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How the students chased perpendicular lines in GeoGebra Classroom

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This paper focuses on the issue of what solution strategies do secondary school students provide while performing non-standard geometric constructions. For this purpose, we developed an interactive electronic environment in GeoGebra Classroom and, in that environment, we let students construct perpendicular lines in multiple ways. We formulated the task for students so that it corresponded with their school geometry curriculum but in a non-standard open-ended way. Then we analyzed students' constructions qualitatively. Findings showed that the students were usually able to solve the task but just a few of them were able to generate multiple strategies. However, creative constructional approach appeared in data indicating the significance of the assignment or instruction where a variety of mathematical approaches are possible.

Keywords: Dynamic geometry environments; multiple solution strategies; plane geometry; secondary school students.

Introduction

Recently, school geometry has got a rather dual role: it seeks to reflect ongoing technological progress but at the same time still arises from fundamental concepts described in ancient Euclid's *Elements*. One of the significant aspects of the *Elements* consists in stressing on acknowledging the definitions and proving the theorems, while one of the goals of school geometry is developing students' justification skills or, in general, students' conceptual knowledge in the domain. In the past decades, educational researchers have been naturally concerned with the impact of the use of modern electronics technologies on learners' mathematical reasoning. They have explored how dynamic geometry environments (DGE) such as Cabri Geometry or GeoGebra could assist students to improve their understanding of proof in geometry (e.g. Marrades & Gutiérrez, 2000). In this context, an important characteristic of DGE lies in the ability to modify geometrical objects and to observe their features during the modifications. It has become clear that this feature of DGE provides students with opportunities for deep explorations and heuristics on their way to argue the properties of geometrical objects (Lawson & Chinnappan, 2000; Sträßer, 2001). Through the dynamic process of observing modifications of geometrical objects, DGE does not only facilitate students' justification skills but can also provide visual information that a particular property does not apply in general (Prusak et al., 2012). Altogether, the research has revealed the importance of DGE for stimulation of formulating conjectures and creating proofs (Komatsu & Jones, 2017).

Along with the use of DGE, other activities in school geometry could contribute to formulating geometrical conjectures, namely geometric constructions (Herbst et al., 2017: see p. 106). However, there is a lack of investigations that would deeply focus on the relation between constructional problems and reasoning in geometry, or, in general, on explaining how performing geometric constructions can contribute to improving students' conceptual and procedural knowledge and flexibility in the domain. In this contribution, we present an introductory exploratory study on the

topic. We proceed from a construction of perpendicular lines, formulate a task that is not standard compared to common activities in school geometry in our country, and analyze students' solutions of such a task. The task is not standard for three reasons:

- (i) we let students construct two perpendicular lines on tablets in an online application GeoGebra Classroom whereas it is standard for them to perform geometric constructions on paper;
- (ii) we ask students to solve the problem in multiple different ways;
- (iii) we allow students to use only two tools: one that creates a line through two points, and another one that creates a circle from a center and point or a center and radius – despite the fact that the standard approach to school geometry also allows them to use a tool that creates a perpendicular line to a given line.

Our explorations correspond with the theoretical framework of flexibility which is defined as the knowledge of multiple solution methods and the ability to generate and perform them appropriately and effectively (Rittle-Johnson et al., 2012). With respect to notes on learning and teaching geometry mentioned above, it becomes clear that developing students' capability to solve mathematical problems in different ways has a significant potential for increasing their conceptual and procedural knowledge (e.g. Star et al., 2015). Since including the facet of multiple solution procedures required from students, we follow up the studies on so-called multiple solution tasks (MSTs) that might be used as a means of developing students' creativity and flexibility in geometry (Levav-Waynberg & Leikin, 2012; Gridos et al., 2019). From the general perspective of research on incorporating an artefact of ICT into the teaching/learning system, we focus on an online application, learners, and knowledge, i.e., on all three edges of the face "ALK" of the corresponding didactic tetrahedron (Donevska-Todorova & Trgalova, 2017).

Within this context, we introduce an exploratory qualitative empirical study with the following research question: "What solution strategies do secondary school students provide when asked for multiple constructions of perpendicular lines while using only circles and lines within the GeoGebra Classroom environment?"

Theoretical background

Geometric constructions

Geometric constructions are understood as a specific category of mathematical problems that ask solvers to draw a geometrical object in a precise and exact way. According to the given circumstances, it requires choosing an appropriate construction method that would correspond to geometrical characteristics of the desired object (Kuřina, 1996). Therefore, geometric constructions are considered a form of mathematical activity combining the process of manipulating objects with the processes of visualization and reasoning (Duval, 2006). What remains to be discussed, is the way how performing geometric constructions could stimulate doing proofs in geometry and, in particular, how it could provide students with significant ideas that would be helpful during the proving process (Herbst & Brach, 2006).

In the Czech Republic, where our research study took place, geometric constructions as a part of school geometry are commonly performed on paper while using a pencil, straightedge, compass, and a special ruler called *a triangle with a guideline*. This special ruler has a form of a transparent piece of plastic in a shape of an isosceles right-angled triangle with a guideline (an impressed line segment) connecting the center of the hypotenuse with the opposite vertex. The tool is frequently used in school geometry from early elementary school grades, where it provides students with an easy way of drawing a line that is perpendicular to a given line – the solver just has to match the guideline with the given line, and then outline the longest side of the ruler with a pencil. From the perspective of the GeoGebra environment, a triangle with a guideline corresponds to the tool *Perpendicular line*.

As mentioned above, we do not allow the use of the tool *Perpendicular line* in our study. It means an obstacle for the students but, on the other hand, it challenges them to come up with an original solution and opens a rich space for multiple correct ways of constructing the object. This arrangement provides us with an opportunity to explore various students' ideas on a problem that is unfamiliar for them but still fully corresponds with the content of school geometry that is appropriate to their age. This way, our approach shifts the issue of geometric constructions into the frame of open-ended problems in mathematics education (Pehkonen, 1997), and thus mediates a suitable environment for investigating students' knowledge.

GeoGebra Classroom

The use of DGE in our research follows the above-mentioned studies that focus on the benefits of the modern technologies for teaching and learning geometry. We work in an online application GeoGebra Classroom provided by GeoGebra software. This software was originally created by Markus Hohenwarter (2002) and has been under continuous development and addition of new elements in the past two decades. These days, GeoGebra has the form of a rich open-source application covering the topics of geometry, algebra, calculus and statistics, in the range from primary school to university levels (GeoGebra, 2021).

GeoGebra Classroom is one of the latest features of GeoGebra software which was introduced to public in May 2020. It allows teachers (or researchers) to assign various tasks to students, then ask them to join the environment via entering a code and solve the tasks individually. Teachers or researchers can observe updated students' advances in the solving process live, and record the step-by-step course of all the constructions contributed by individual students. Such interactive features provide us with complex data on students' work. In this contribution, we show how these features might be used in research focusing on student knowledge.

Design of the study

Participants

Participants of our research study were 19 students from the same ninth-grade class (age 15 to 16 years) at a suburban school. At the time of the study, they had already discussed whole mathematical curriculum belonging to the lower-secondary school level (grades 6 to 9), including prescribed parts of Euclidean planar geometry and geometric constructions such as copying a line segment, an angle, or a triangle, constructing circles, triangles and quadrilaterals on the basis of certain requirements, or

constructing an image of an object in particular symmetry. During some of their previous mathematics lessons, they had also gotten to experience exercises in the GeoGebra environment. Nevertheless, they had not been asked to perform geometric constructions in GeoGebra Classroom before.

Data collection

As indicated above, we prepared GeoGebra Classroom based on three activities with GeoGebra applet where we let the students construct perpendicular lines. The first activity was named “Right angle for the first time” and its assignment was formulated as “Make a right angle! I.e., construct two lines, about which you can safely say that they are perpendicular to each other.” The second activity was named “Right angle differently” and it requested to “Make a right angle again, but now construct two perpendicular lines in a different way than in the previous task.” The third activity was named “Right angle still differently?” and it asked “Can you do it again? Construct a right angle by a different method than in the two previous tasks.”

For the purpose of our research, the GeoGebra toolbar was customized in all three activities. Solvers were able to use only the tools *Move*, *Point*, *Intersection*, *Line segment*, *Ray*, *Line*, *Circle with center through point*, *Compass* and *Delete*. They could also go through the steps of their construction using *Navigation bar*, return to the previous step pressing *Undo*, redone an undone action by pressing *Redo*, or start again with the button *Reset construction*.

Data collection was conducted during a mathematics lesson, while working face-to-face with the participants in the classroom. At the beginning of the lesson, each participant has got its own school tablet, i.e., a device with touchscreen and internet connection. They were used to working with this equipment. We gave them the code of the arranged GeoGebra Classroom to type in the box and join the environment. The participants had 20 minutes to complete all three assignments. They worked individually and independently; we provided them only with technical support.

Task analysis

As a preparation for data analysis, we investigated the assigned problem from the geometrical point of view. We looked for different construction strategies that would lead to creating perpendicular lines using only the given tools. We identified eight basic strategies; others might be derived from them by minor changes (e.g. in the order of steps) or by combining various basic strategies together. In their background, they all have a construction of a geometrical object that contains a right angle as a general property of its basic attributes (e.g. between a line segment and its axis, between diagonals in a rhombus, between a side and its median in an equilateral triangle, between a base and its median in an isosceles triangle) or as a consequence of a general principle used to construct the object (the Thales’s or Pythagorean theorem – Proposition 31 in Book 3 or Proposition 47 in Book 1 of Euclid's *Elements*). We ordered the strategies according to the interrelations between them. In the next paragraph, we illustrate the issue by introducing three of the identified strategies and their interrelations.

The strategy that can be considered as the simplest one is based on the construction of the perpendicular bisector of a line segment consisting of creating two circles with the centers at the end points of the line segment, the circles having the same radii greater than half of the line segment

length. This construction had been well-known to the participants as a method of finding the axis (perpendicular bisector) of a line segment. Such an approach leads to obtaining two points (the intersections of the circles) that are at the same distance from the end points of the line segment. The set of all such points is exactly the same as the line that divides the given line segment into two halves forming right angles at the intersection point. This, in its substance, is a geometrical theorem that can be proven on the basis of the congruence of triangles or the properties of the diagonals of a rhombus. When the radii of the circles are equal to the length of the line segment, the circles go through the end points; we labeled this strategy as PL1. The general case when the radii are arbitrary but same and not equal to the length of the line segment, we labeled as PL2. The modification of the general strategy, where the radii are not the same, we labeled as PL3. The PL3 case leads to properties of diagonals of a kite (a quadrilateral with two pairs of adjacent sides of the same length has its diagonals perpendicular).

Data analysis

We analyzed collected data qualitatively, using open coding and constant comparison (Miles, Huberman & Saldaña, 2014). We carefully observed all students' solutions from the perspective of correctness and relevance of (i) individual construction steps, (ii) the figure presented as final in the construction, (iii) the sequence of the GeoGebra tools used during the construction. The latter information was available through the GeoGebra functionality *Construction protocol*. We were also comparing the ongoing findings with our list of basic strategies.

Findings

Strategies provided by participants – an overview

During data analysis, we identified three different basic strategies used by the respondents: PL1 mentioned above, a strategy based on the Thales's theorem (an angle inscribed across a circle's diameter is a right angle) that had also been stated in our list of strategies, and a new strategy that had not been stated in the list. The new strategy consists of the construction of three circles with collinear centers and the same radii, and two rays passing the intersections of the circles and forming the equilateral triangle. Generally speaking, the method is based on the properties of triangular lattice. We added this new strategy to our list, and reordered the list of basic strategies to still follow the sequence of relations between them. After the reordering, PL1 stayed PL1, the new strategy became the fifth one, i.e. PL5, and the strategy based on the Thales's theorem became PL9.

For details on the three strategies see Fig. 1: for each of the strategies, we present the number of respondents that provided the strategy (in square brackets), the list of GeoGebra icons that were available for the respondents during the task, the final figure, the sequence of the construction steps expressed through the GeoGebra tools icons, an explanatory drawing proving the perpendicularity, and a note on the geometrical background of the method.

In all of the assigned activities together (i.e., among the $3 \cdot 19 = 57$ attempts to create perpendicular lines), 16 students completed the strategy PL1 (one of them twice, with two different orders of construction steps), one student completed PL5, and four students completed PL9. All other attempts were unsuccessful: the participants either did not present any perpendicular lines, or presented just a

freehand sketch (they drew two lines that looked like they were perpendicular but, in reality, they were not).

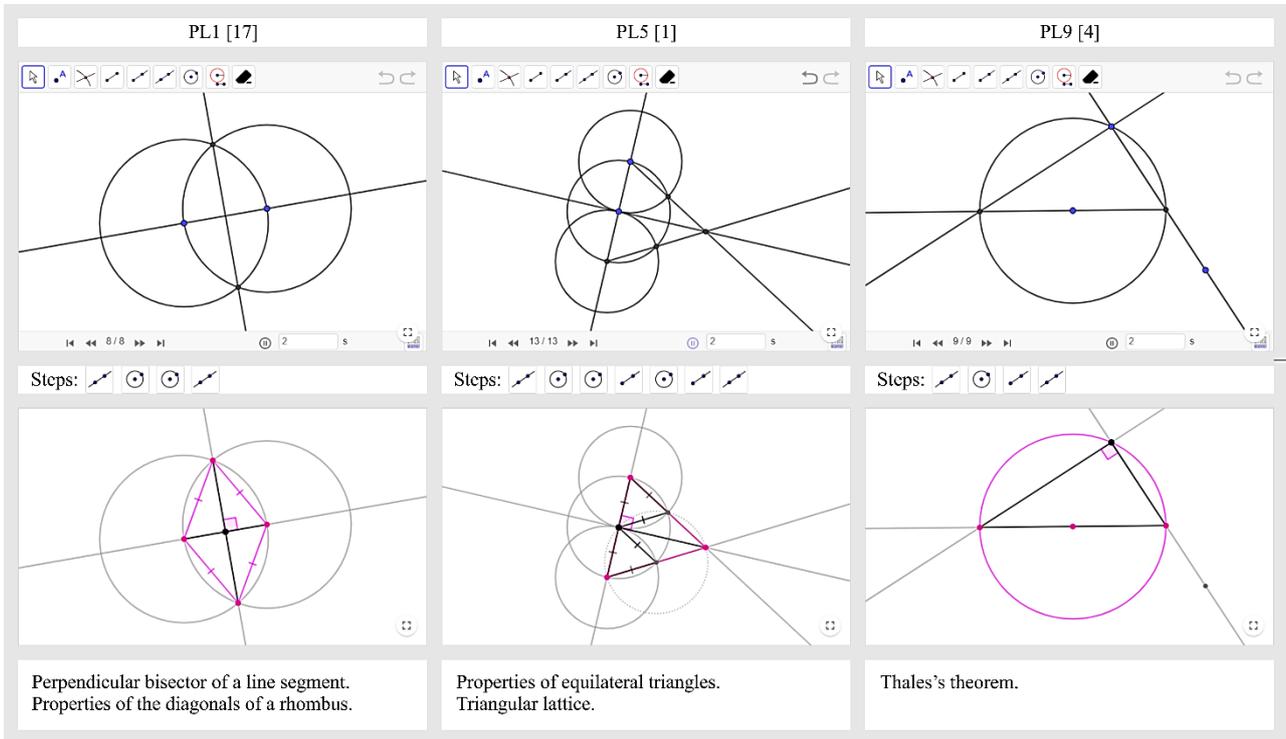


Figure 1: Three constructional strategies that appeared in data with, in square brackets, the number of respondents that provided the strategy

Individual participants

From the perspective of individual participants across the three activities, nobody was able to provide three different strategies. One student provided three different constructions that were based on two different strategies (one of the strategies was presented twice, with two different orders of steps), three students provided two different strategies, 12 students provided one strategy (all of them PL1), and three students did not manage to provide any strategy. The diagram of individual strategies within individual assigned activities that also captures individual participants' shifts in strategies across the three activities is shown in Fig. 2.

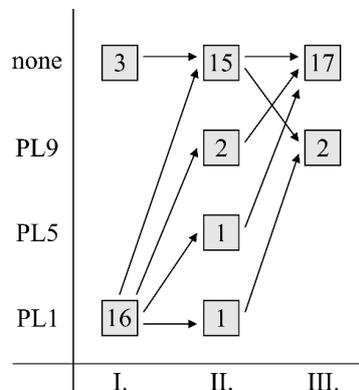


Figure 2: The number of students using individual strategies (PL1, PL5, PL9, none) within individual activities (I., II., III.), the arrows indicate participants' shifts in strategies

Discussion and conclusion

In this study, we focused on the variety of solution strategies that secondary school students provided while performing non-standard geometric constructions. For this purpose, we developed an interactive electronic environment in GeoGebra Classroom, and asked the students to construct perpendicular lines in multiple ways. To accomplish the non-standard nature of the task, we allowed just lines and circles to be used as tools during the construction. Using the GeoGebra Classroom environment, we obtained detailed complex data on students' work which confirmed the potential of DGE in rendering students the opportunities for deep explorations and heuristics (Lawson & Chinnappan, 2000). On the other hand, most of the students provided just one construction strategy, although we asked them to come up with three different ways of constructing perpendicular lines. The variability of the strategies was also low, we detected only three different students' approaches whereas we found at least eight solving methods available and appropriate for the given students. Therefore, as a plan for the future, we see the need to investigate further the relationship between students' performance in non-standard geometric constructions and multiple solution tasks, their conceptual understanding and their justification skills.

Among the construction strategies provided by students, two were based on a fundamental feature of a basic geometric figure or a basic construction – a perpendicular bisector of a line segment (PL1), Thales's theorem (PL9). These methods could be considered mere applications of a known geometrical shape property or construction procedure. However, the approach PL5 that emerged from data represents a shift from the use of the known individual object or construction procedure to a creative synthesis of various geometrical objects and constructions. We could perceive this method as a construct that arises from an enhanced connectedness of student's knowledge in the domain (Levav-Waynberg & Leikin, 2012) or from student's ability to use concepts and procedures in a flexible way (Rittle-Johnson et al., 2012).

The strategy PL5 also highlights the significance of open-ended approach in mathematics teaching and learning (Pehkonen, 1997) that goes hand in hand with the need for stimulation of using various construction strategies as in our assignment. It can be concluded that, without this request, the strategy PL5 would not have been explored, contemplated and developed by the student. For the future, it would be helpful to investigate which other impulses can lead students to such innovative and original solutions of geometric problems.

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