν δεδομένον ὑπόλογον τοῦ λοιποῦ πρὸς ἄλλον τινά, καὶ οὕτως *{{corr. ex -ως m. I}}* ἂν εἶη ὁ ζητούμενος πρόλογος τοῦ λοιποῦ, ὃς καὶ δέδοται διὰ τῆς τοιαύτης μεθόδου. τὴν δὲ τούτου ἀπόδειξιν ζητεῖ ἐν τῷ τελευταίῳ θεωρήματι τοῦ ε^ο βιβλίου τῆς λογιστικῆς.

marg. Βαρλαάμ γ^ο

εἰ βούλει λόγον ἐκ λόγου ἀφελεῖν ἔχοντα πρόλογον \τὸν/ τοῦ ἡγουμένου λόγου ὑπόλογον ὥστε τὸν καταλειπόμενον πρὸς ὄρω καταλείπεσθαι δεδομένῳ ὑπολόγῳ ὄντι, ὃς ὁ αὐτὸς ἔσται τῷ ὑπολόγῳ τοῦ ἡγουμένου, τούτων ὑποκειμένων ποίει οὕτως. ὡς τὸν πρόλογον τοῦ ἐπομένου πρὸς τὸν ὑπόλογον αὐτοῦ οὕτω τὸν πρόλογον τοῦ ἡγουμένου πρὸς ἄλλον τινά, οἷον τυχὸν τὸν ν, καὶ οὕτως ἂν εἶη ὁ ζητούμενος πρόλογος τοῦ λοιποῦ, δοθεὶς διὰ τῆς τοιαύτης μεθόδου. τὴν δὲ τοῦ τοιούτου ἀπόδειξιν ζητεῖ ἐν τῷ κ^β τοῦ πέμπτου βιβλίου τῆς λογιστικῆς.

marg. Βαρλαάμ δ^ο

εἰ βούλει λόγον ἐκ λόγου ἀφελεῖν ἔχοντα τὸν αὐτὸν ὑπόλογον ὥστε τὸν καταλειπόμενον καταλείπεσθαι πρὸς ὄρω δεδομένῳ ὑπολόγῳ ὄντι, ὃς ὁ αὐτὸς ἔσται τῷ κοινῷ ὑπολόγῳ τοῦ ἡγουμένου καὶ ἐπομένου, ποίει οὕτως. ὡς τὸν πρόλογον τοῦ ἐπομένου πρὸς τὸν πρόλογον τοῦ ἡγουμένου οὕτως αὐτὸν τὸν δεδομένον ὑπόλογον τοῦ λοιποῦ πρὸς ἄλλον τινά, οἷον τυχὸν τὸν ν, καὶ οὕτως ἂν εἶη ὁ ζητούμενος πρόλογος τοῦ λοιποῦ, ὃς καὶ δέδοται διὰ τῆς τοιαύτης μεθόδου. τὴν δὲ τοῦ τοιούτου ἀπόδειξιν ζητεῖ ἐν τῷ κ^ο τοῦ ε^ο τῆς λογιστικῆς.

marg. Βαρλαάμ ε^ο

εἰ βούλει λόγον ἐκ λόγου ἀφελεῖν ἐν ἰδίῳ ὄροις ὥστε τὸν καταλειπόμενον καταλείπεσθαι πρὸς ὄρω δεδομένῳ ὑπολόγῳ ὄντι, ὃς ὁ αὐτὸς ἔσται τῷ ὑπολόγῳ τοῦ ἡγουμένου, ποίει οὕτως. ὡς τὸν πρόλογον τοῦ ἐπομένου πρὸς τὸν ὑπόλογον αὐτοῦ οὕτω τὸν τοῦ ἡγουμένου πρόλογον πρὸς ἄλλον τινά, οἷον τυχὸν τὸν ν, καὶ οὕτως ἂν εἶη ὁ ζητούμενος πρόλογος τοῦ λοιποῦ, ὃς καὶ δέδοται διὰ τῆς τοιαύτης μεθόδου. τὴν δὲ τούτου ἀπόδειξιν ζητεῖ ἐν τῷ τελευταίῳ τοῦ ε^ο τῆς λογιστικῆς.

marg. τοῦ Καβάσιλα κυροῦ Νικολαίου

ἔστω κύκλος ὁ ΑΒΓ περὶ κέντρον τὸ Ε, καὶ πρὸς αὐτῷ βεβηκέτω γωνία ἐπὶ τῆς ΑΒ περιφερείας ἢ ὑπὸ ΑΕΒ. λέγω ὅτι ὡς ἡ ΑΒ περιφέρεια πρὸς ὅλην τὴν τοῦ κύκλου περιφέρειαν οὕτως ἡ ὑπὸ ΑΕΒ γωνία πρὸς δ ὀρθάς, διήχθω ἡ ΑΕ ἐπὶ τὸ Γ. ἔστιν ἄρα ὡς ἡ ΓΒ περιφέρεια πρὸς τὴν ΒΑ περιφέρειαν οὕτως ἡ ὑπὸ ΓΕΒ γωνία πρὸς τὴν ὑπὸ ΒΕΑ· καὶ συνθέντι ὡς τὸ ΓΒΑ ἡμικύκλιον πρὸς τὴν ΑΒ περιφέρειαν οὕτως αἱ ὑπὸ ΓΕΒ ΒΕΑ γωνία, τούτεστιν αἱ β ὀρθαί, πρὸς τὴν ὑπὸ ΑΕΒ· καὶ ἀνάπαλιν ὡς ἡ ΑΒ περιφέρεια πρὸς τὸ ἡμικύκλιον οὕτως ἡ ὑπὸ ΑΕΒ γωνία πρὸς δύο ὀρθάς· ἀλλὰ μὴν καὶ ὡς τὸ ἡμικύκλιον πρὸς ὅλην τοῦ κύκλου περιφέρειαν ὡς *{{lege οὕτως}}* αἱ δύο ὀρθαί πρὸς δ ὀρθάς· διΐσου ἄρα ὡς ἡ ΑΒ περιφέρεια πρὸς τὴν ὅλην περιφέρειαν τοῦ κύκλου οὕτως ἡ ὑπὸ ΑΕΒ γωνία πρὸς δ ὀρθάς.

|109v marg. τοῦ αὐτοῦ

πάλιν ἐπεξεύχθω ἡ ΒΓ. λέγω ὅτι ὡς ἡ ΑΒ περιφέρεια πρὸς τὴν ὅλην περιφέρειαν τοῦ κύκλου οὕτως ἡ ὑπὸ ΑΓΒ γωνία πρὸς δύο ὀρθάς. ἐπεὶ γάρ ἐστιν ἡ ὑπὸ ΑΕΒ γωνία διπλῆ τῆς ὑπὸ ΑΓΒ, ἔστιν ἄρα ὡς ἡ ὑπὸ ΑΕΒ πρὸς τὴν ὑπὸ ΑΓΒ οὕτως αἱ δ' ὀρθαὶ πρὸς δύο ὀρθάς· ἐναλλάξ ἄρα ὡς ἡ ὑπὸ ΑΕΒ πρὸς δ' ὀρθάς ἢ ὑπὸ ΑΓΒ πρὸς δύο ὀρθάς· ἀλλ' ὡς ἡ ὑπὸ ΑΕΒ πρὸς δ' ὀρθάς οὕτως ἦν ἡ ΑΒ περιφέρεια πρὸς ὅλην τὴν τοῦ κύκλου περιφέρειαν· καὶ ὡς ἄρα ἡ ΑΒ περιφέρεια πρὸς ὅλην τὴν τοῦ κύκλου περιφέρειαν οὕτως ἡ ὑπὸ ΑΓΒ γωνία πρὸς δύο ὀρθάς, ὅπερ ἔδει δεῖξαι.

marg. τοῦ αὐτοῦ

τούτων προποκειμένων δεῖξομεν ὅτι ἐὰν ὀρθογωνίου τριγώνου δοθῆ ὀποτέρα τῶν ὀξείων γωνιῶν, δοθήσονται καὶ αἱ πλευραί. ὀρθογωνίου γὰρ τριγώνου τοῦ ΑΒΓ ὀρθὴν ἔχοντος τὴν πρὸς τῷ Β γωνίαν δεδόςθω ἢ πρὸς τῷ Α. λέγω ὅτι δοθήσονται καὶ αἱ πλευραί. γεγράφθω γὰρ [[περι]] ἐπὶ τῆς ΑΓ διαμέτρου ἡμικύκλιον τὸ ΑΒΓ, καὶ κέντρου ὄντος τοῦ Δ ἐπεξεύχθω ἡ ΒΔ. ἐπεὶ οὖν δέδοται ἡ ὑπὸ ΒΑΓ γωνία, διπλῆ δὲ αὐτῆς ἢ ὑπὸ ΒΔΓ, καὶ αὕτη ἄρα δέδοται· καὶ ἔστιν ὡς ἡ ὑπὸ ΒΔΓ πρὸς δ' ὀρθάς οὕτως ἢ ΒΓ περιφέρεια πρὸς ὅλην τὴν τοῦ κύκλου περιφέρειαν· δέδοται ἄρα καὶ ἡ ΒΓ περιφέρεια· καὶ λοιπὴ ἄρα εἰς τὸ ἡμικύκλιον ἢ ΑΒ δέδοται· ὥστε καὶ αἱ ὑπ' αὐτὰς εὐθεῖαι δεδομένα εἰσιν, ὥστε εἶναι τοιούτων τινῶν οἶων ἢ ΑΓ ρκ· δέδονται ἄρα αἱ πλευραί, ὅπερ ἔδει δεῖξαι.

λέγω δὴ ὅτι κὰν ὁ τῶν πλευρῶν λόγος δοθῆ, δοθήσονται καὶ αἱ πλευραὶ καὶ ἑκατέρα τῶν ὀξείων γωνιῶν. ἐπὶ γὰρ τῆς αὐτῆς καταγραφῆς πολλαπλασιάσαντες εἰς ἑαυτοὺς τὸν τε τῆς ΑΒ ἀριθμὸν καὶ τὸν τῆς ΒΓ καὶ ἀμφοτέρους συνθέντες καὶ τούτου συναμφοτέρου τὴν τετραγωνικὴν πλευρὰν λαβόντες καὶ τῆς |110r ΑΓ εἶναι τὸν ἀριθμὸν νομίσαντες ποιήσομεν ὡς μὲν τοῦτον τὸν ἀριθμὸν πρὸς τὸν ἀριθμὸν τῆς ΑΒ οὕτω τὸν ρκ πρὸς ἄλλον τινά, ὃν ἐπὶ τῆς ΑΒ δηλονότι τάξομεν. τὸ αὐτὸ δὲ καὶ ἐπὶ τῆς ΒΓ ποιήσαντες καὶ ἐκ τοῦ τῶν ἐν κύκλῳ εὐθειῶν κανόνος τὰς τῶν ΑΒ ΒΓ περιφερειῶν μοίρας λαβόντες ἔξομεν δηλονότι καὶ τὰς πρὸς τῷ Δ κέντρῳ γωνίας δεδομένας καὶ τὰς αὐτῶν ἡμισείας, τουτέστιν ἑκατέραν τῶν ὑπὸ ΒΑΓ ΒΓΑ, ὅπερ ἔδει δεῖξαι.

ἔστιν εἶναι τὸν λόγον τῶν μεγεθῶν δεδομένον μὴ ὄντων τῶν μεγεθῶν δεδομένων τῆ πρὸς τὰς οἰκείας ὀλότητος ἀναφορᾶ, ὡς ὅταν τις ἀπολαβὼν τμῆμα περιφερείας ἀποδείξῃ ταύτην τοιούτων ξ οἶων ἄλλη τις περιφέρεια ἐν τῷ αὐτῷ κύκλῳ μ, ἀλλ' οὕτω δηλον εἰ οἶων ἐστὶν ὁ ὅλος κύκλος τξ τοιούτων ἐστὶν ἢ μὲν ξ ἢ δὲ μ, ὥστε ὁ μὲν λόγος αὐτῶν δέδοται ἡμίολιος ὢν, πηλίκη δὲ ἐστὶν ἑκατέρα ὡς πρὸς τὴν ὅλην τοῦ κύκλου περιφέρειαν, οὕτω δηλον. ὄντων δὲ τῶν μεγεθῶν δεδομένων μὴ εἶναι τὸν λόγον δεδομένον αὐτῶν ἀδύνατον.

ἔστω ἔχειν τὸ Α πρὸς τὸ Β λόγον ὁποιοῦδήτινα, καὶ μείζονος ὄρου ὑποκειμένου τοῦ Α. λέγω ὅτι ὁ τοῦ Α πρὸς τὸ Β λόγος μείζων ἐστὶν ἢ ὁ τοῦ Β πρὸς τὸ Α. γεγονέτω γὰρ ὡς ὁ Α πρὸς τὸν Β οὕτως ὁ Β πρὸς τὸν Γ· ἀνάπαλιν ἄρα ὡς ὁ Γ πρὸς τὸν Β ὁ Β πρὸς τὸν Α. καὶ ἐπεὶ ὁ Α μείζων ἐστὶ τοῦ Β ὡς δὲ ὁ Α πρὸς τὸν Β ὁ Β πρὸς τὸν Γ, πολλῶ ἄρα ὁ Α μείζων τοῦ Γ. καὶ ἐπεὶ δύο εἰσὶ τὰ Α Β ἄλλο δέ τι τὸ Γ τὸ δὲ μείζον πρὸς τὸ αὐτὸ μείζονα λόγον ἔχει ἢπερ τὸ ἔλαττον, ὁ Α ἄρα πρὸς τὸν Β μείζονα λόγον ἔχει ἢπερ πρὸς αὐτὸ τὸ Β τὸ Γ· ὡς δὲ τὸ Γ πρὸς τὸ Β τὸ Β πρὸς τὸ Α· καὶ τὸ Α ἄρα πρὸς τὸ Β μείζονα λόγον ἔχει ἢπερ τὸ Β πρὸς τὸ Α, ὅπερ ἔδει δεῖξαι.

|110v ἔστω τὸν μὲν Α ἔλαττον τοῦ Β τὸ δὲ Γ μείζον τοῦ Δ. λέγω ὅτι τὸ Α πρὸς τὸ Β ἐλάττονα λόγον ἔχει ἢπερ τὸ Γ πρὸς τὸ Δ. γεγονέτω γὰρ ὡς τὸ Β πρὸς τὸ Α τὸ Δ πρὸς τὸ Ε· μείζον ἄρα καὶ τὸ Δ τοῦ Ε· ἔστι δὲ καὶ τὸ Γ τοῦ Δ μείζον· πολλῶ ἄρα τὸ Γ τοῦ Ε μείζον ἐστὶν· ἄλλο δέ τι τὸ Δ· ἐλάττονα ἄρα λόγον ἔχει τὸ Ε πρὸς τὸ Δ ἢπερ τὸ Γ πρὸς τὸ Δ· ὡς δὲ τὸ Ε πρὸς τὸ Δ τὸ Α πρὸς τὸ Β· τὸ Α ἄρα πρὸς τὸ Β ἐλάττονα λόγον ἔχει ἢπερ τὸ Γ πρὸς τὸ Δ, ὅπερ ἔδει δεῖξαι.

Translation

On the square root¹⁴¹

¹⁴¹ This text transforms the sequence of constructions in Barlaam, *Logistikē* II 39 into an iterative procedure, and alludes (“it is readily agreed, etc.”) to the main result proved in this proposition, namely, that each iteration of the Heronian algorithm for finding an approximate square root of a given number gives a better approximation. Our text also introduces a terminology for the numbers found at the intermediate steps of an assigned iteration. For the Heronian algorithm, see ACERBI – VITRAC,

Set out the sought number that is not a square, of which you seek the approximate square root, next set the greater nearby square, next its square root, next divide the sought <number> by the square root of the square, and write down the <number resulting> from the division, and let it be called “first from the division”. Uniting the square root of the square and the <number> from the division take $\frac{1}{2}$ of these, and call this “first $\frac{1}{2}$ ”, and multiply this by itself, and call what is completed “first”. It is readily agreed that this will be greater than the sought <number> and less than the square.

Then again, as from a fresh start, divide the sought number by the “first half” taken from both the square root of the square and the <number resulting> from the 1st division, and note down what <results> from the division. Next, unite this with the just-mentioned half, and take $\frac{1}{2}$ of these, which also call “second half”, and multiply this by itself, and call what is completed “second”. It is readily agreed that this will be greater than the sought <number> and less than the “1st”.

Then again, as from a fresh start, divide what is sought by the “2nd half”, and arrange what results from the division. Next, unite this with the “2nd half” and set out what has been completed, and taking $\frac{1}{2}$ of this, which is also called “third $\frac{1}{2}$ ”, multiply by itself, and it is agreed that what is completed will be greater than the sought <number> and less than the “2nd”, which also call “third”.

And do this until you see that what is completed last is approximately equal to the sought <number>, and claim that its square root is approximately the same as <that of> the sought <number>. Seek a proof of this in the *Logistikē*.

1st, Barlaam’s

If you wish to remove from a ratio a ratio that has the antecedent of the leading ratio as consequent in such a way that the <ratio> which is left out is left out with respect to a given term serving as the antecedent, which will also be the antecedent of the leading <ratio>, this being supposed, do as follows. <Make,> as the antecedent of the leading <ratio> to the antecedent of the following <ratio>, so the consequent of the leading <ratio> to some other <term>, for instance, 50, and this will also be the consequent of the <ratio> which is left out, which is also given by means of such a procedure. Seek the proof of such a <statement> in the 21st of the 5th book of the *Logistikē*.

2nd, Theon’s¹⁴²

If you wish to remove from a ratio a ratio in specific terms in such a way that the <ratio> which is left out is left out with respect to a given term serving as the consequent, which will not be the same as any of the original <terms>, do as follows. <Make,> as the consequent of the leading <ratio> to its antecedent, so the consequent of the following <ratio> to some other <term>, for instance, 50, arranging the greater of which as the antecedent of the following <ratio> make again, as the antecedent of the following <ratio> to 50, so the given consequent of the remainder to some other <term>, and this will also be the sought <term>, which is also the antecedent of the remainder, given by means of such a procedure.

Barlaam’ to the same

If you wish to remove from a ratio a ratio in specific terms in such a way that the <ratio> which is left out is left out with respect to a given term serving as the consequent, which will not be the same as any of the original <terms>, do as follows. First, <make,> as the antecedent of the leading <ratio> to its consequent, so the antecedent of the following <ratio> to some other <term>, for instance, grant that, 50. Again, <make,> as 50 itself to the consequent of the following <ratio>, so the given consequent itself of the remainder to some other <term>, and this will also be the sought antecedent of the remainder, which is also given by means of such a procedure. Seek the proof of this in the last theorem of the 5th book of the *Logistikē*.

3rd, Barlaam’s

Héron d’Alexandrie 164–165 and Annexes 3 and 5. For the meaning of “procedure” here intended, see ACERBI, *The Logical Syntax*, sect. 1, § 2.

¹⁴² This is the procedure Theon employs at ROME, *Commentaires* II 591, 5–594, 7 (*in Alm.* I 16), 595, 18–596, 4 (*in Alm.* I 16), 622, 5–623, 7 (*in Alm.* II 3).

If you wish to remove from a ratio a ratio that has the consequent of the leading ratio as antecedent in such a way that the <ratio> which is left out is left out with respect to a given term serving as the consequent, which will be the same as the consequent of the leading <ratio>, this being supposed, do as follows. <Make,> as the antecedent of the following <ratio> to its consequent, so the antecedent of the leading <ratio> to some other <term>, for instance, grant that, 50, and this will also be the sought antecedent of the remainder, given by means of such a procedure. Seek the proof of such a <statement> in the 22nd of the fifth book of the *Logistikē*.

4th, Barlaam's

If you wish to remove from a ratio a ratio that has the same consequent in such a way that the <ratio> which is left out is left out with respect to a given term serving as the consequent, which will be the same as the consequent common to the leading and following <ratios>, do as follows. <Make,> as the antecedent of the following <ratio> to the antecedent of the leading <ratio>, so the given consequent itself of the remainder to some other <term>, for instance, grant that, 50, and this will also be the sought antecedent of the remainder, which is also given by means of such a procedure. Seek the proof of such a <statement> in the 20th of the 5th of the *Logistikē*.

5th, Barlaam's

If you wish to remove from a ratio a ratio in specific terms in such a way that the <ratio> which is left out is left out with respect to a given term serving as the consequent, which will be the same as the consequent of the leading <ratio>, do as follows. <Make,> as the antecedent of the following <ratio> to its consequent, so the antecedent of the leading <ratio> to some other <term>, for instance, grant that, 50, and this will also be the sought antecedent of the remainder, which is also given by means of such a procedure. Seek the proof of this <statement> in the last of the 5th of the *Logistikē*.

Sir Nicholas Kabasilas'

Let there be a circle, $AB\Gamma$, about center E , and at it let an angle, AEB , happen to stand on arc AB . I claim that, as arc AB is to the whole circumference of the circle, so angle AEB is to 4 right angles¹⁴³. Let a <straight line>, AE , be drawn through as far as Γ . Therefore, as arc ΓB is to arc BA , so angle ΓEB is to <angle> BEA ; and by composition, as semicircle ΓBA is to arc AB , so the angles ΓEB BEA , that is, 2 right angles, are to <angle> AEB ; and by inversion, as arc AB is to the semicircle, so angle AEB is to two right angles; but of course, as the semicircle is to the whole circumference of the circle, so two right angles are to 4 right angles too; therefore, through an equal, as arc AB is to the whole circumference of the circle, so angle AEB is to 4 right angles.

Of the same

Again, let a <straight line>, $B\Gamma$, be joined. I claim that, as arc AB is to the whole circumference of the circle, so angle $A\Gamma B$ is to two right angles¹⁴⁴. In fact, since angle AEB is double the <angle> $A\Gamma B$, therefore, as <angle> AEB is to <angle> $A\Gamma B$, so 4 right angles are to 2 right angles; therefore, by inversion, as <angle> AEB is to 4 right angles, <angle> $A\Gamma B$ is to two right angles; but as <angle> AEB is to 4 right angles, so arc AB was to the whole circumference of the circle; therefore, as arc AB is to the whole circumference of the circle, so <angle> $A\Gamma B$ is to 2 right angles too, which it was really required to prove.

Of the same

This being presupposed, we shall prove that, if either of the acute angles of a right-angled triangle be given, the sides will also be given¹⁴⁵. In fact, let there be given the <angle> at A of a right-angled triangle, $AB\Gamma$, that has

¹⁴³ This is true, but Kabasilas' argument begs the question. For he assumes that "as arc ΓB is to arc BA , so angle ΓEB is to <angle> BEA ". This is equivalent to assuming that proportionality between arcs and subtending angles holds in general, a property a particular case of which Kabasilas has set out to prove. His argument applies in succession *Elem.* V 18 ("by composition"), 7 porism ("by inversion"), 22 ("through an equal").

¹⁴⁴ This is true. Kabasilas' argument applies in succession *Elem.* III 20, V 7 porism ("by inversion"), 11 (transitivity of sameness of ratio).

¹⁴⁵ This is false. In his next-to-last step and contrary to his initial assumptions, Kabasilas also assumes that the hypotenuse $A\Gamma$ of the right-angled triangle $AB\Gamma$ is given, which indeed—together with the initial assumptions—makes all sides of the

the angle at B right. I claim that also the sides will be given. In fact, on diameter $A\Gamma$ let a semicircle be described, $AB\Gamma$, and Δ being the center let a <straight line>, $B\Delta$, be joined. Then since angle BAG is given, and <angle> BAG is the double of it, therefore this is also given; and as <angle> BAG is to 4 right angles, so arc $B\Gamma$ is to the whole circumference of the circle; therefore arc $B\Gamma$ is also given; therefore <arc> AB is also given as a remainder to a semicircle; so that the straight lines under them are also given, so has to be so many <degrees> where $A\Gamma$ is 120; therefore the sides are given, which it was really required to prove.

I now also claim that, if the ratio of the sides be given, the sides will also be given and each of the acute angles¹⁴⁶. In fact, on the same diagram, multiplying by themselves both the number of <straight line> AB and that of <straight line> $B\Gamma$ and composing both of them and taking the square root of this both-together and recognizing that it is the number of <straight line> $A\Gamma$ we shall make, as this number to the number of <straight line> AB , so 120 to some other <number>, which, clearly, we shall arrange above AB . Doing the same also for $B\Gamma$ and taking the degrees of arcs AB , $B\Gamma$ from the table of the chords in a circle we shall also have, clearly, the angles at the center Δ given and their halves, that is, each of BAG , BGA , which it was really required to prove.

It is possible that, by reference to suitable wholes, the ratio of magnitudes be given even if the magnitudes are not given¹⁴⁷, in the same way as when someone, cutting off a segment of a circumference, proves that this is 60 where some other arc in the same circle is 40, but it is not yet clear whether this is 60 and that is 40 where the whole circle is 360, so that their ratio is given because it is hemiolic, but what is the value of each with respect to the whole circumference of the circle, this is not yet clear. On the other hand, it is impossible that, two magnitudes being given, their ratio is not given.

Let there be that A has to B a ratio whatsoever, and let A be supposed to be the greater term. I claim that the ratio of A to B is greater than that of B to A ¹⁴⁸. In fact, let it happen to come to be that, as A is to B , so B is to Γ ; therefore, by inversion, as Γ is to B , B is to A . And since A is greater than B and, as A is to B , B is to Γ , therefore A is much greater than Γ . And since A , B are two and Γ is some other, and that which is greater has a greater ratio to the same than that which is lesser, therefore A has to B a greater ratio than Γ has to B itself; and as Γ is to B , B is to A ; therefore A also has to B a greater ratio than B has to A , which it was really required to prove.

Let A be less than B and Γ greater than Δ . I claim that A has to B a lesser ratio than Γ has to Δ ¹⁴⁹. In fact, let it happen to come to be that, as B is to A , Δ is to E ; therefore Δ is greater than E ; and Γ is also greater than Δ ; therefore Γ is much greater than E ; and Δ is some other; therefore E has to Δ a lesser ratio than Γ has to Δ ; and as E is to Δ , A is to B ; therefore A has to B a lesser ratio than Γ has to Δ , which it was really required to prove.

Commentary

In the second sequence of texts, six procedures are expounded for removing a ratio from a ratio, five of which refer to *Logistikē* V 21, 23, 22, 20, 23, respectively, and the sixth to Theon's commentary on the *Almagest*. Com-

triangle given. The rest of the argument is correct, and applies in succession *Data* 2 (twice), 1, 4, and, implicitly, the table of chords set out in *Almagest* I 11.

¹⁴⁶ This is also false. In his argument, Kabasilas assumes that all sides of triangle $AB\Gamma$ are numerically given. He also applies *Elem.* I 47 (the Pythagorean theorem) and refers to using the table of chords set out in *Almagest* I 11.

¹⁴⁷ This is true. The example is again grounded on notions typical of the *Almagest*.

¹⁴⁸ This is true. Kabasilas' argument applies in succession existence of a third proportional in magnitudes, *Elem.* V 7 porism ("by inversion"), 14 and transitivity of "being greater than", 8 (a part of whose enunciation is cited; this is the clause without denotative letters), 13.

¹⁴⁹ This is true, and entails the previous result. Kabasilas' argument applies in succession existence of a fourth proportional in magnitudes, *Elem.* V 14 and 16 (in the *Elements*, the very simple property used by Kabasilas is not proved but it is frequently applied by combining V 14 and 16, as I have indicated; this is a minor weakness of the deductive structure of the Euclidean treatise, which has triggered some literature: see J.-L. GARDIES, La proposition 14 du livre V dans l'économie des *Éléments* d'Euclide. *Revue d'Histoire des Sciences* 44 (1991) 457–467; K. SAITO, Proposition 14 of Book V of the *Elements* – A Proposition that remained a Local Lemma. *Revue d'Histoire des Sciences* 47 (1994) 273–282), transitivity of "being greater than", V 8, 13.

pounding and removing ratios was a thorny issue in Greek and Byzantine mathematics¹⁵⁰. As Greek mathematicians conceptualized ratios—even numerical ratios—as relations and not as numbers (that is, as our fractions), trivial-looking mathematical tools as multiplying and dividing ratios were not available to them. These operations were replaced by the notion of compounded ratio and by the procedure of removal of a ratio from a ratio, respectively. Late authors, and Barlaam better than any other, reconceptualized ratios and the just-mentioned notion of compounded ratio and procedure of removal so as to make them virtually coincide with our notions of fractions and their multiplication and division. Crucial to this change of paradigm was the notion of “value” (πιηλυκότης) of a ratio, sanctioned in *Elem.* VI def. 5 but never fully exploited in Greek antiquity.

Book V of Barlaam's *Logistikē* treats systematically the subjects of compounded ratios and of removal of a ratio from a ratio; by exploiting the resource of the “value”, he provides them with a flawless basis. Barlaam's propositions are theorems and problems, which are proved and constructed in impeccable Euclidean style. In particular, propositions 15–23 are problems: 15–16 explains how to find the ratio compounded of a number of given ratios, whose antecedent or consequent is a given number; 17 shows, as an immediate consequence of proposition 14, that the result of a removal of one ratio from another is simply the ratio of the “values” of the two ratios. Propositions 18–23 require to find, in a plurality of cases of removal, a remainder having a given extreme; the involved constructions operate first the removal and then, by taking a suitable fourth proportional, transform the remainder to a ratio having the given extreme. The constructions of some problems in this last group were transformed into procedures by the author of the second sequence of texts in the *varia mathematica*. To better appreciate Barlaam's feat and get acquainted with his terminology, let us read the definitions and the enunciations of propositions 15–23 of *Logistikē* V.

Definitions

1. A value of a ratio is a number that, multiplied by the consequent term of the ratio, makes the antecedent.
2. A ratio is said to be compounded of ratios when the values of the ratios, multiplied by one another, make the value of the ratio.
3. When I remove a ratio from a ratio, I call “leading [ἡγούμενον] ratio” that from which I make the removal, “following” [ἐπόμενον] that <ratio> which I remove. The <ratio> that with the following <ratio> makes the leading <ratio>, this is said “remainder” [λοιπός] or “left out” [καταλειπόμενος] ratio.
4. When I prescribe to remove a ratio from a ratio, then I prescribe to find the remainder.
5. When someone, being prescribed to remove a ratio from a ratio, find the <ratio> left out, the prescription turns out to be accomplished.

Propositions

15. As many ratios as one pleases and a number being given, find which is the ratio compounded of them whose antecedent will be a given number.
16. As many ratios as one pleases and a number being given, find which is the ratio compounded of them whose consequent will be a given number.
17. A given <ratio> being removed from a given ratio, find which is the ratio left out.
18. Remove from a given ratio the inverse ratio in the same terms, so that the <ratio> which is left out is left out with respect to a given term.
19. Remove from a given ratio a given <ratio> that has the same antecedent, so that the <ratio> which is left out is left out with respect to a given term.
20. Remove from a given ratio a given <ratio> that has the same consequent, so that the <ratio> which is left out is left out with respect to a given term.

¹⁵⁰ On this bewildering chapter of Greek and Byzantine mathematics see F. ACERBI, *Composition and Removal of Ratios in Geometric and Logistic Texts from the Hellenistic to the Byzantine Period*, in: *Revolutions and Continuity in Greek Mathematics*, ed. M. Sialaros. Berlin 2018, 131–188, which I partly use in what follows.

21. Remove from a given ratio a given <ratio> that has the antecedent of the leading ratio as consequent, so that the <ratio> which is left out is left out with respect to a given term.
22. Remove from a given ratio a given <ratio> that has the consequent of the leading ratio as antecedent, so that the <ratio> which is left out is left out with respect to a given term.
23. Remove from a given ratio a given <ratio> in specific terms, so that the <ratio> which is left out is left out with respect to a given term.

We readily see that the six procedures use Barlaam's terms "leading" and "following", which are unique to him, and cite *verbatim* the enunciations of the problems whose constructions are transformed into procedures.

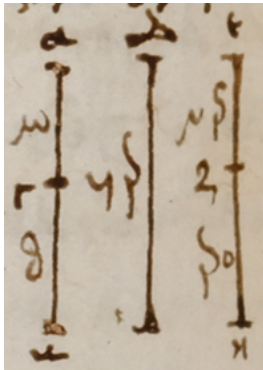
Crucial to understanding our six procedures is to recall that the ratios compounding a compounded ratio are not "multiplied": what is "multiplied", according to *Elem.* VI def. 5, are their "values". For this reason, when writing a compounded ratio in symbolic form, I put the standard mathematical sign of composition "°" between the two compounding ratios, thus: $a:b = (c:d)°(e:f)$. A ratio is said to be "compounded" of two ratios: it neither "is" the two compounding ratios, nor is it "equal" or "identical" to them. The sign "=" in the above formula is thus partly misleading. Despite the fact that *Elem.* VI def. 5 reduces "composition" to a multiplication, a compounded ratio was never conceived of as the result of an operation on ratios. With reference to the formula above, the two ratios $c:d$ and $e:f$ are not "compounded" to produce $a:b$; what is "compounded" is $a:b$ itself, while $c:d$ and $e:f$ are the "compounding" ratios. If an operation is at issue here, this is not a "composition", but a "de-composition" of $a:b$ into compounding ratios.

The operation of removal was normally performed on ratios that were already provided in compounded form. Suppose that one is required to remove $c:d$ from $a:b$. To this end, one writes the ratio $a:b$ in compounded form in such a way that one of the compounding ratios is $c:d$, say $a:b = (x:y)°(c:d)$. Next, it suffices to literally "remove" $c:d$ from the right-hand side; ratio $x:y$ is, in a most concrete sense, the "remainder". Usually, one is not interested in ratio $x:y$, which is an equivalence class both in the Greek conceptualization and in ours, but in a particular ratio in this class, identified by the fact that one of the terms of $x:y$ is "given". The problem reduces to that of finding the term of ratio $x:y$ that is not given. Accordingly, five of the six terms in the expression $a:b = (x:y)°(c:d)$ are given, and one must find the term that is not given. This is solved by once or twice taking a suitable fourth proportional of three terms among the given ones. So conceived, the procedure of removal presents several cases, according to assigned mutual relations between the given terms of the involved ratios (read the enunciations of *Logistikē* V 18–23 above). Some of these cases are formulated as procedures in the *varia mathematica*. These procedures can be written in symbolic form, as follows (x is the sought term):

1. Require $a:b = (a:y)°(c:a)$; make $a:c::b:y$. *Logistikē* V 21 is referred to.
2. Theon. Require $a:b = (x:g)°(c:d)$; make $b:a::d:w$ and $c:w::g:x$.
2. Barlaam. Require $a:b = (x:g)°(c:d)$; make $a:b::c:w$ and $w:d::g:x$. *Logistikē* V 23 is referred to.
3. Require $a:b = (x:b)°(b:d)$; make $b:d::a:x$. *Logistikē* V 22 is referred to.
4. Require $a:b = (x:b)°(c:b)$; make $c:a::b:x$. *Logistikē* V 20 is referred to.
5. Require $a:b = (x:b)°(c:d)$; make $c:d::a:x$. *Logistikē* V 23 is referred to.

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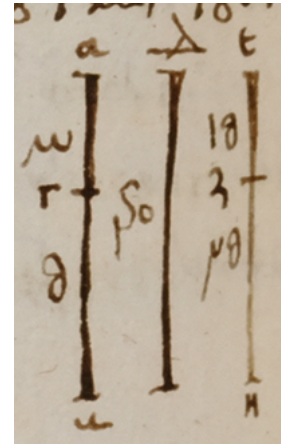
II. Marc. gr. Z. 332, ff. 61v-66v



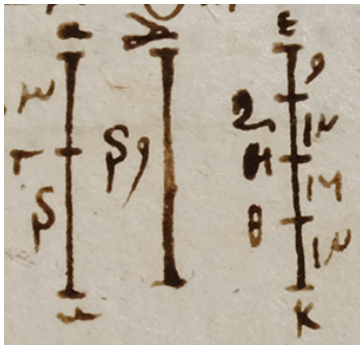
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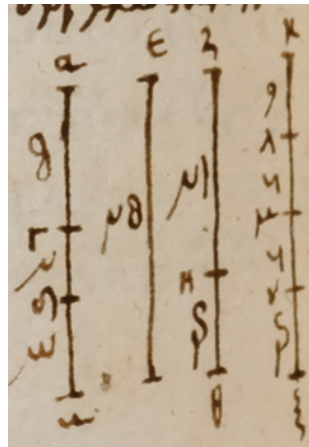
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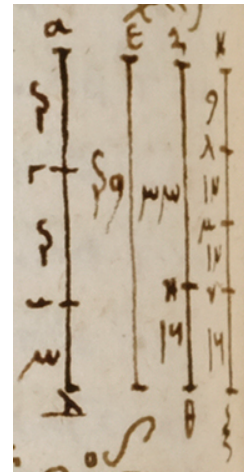
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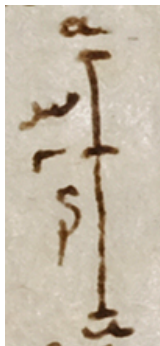
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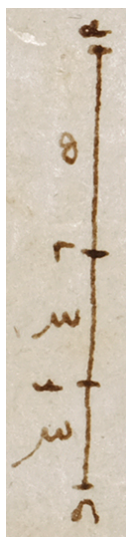
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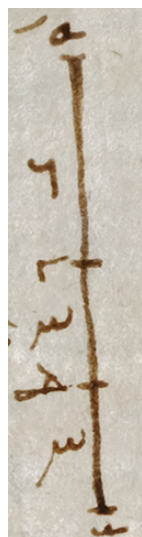
Prop. 6



Prop. 7



Prop. 8

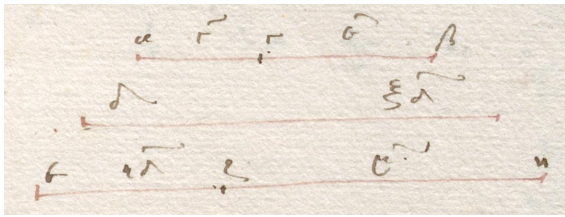


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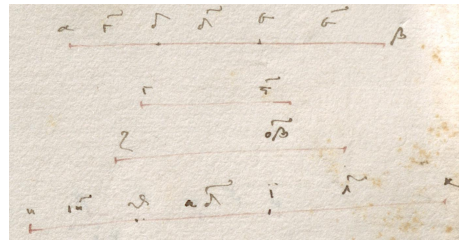


Prop. 10

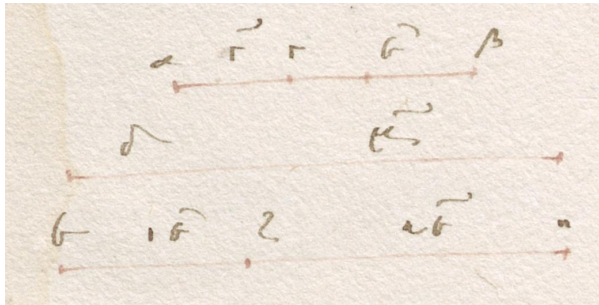
III. Krems. 343, ff. 127r-132v



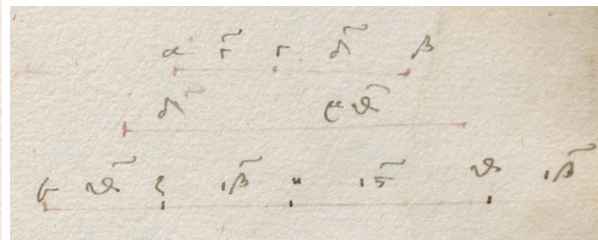
Prop. 1 (= 2)



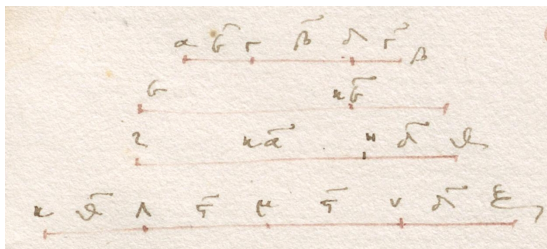
Prop. 2 (= 1)



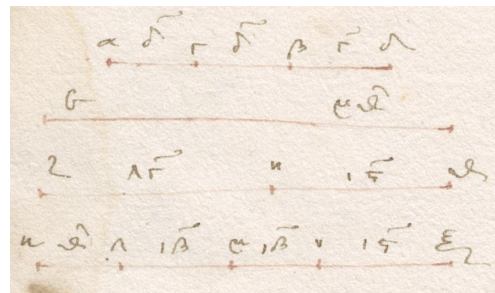
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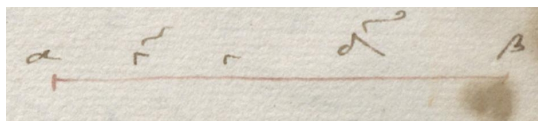
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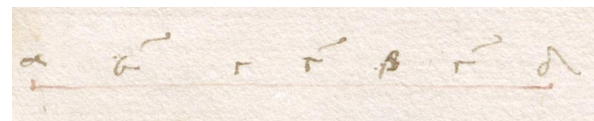
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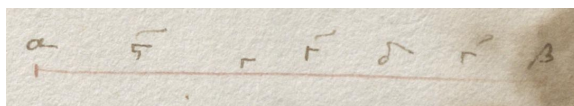
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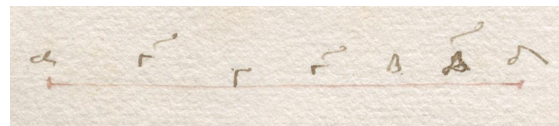
Prop. 7



Prop. 8



Prop. 9



Prop. 10

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- III Kremsmünster, Stiftsbibliothek, Ms. 343 (*Diktyon* 37295), ff. 127r–132v, the diagrams of Barlaam's *Demonstratio*. Courtesy of Stiftsbibliothek Kremsmünster.