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# Construction of Mathematical knowledge in digital-collaborative settings

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*There are many different ways in which technology can enhance learning and teaching of mathematics. Therefore, a specific research focus is needed. One of the foci is to look at the role technology plays in learning mathematics through the lens of communication – especially the way in which it influences the construction of mathematical knowledge in interaction. To do this, student teachers were filmed while using Padlet to collaboratively summarise and discuss characteristics of different mathematical functions. First results hint at Padlet being a useful tool to induce and support meaningful mathematical discourse and mathematical thinking.*

*Keywords: Communication, technology, digital media, collaboration, knowledge building.*

## Introduction

Although the use of digital media for educational purposes has become an omnipresent topic – whether it is in research, politics or every-day-conversation – teaching with technology is often based on best-practice approaches or personal preference (Rink & Walter, 2020). Research also shows that while students use technology on a daily basis, they do not use it for academic purposes as often (Zawacki-Richter, 2021; Dolch & Zawacki-Richter, 2018). Although the academic use of technology in schools and universities has significantly increased over the last year, it often remains on a ‘consuming’ level (e.g. reading documents or watching videos) and rarely leads to subject-rich learning processes (Zawacki-Richter 2021; Biermann & Kommer, 2012). Considering that communication is fundamental for learning in general (e.g. Miller, 2002) and developing mathematical knowledge in particular (Steinbring, 2015), this problem is addressed by making communication the key focus when looking at digital media used in education (Ball & Barzel, 2018).

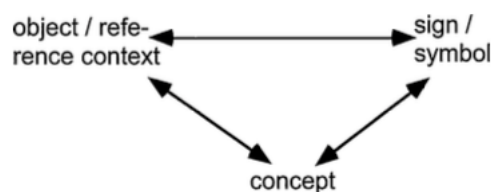
In this paper, we focus on the question of how technology influences communication and, thus, the construction of mathematical knowledge in collaborative settings. In what follows below, a brief theoretical background is given on *Construction of Mathematical Knowledge in Communication and Technology in Mathematics Education*. Afterwards the *Methodology* as well as some initial *Results* are presented.

## Theoretical Background

Studies by Sung, Yang and Lee (2017) show evidence that digital-collaborative learning may lead to an increased learning performance and a more positive attitude towards learning in general. However, one should not assume that using technology will always be beneficial as “all [digital tools][...] come with affordances and limitations, with opportunities and constraints” (Drijvers, 2019, p. 9). With a view to university levels, research indicates that technology is in fact used a lot to share documents or to collect data (e.g. in online courses), but is only infrequently used to encourage collaborative learning (Zawacki-Richter 2021; Dolch &

Zawacki-Richter, 2018; Biermann & Kommer, 2012). This is problematic since working on tasks collaboratively in small groups is one of the best ways to induct productive mathematical thinking (Nührenböcker & Steinbring, 2009). Students working and learning together are forced to communicate and interact with each other. “Statements and suggestions are offered for joint consideration. These may be challenged and counterchallenged, but challenges are justified and alternative hypotheses are offered” (Howe & Mercer, 2007, p. 6). These situations are of uttermost importance, because – following enactivism – learning itself is manifested in communication (e.g. Miller, 2002).

Learning mathematics is somewhat unique since it is not accessible by senses (Ball & Barzel, 2018). In fact, *mathematical knowledge* is not a given set of definitions and theorems, but is only accessible “using signs, words or symbols, expressions or drawings” (Duval, 2000, p. 61). The meaning of those signs, symbols or expressions has to be constructed by the learners themselves during the act of communication (Steinbring, 2006). Therefore, communication is at the center of constructing any mathematical knowledge. Based on this underlying interpretation of learning, the construction of mathematical knowledge can be modeled using Steinbring’s *epistemological triangle*:



**Figure 1: Epistemological Triangle**

“Mathematics requires *certain sign or symbol systems* in order to keep a record of and code the knowledge. [...] [The meaning of those signs] has to be produced by the learner by means of establishing a mediation to suitable *reference contexts*” (Steinbring, 2006, p. 135). A sequence of such triangles can be used to cover the whole learning process (Ball & Barzel, 2018) and to examine the impact of technology on those processes. “To use technology for effective collaboration and communication in mathematics classrooms it is necessary to consider the role of technology in the epistemological process” (Ball & Barzel, 2018, p. 229).

There is a great variety of digital tools that can be used in education. To gain some orientation it seems appropriate to characterise these tools with regards to how they are used in communication. Ball and Barzel (2018) distinguish between *communication through technology*, *communication with technology* and *communication of technology displays*. The first one involves use of technology to directly support online synchronous communication between people (e.g. Skype, Zoom). Communication *with* technology “considers the entry of syntax, selection of menu items, programming or any command that drives the technology to produce a display” (ibid.) (e.g. Apps like “digital-twenty-frame”). The third one describes a situation in which the technology display is a stimulus for discussion. “This discussion could occur in a range of contexts, for example, through two students’ consideration of one shared screen or through public display of student work via technology such as an interactive whiteboard or data projector” (ibid.) (e.g. Padlet). While those categories are useful to distinguish and analyse the use of technology in communication, they may overlap and do not occur in isolation (ibid.). For example, students may discuss a technology display while at the

same time interacting with the display itself. Thus, they may be even interacting with each other through the input itself. This shows how complex the connection between communication and technology is. Naujok (2012) and Knopf & Abraham (2016) mention that those discussions and interactions evolving while working with technology may be especially important and fruitful for deep learning processes.

Combining those three ideas – *constructing mathematical knowledge in communication, technology and collaboration* – it becomes evident that there are specific affordances and opportunities in teaching and learning mathematics in *digital-collaborative* settings. One such setting and initial results are presented on the following pages.

## **Methods**

The research presented in this paper is part of a research project called K4D ('Collaboration for Digitisation'), funded by the German Federal Ministry of Education and Research. The project started in early 2020 at TU Dortmund University and aims at a better understanding of teaching and learning with technology in higher education.

During their first year at TU Dortmund University, students of mathematics education are obliged to attend the lectures '*Arithmetic and its Didactics I*' and '*Arithmetic, Functions and its Didactics II*'. Those lectures are accompanied by a mandatory seminar once a week in which students tackle mathematical tasks in small groups. Due to the Covid-situation, students worked remotely from home using Zoom. During some of those exercises, students were given tasks they had to collaboratively work on, while using Padlet, which is a tool considered a 'digital pinboard' enabling users to share, connect and sort documents (e.g. videos, recordings, pictures, text). Users can comment and react on those documents in real-time. Padlet can be categorized as a general educational technology and – in contrast to subject-specific technology – could be used for many different activities. The usage of Padlet to foster collaboration was planned before the pandemic, but within the usual seminar settings. Data was collected by screen-capturing and voice-recording those situations. The sessions were 30 to 60 minutes long and about 20 groups have been recorded. Additionally, questionnaires were given to the students to better understand how they themselves experienced the collaborative work with Padlet. Research is intended to continue in 2021 and 2022 to gain more data. At this stage of the research process, only exemplary insight into the data and findings can be given.

### **Tasks for working with Padlet**

The topic of the recorded session is basic functions (linear, proportional, reciprocal) and the tasks (listed below) are supposed to engage the students in discussions about their specific content knowledge.

(1) *Find a context/situation to those functions (linear function, proportional function, reciprocal function) and upload it to Padlet. Don't name the function in your context/situation.*

[Additional information: (1) done by each student on their own the day before the group exercises took place. In each group, about 25 students uploaded their contexts in one Padlet.]

(2) *Cluster the different contexts/situations and discuss which situation belongs to which type of function.*

[Additional information: (2) done in small groups (3-5 students) during the exercises. The original Padlets were copied so that all groups could create and discuss their own cluster.]

(3) Name characteristics for each type of function and upload them to Padlet.

[Additional information: (3) done in small groups (3-5 students) during the exercises. One Padlet was created for each type of function. The students kept working in small groups (3-5 students), but those Padlets were not copied for each group, but filled by different groups of students simultaneously.]

## Questionnaires

In order to gain additional insights into the processes, questionnaires were conducted to better understand how the students themselves experienced the collaborative work using Padlet. Items were given to the students ( $n = 220$ ) which they could agree or disagree with on a 1 to 5 scaling. Some exemplary examples of those items are as follows: “Working with Padlet was very intuitive.”; “Other students’ postings confronted me with new ideas/approaches/representations.”; “Other students’ postings led to more intense discussions within our group.”

## Selected Results

The following transcript shows four students working on task (2) and (3). At the beginning, they discuss which type of function is represented in a given context-situation.

(Transcripts are translated)

- 1 Student 3: [...] I don’t really get (...) get the difference. Between a reciprocal function (..) and the others.
- 2 Student 2: [...] There is that mnemonic (*for reciprocal functions*): “the more, the less”, so#
- 3 Student 3: #so it’s decreasing?
- 4 Student 2: Exactly. Exactly.
- 5 Student 3: [...] Okay, that makes sense.

The students proceed to cluster the given context-situations and decide – using the mnemonic: “the more, the less” – that the following is a reciprocal function:

“Peter spends a fixed amount of money each month. [...] He got 3000€.  
He withdraws 100€ each month.”

Later during the group-exercise, the students start working on task number (3). While doing so, they are confronted with other groups’ posts in Padlet. Two of those posts and the unfolding discussions can be summarised as below:

Post I:

“a reciprocal function has no zero point  
and no intercept.”

Post II:

“ $x * y$  is always the same for each  
coordinate.”

Confronted with those posts, a discussion emerges:

- 6 Student 3: [...] But they wrote “it (*a reciprocal function*) has no zero point and no intercept” (..) but it could (..) it could start with an intercept, couldn’t it? (..) Or what do they mean?”
- 7 Student 1: Yes.

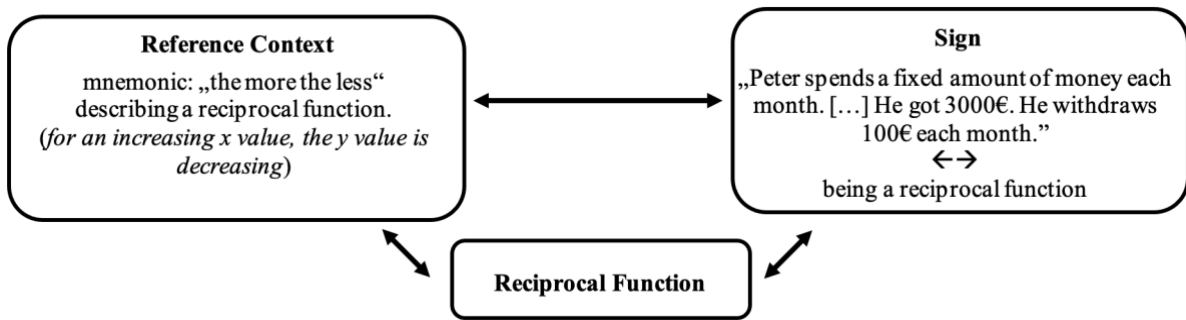
- 8 Student 2: Yes.
- 9 Student 1: [...] It's like with that money. If it starts at 3500€ (3000€), then that is on the y-axis."
- 10 Student 3: Sure.
- 11 Student 3: [...] It's the same with those workers (*Referring to another context-situation: "For the construction of a new [building] a single worker needs 120 days. Two workers need 60 days. [...]"*)
- 12 Student 1: Sure.
- 13 Student 3: Yes, and then you would still start with one, two, three, four workers on the y-axis.
- 14 Student 1: But you have no zero point, like, like the intercept means that there is something like (..) zero, three thousand ( (0/3000)). So x is always zero. And I think in that worker-context, that is not (..) it is not possible, because (..) one (..) zero workers, you can't say that, that they need twice as long as one worker. Because that makes no sense. Because if no worker is working, nothing ever happens.
- 15 Student 3: Mhm.
- 16 Student 1: So maybe the other context is wrong.
- 17 Student 3: Maybe (*laughing*). But I don't get what they mean with "x \* y is always the same for each coordinate".
- 18 Student 3: [...] Especially "for each coordinate" (..) "for each coordinate". What does that even mean? (...) Maybe something like it's linear (..) so that (..) like it's (the function) increasing all the time (..) But#
- 19 Student 1: #Ah, I think I know what they mean. They mean (..) like what you said before. That if one worker needs 120 days and two workers need 60 days, then it's still 120 days in total.
- 20 Student 3: Ah okay, that's possible.
- 21 Student 1: So for example one worker is your x and 120 is your y.
- 22 Student 3: Yes, yes.
- 23 Student 1: And 60 times two equals 120.
- 24 Student 3: Mhm, okay. Nice.

The group decides that the first context (the one with the "money") does not represent a reciprocal function, but the latter (the one with the "workers") does.

## Interpretation and discussion

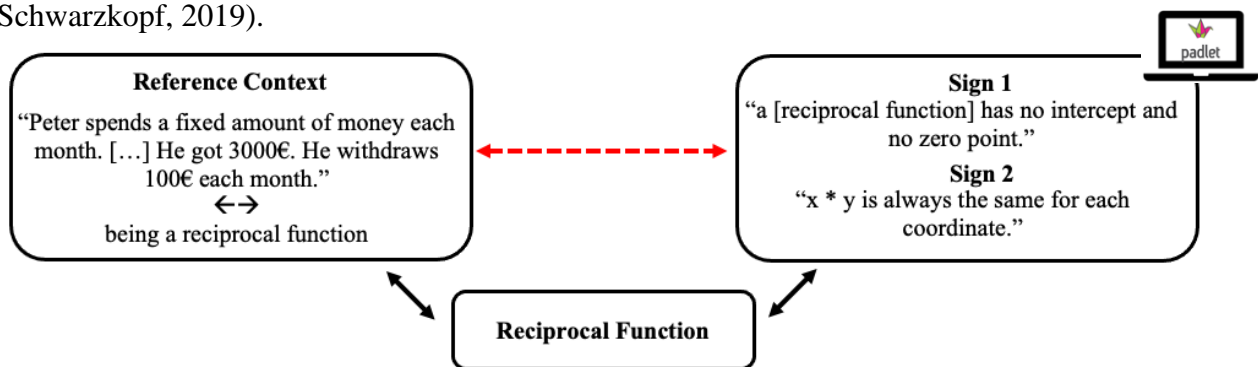
The scene summarised above is analysed using an epistemological perspective. Since that analysis is open for discussion, we will try to re-construct the underlying knowledge and ideas, using Steinbrings epistemological triangle and focusing on how the process is shaped by the use of technology.

Clustering the different context-situations in Padlet proves to be a solid way to force students to interact. They engage in meaningful discussions about whether or not a given situation can be linked to a specific type of function. By doing so, they create a common ground for different *concepts* – for example, what they understand of a reciprocal function (1-5). The students link the situation (*'Peter and his money'*) to a reciprocal function. This link acts as the *sign/symbol* the students try to interpret, using their *reference context*. In this specific case, the reference context is the mnemonic ("the more, the less" – meaning: for an increasing x value, the y value is decreasing) which S2 mentions to legitimate the situation describing a reciprocal relation (2) (Fig. 2).



**Figure 2: Mechanic significance with a mnemonic**

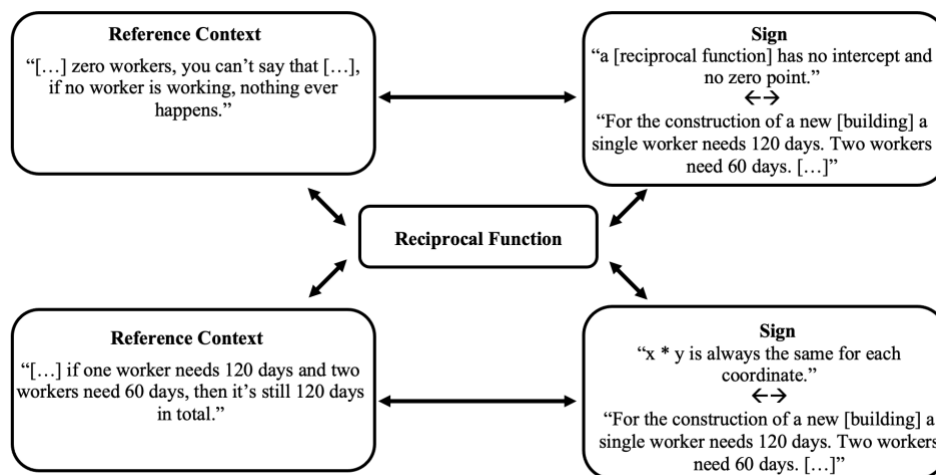
Since no group member protests against the above interpretation, it appears to be a shared one within the group. Now, the use of Padlet comes into play: the posts by other groups inflict a conflict to the formerly shared interpretation – a “productive irritation” (Nührenbörger & Schwarzkopf, 2019).



**Figure 3: Productive irritation through the confrontation with a content post**

The new sign is the statement (“a [reciprocal function] has no intercept and no zero point”) in Padlet. The group tries to understand that statement by mediating it to a reference context. In this case, the reference context is the former sign. However, the group is irritated in a twofold mode. On the one hand, the mediation cannot be done successfully (6). On the other hand, they have to interpret the meaning of the second statement (“ $x * y$  is always the same for each coordinate”) (17) (Fig. 3). When the group fails to interpret the new signs in relation to their reference context, they question their original assumption (16). Therefore, the group looks at another one of the given context-situations: The one with the “workers”. During their discussion, they realise that both of the statements made in Padlet can be explained using this context (14 & 19). The connection between the “new” context situation and those “new” statements acts as the new signs (Fig. 4). Finally, the students successfully mediate them to fitting reference contexts (Fig 4). The epistemological analysis of these sequences clearly shows a shift from a vague understanding and interpretation of functions to a more sophisticated one – for example by (unknowingly) referring to the anti-proportionality factor (Heiderich & Hußmann, 2013). Padlet seems to induce these mathematical learning processes by confronting students with different ideas and interpretations.

An evaluation of the questionnaires ( $n = 220$ ) supports those findings: 88% of students said that when using Padlet “they were confronted with new ideas, approaches and representations” and 75% said it led to “more intense discussions”.



**Figure 4: Structural significance on reciprocal functions**

## Conclusion

This paper highlights specific potentials of the use of technology (by using Padlet) in mathematics education. The first results hint at Padlet being a useful tool to induce and to support mathematical discourse and mathematical thinking. With Padlet, students can be confronted with multiple different ideas and approaches, while at the same time communicating and discussing in a small group. This combines aspects of collaborative learning in smaller groups and class-wide discussions at the same time, resulting in manifold occasions for productive irritations and mathematical discourse. As with every other media or technology, it has to be carefully considered when and how to use them in order to do so most efficiently. Therefore, more research will be done to identify specific design elements and to deepen the understanding of how the use of interactive pinboards, like Padlet, affects mathematical knowledge building.

## Acknowledgment

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