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► **To cite this version:**

Petra Surynková. An impact of 3D computer and 3D printed models on the students' success in spatial ability and geometry testing. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03748981

HAL Id: hal-03748981

<https://hal.science/hal-03748981v1>

Submitted on 10 Aug 2022

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An impact of 3D computer and 3D printed models on the students' success in spatial ability and geometry testing

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The aim of the presented study was to measure whether 3D computer and 3D printed models could improve students' performance in testing their spatial ability and what brings the greater benefits. This study employed a quasi-experimental approach with a total of 25 secondary school students. A pre-test was employed to determine each student's level of spatial ability, namely the Mental Rotations Test. All students were taught the topic of three-dimensional geometry using 3D physical printed models, 3D virtual computer models or 2D drawings of 3D objects. A newly designed 3D geometry post-test was used in two groups of students - the first group was allowed to manipulate 3D printed models, the second group was allowed to use 3D computer models during the test. The group with 3D computer models outperformed peers with 3D printed models but a statistically significant difference was not found. Afterwards, the same students were tested again using the identical 3D geometry test without any visual aids. It was concluded that the 3D computer models provided statistically significant higher scores in comparison to the absence of any visual aid.

Keywords: Spatial ability, 3D printed models, 3D virtual models, geometry.

Introduction

Secondary school geometry is traditionally focused on studying geometric relationships in the two-dimensional space, i.e. planar geometry, and on studying solid geometry, i.e. spatial geometry. To study geometry is a big challenge and even if we live in the three-dimensional world it is difficult to work with three-dimensional objects especially when they are depicted on two-dimensional display. To be successful in the solving geometry problems spatial ability plays the main role. According to Lean and Clements (1981), *spatial ability* is the ability to formulate mental images and to manipulate these images in the mind. Very similar definition is provided by Linn & Petersen (1985) that is spatial ability generally refers to skill in representing, transforming, generating, and recalling symbolic, nonlinguistic information. Sorby (1999) distinguished *spatial ability* and *spatial skills*. Spatial ability is considered as the innate ability to visualize, whereas spatial skills are learned and acquired through training. Anyway, the both terms are very closely related and it is very difficult to distinguish them. In compliance with the literature, I will uniformly use the term spatial ability and will not examine the way in which the ability was acquired.

Spatial ability is a very essential part of intelligence and it is proved that continuous training has a great effect on its enhancement (Maresch & Posamentier, 2019, p. xv). Many studies undoubtedly proved that spatial ability of persons of different age can be trained (Maier, 1998). According to Sorby & Baartmans (2000) spatial ability can be improved through the targeted engineering graphics courses where diverse spatial activities are used ranging from manipulation of concrete models to computer visualization activities. Alias et al. (2002) state that spatial ability can be fostered through activities predominantly consisting of free-hand sketching and object manipulation. Spatial

ability is important for engineering, design, or other technology disciplines. It has been found to be very useful to a students' success in engineering related subjects as mathematics, engineering drawing, or computer-aided design (Alias et al., 2002). Moreover, spatial ability involves our mathematical, verbal, and logical capabilities and it is even crucial for our everyday life. We need the spatial ability to be able to orient ourselves in the environment, to understand the spatial relations among objects, to solve everyday tasks such as packing, moving, and many more.

The extensive research on spatial ability mainly based on factor analysis studies has resulted in the detailed categorization of spatial ability and its factors (Lohman, 1979). However, there was not a clear consistent model of different subcomponents of spatial ability. To illustrate this fact, let us mention that based on mental processes which are used for solving certain tasks McGee (1979) described two major components (factors) of spatial ability – *spatial visualization* and *spatial orientation*. On the other hand, the classification proposed by Lohman (1979) consists of three basic spatial ability factors – *spatial relation*, *spatial orientation*, and *visualization*. Five major factors of spatial ability – *visualization*, *spatial relations*, *closure speed*, *flexibility of closure*, *perceptual speed* were detected by Carroll (1993). Yilmaz (2009) provides a picture of a comprehensive model also with some another components of spatial ability. It seems that the number of underlying factors of spatial ability varies from study to study. However, *visualization*, *spatial relation*, *mental rotation*, and *spatial orientation* are subcomponents of spatial ability which are nowadays often designated as the relevant (Maresch & Posamentier, 2019). I will consider these subcomponents also in my research.

If we need to move or to alter in our mind some parts or the whole mentally presented objects, then it is considered as a spatial visualization task. Spatial visualization is the ability to imagine manipulating, moving, rotating, twisting, or inverting objects without reference to one's self. It means that the imagined object or its parts are moved or changed in our minds. Spatial relation works with the mental comparing of objects and with the identification of object parts which fits together. This subcomponent is not completely independent from the visualization subcomponent. The subcomponent mental rotation is the ability to imagine rotating a two and three-dimensional object or figure. Finally, the factor spatial orientation requires one's ability to imagine the appearance of an object from different perspectives. In other words, the imagined object does not mentally move, it remains the same and we mentally move ourselves to different viewpoints. Nowadays, the identification and description of the strategies for solving spatial problems emerge as a very interesting topic and it is brought into focus of the researchers. The classic research methods on factors of spatial ability assume that the spatial ability tasks of some category are solved using the same intended strategy. From the literature (Maresch & Posamentier, 2019) and also according to my own experiences with students, we know that geometric tasks are solved differently by different individuals.

In my research, I focus on the measurement and improvement of spatial ability of secondary school students. On the ground of the study, I include the optimal training and teaching methods into mathematics instructions to improve students' spatial ability. The objective of my research is to measure whether the 3D computer models and 3D printed models could improve students' performance in testing their spatial ability.

Spatial ability and the use of dynamic geometry and 3D printed models in mathematics education

In the presented research, let us focus on secondary school mathematics, namely on the topic of solid geometry. This traditional geometric topic covers the study of two-dimensional and three-dimensional Euclidean space. It includes the study of properties of and relationships between geometrical objects in the plane and in the three-dimensional space. It includes the measurements of volumes of various solid figures such as pyramids, prisms, polyhedrons, cylinders, cones, truncated cones, or spheres, cross-section of solids, transformations of two- and three-dimensional shapes in the plane and the three-dimensional space. Students determine the relative positions between two geometrical figures – lines and planes. Very typical tasks of solid geometry are to determine the size of an angle formed by two rays, to find the size of a dihedral angle, i.e. the angle between two intersecting plane, or to compute the size of an angle between a plane and intersecting straight line. Usually students solve such tasks depicted in two dimensional situations; so the proper visualization and its correct interpretation are crucial here. In the Czech mathematics textbook, usually the oblique projections are used. An oblique projection is a simple type of parallel projection which produces two-dimensional images with the specific properties (Carlbon & Paciorek, 1978). It holds that parallel lines are projected into parallel lines. If a polyhedron is projected, usually some its face or faces are parallel to the image plane (then these faces are projected in true shapes and sizes and are not distorted). I insist on working with an arbitrary position of projected solids, i.e. solids can be viewed from above from the right, from above from the left, from below from the left, and from below from the right. This is usually neglected in mathematics textbooks and only one position of the viewpoint is used.

I use dynamic software GeoGebra in my mathematics instructions. Indisputably, GeoGebra belongs among DGS (dynamic geometry systems) which are the most widespread all over the world among teachers and students. It is open-source software which is easy to use and understandable even for the absolute beginners. According to my experiences, GeoGebra software can be used with a potentially positive impact in teaching and learning process especially in such cases where its dynamic features can be used. It offers basic functions to model solid figures or more complex three-dimensional situations. Moreover, the modeled situation can be arbitrary rotated so it is viewed from different viewpoints; so it can substitute the real physical model to a certain extent.

Although, the computer-aided education is very modern and popular and brings indisputably advantages to the process of education, physical object manipulation plays the important role in the learning geometry and enhancing spatial ability. Especially action oriented training methods that work with real models have always shown good results in the improvement of spatial ability (Maier, 1998). This can be based on the approach of embodied cognition which emphasizes that cognition involves a motor behavior (Schneegans & Schöner, 2008).

In my mathematics instructions related to geometry, I use 3D computer models together with physical 3D printed models of solid figures and of three-dimensional situations as a visual aid. The properties of geometrical objects are demonstrated in my instructions using 3D virtual models and physical 3D

printed models or students can use these models as a visual aid when they are solving the geometric tasks.

Methodology

As has been already pointed out, my aim is to measure students' success rate in geometric tasks; specifically whether and how much the 3D computer models and 3D printed models could improve their performance in testing their spatial ability. In order to explicate the effectiveness of visual aids, a quasi-experimental approach was chosen. 25 students (one class) in the third grade (17-18 years old) of a secondary school (grammar school) in the Czech Republic were involved in the experiment. The topic of solid geometry was taught for two months at the end of the school year 2020/2021. Partially it was a distance learning (3 weeks) which was caused by the worldwide pandemic situation; the rest was standard face-to-face education. All students were taught the topic of three-dimensional geometry (solid geometry) using 3D virtual computer models, 3D physical printed models (this visual aid was used only at school), or 2D drawings of 3D objects. The teaching method was the same for all students in the experiment realized by one teacher.

Firstly, the students took the *Mental Rotations Test* as a pre-test to determine their level of spatial ability. The students received the test approximately in the middle of the period when the teaching of solid geometry was realizing. The Mental Rotations Test is one of the most common instruments for measuring spatial ability. The original test was developed by Vandenberg & Kuse (1978) and it contains 20 items in five sets of four items. Each item consists of a criterion figure, two correct alternatives and two incorrect ones. The alternatives are always shown in a rotated position. Each item in the test is counted as correctly answered if both choices are correctly chosen. This eliminates the need to correct for guessing. The reliability of the test has been found satisfactory. In a sample of 3,268 adults and adolescents of age 14 year or older, was .88; in a similar sample of 336 subjects, the test-retest correlation was .83 after an interval of one year or more, and in an age corrected sample of 456 the test-retest reliability after a year or more was .70 (Vandenberg & Kuse, 1978). For the purposes of the research, I used a redrawn modified version of the Mental Rotations Test called MRT-A (Peters et al., 1995) with the official permission from the author Michael Peters. This modified version of the test consists of 24 items and the nature of the test is completely the same. The maximum score of the test was then 24 points.

Secondly, the students divided into two coherent groups took a newly designed post-test on solid geometry at the end of that two month period. According to the results from the Mental Rotations Test, it was shown that the both groups of students were close to equal. The first group of students (Group 1, $n_1=13$) was allowed to manipulate 3D printed models during the test as the visual aids; the second group of students (Group 2, $n_2=12$) was allowed to use 3D computer models during the test as the visual aids. Students of Group 1 are denoted by G1, students of Group 2 are denoted by G2. The 3D geometry post-test consists of five types of geometric problems in the three-dimensional space – the relative positions between two lines (12 individual tasks), the relative positions between a line and a plane (6 individual tasks), the size of an angle between two lines (3 individual tasks), the size of an angle between a line and a plane (3 individual tasks), and the size of an angle between two planes (3 individual tasks). All the geometric problems were depicted in two dimensional situations,

i.e. projected to the plane; the oblique projection was used in all cases and different positions of the viewpoint were considered. In Figure 1 you can see three concrete tasks of the relative positions of two lines. Students were asked whether the depicted lines (drawn in the auxiliary cube) are parallel, intersecting, or skew. In Figure 2 you can see three concrete tasks of the size of an angle between a line and a plane (again drawn in the auxiliary cube). Students were asked to determine the size of an angle without any algebraic calculation. Every single task was for one point, so the maximum score of the test was then 27 points.

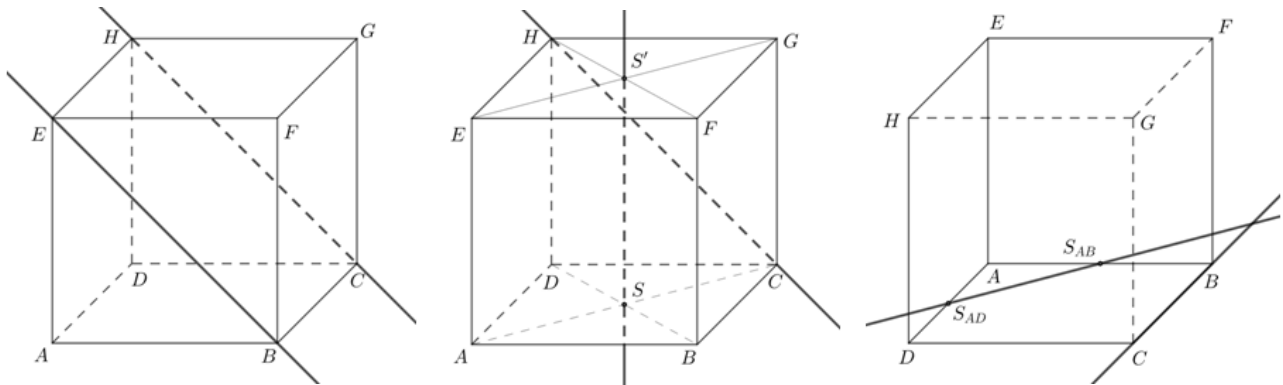


Figure 1: The relative positions of two lines – parallel, skew, intersecting, respectively

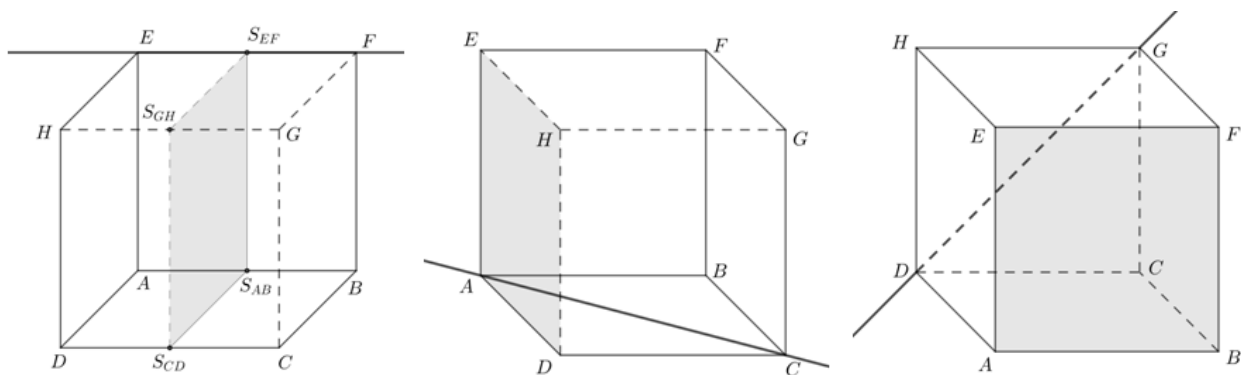


Figure 2: The size of an angle between a line and a plane – 90°, 45°, 0°, respectively

3D virtual computer models were created in GeoGebra software and 3D printed models were made on the 3D printer (Felix 3.0); the both by the author. The 3D virtual computer model is shown in Figure 3, on the left. Students were allowed to draw arbitrary lines into the prepared cube and they could rotate with the three-dimensional situation in GeoGebra software. 3D printed models are shown in Figure 3, on the right. Students were allowed to use them together with the sticks to model lines.

Thirdly, the same students were tested again using the identical 3D geometry test without any visual aids at the beginning of a new school year 2021/2022. For this final testing and for the interpretation of the results, the G1 is newly designated G1' and the G2 is newly designated G2'.

The relationship between the use of 3D computer models and 3D printed models and students' spatial ability was investigated in this research by focusing on the following research questions: What visual aid brings greater benefit to students when they solve geometric problems? Can 3D computer models and 3D printed models improve the students' performance in testing their spatial ability?

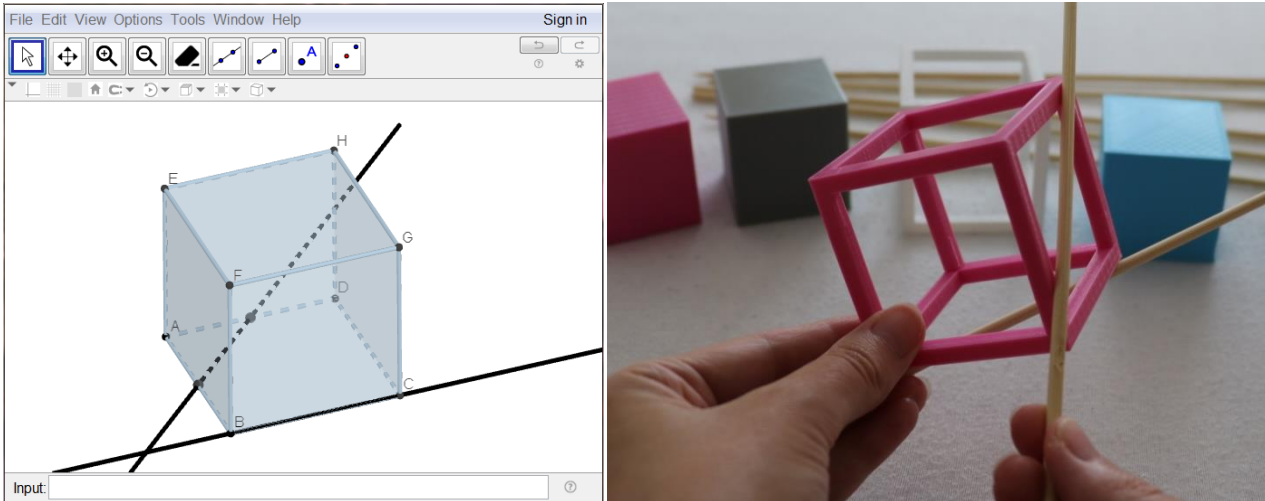


Figure 3: 3D virtual computer model in GeoGebra with allowed functions for drawing an object (a point, a line, a segment line, a midpoint) and functions for control (move graphics view, zoom in, zoom out, delete) on the left, 3D printed models with auxiliary sticks on the right

Students' scores in the Mental Rotations Test and in 3D geometry test were interpreted using the Mann Whitney U or the dependent *t*-test for paired samples. The significance level was chosen 0.05 in each test.

Results

There were no significant differences between the two groups ($G1, n_1=13$ and $G2, n_2=12$) of students as measured by the Mental Rotations Test. The $G1$ had a mean score of 16.92 and the $G2$ had a mean score of 16.25. The second method of data collection involved students' scores in 3D geometry test. The $G1$ that was allowed to manipulate 3D printed models during the test had a mean score of 21 and the $G2$ that was allowed to use 3D computer models during the test had a mean score of 22.92. The Mann-Whitney U test was used to compare differences in students' scores between the groups. The critical value of U at $p < 0.05$ is 41 which is lower than the test criterion of 52.5. Therefore, the result is not significant at $p < 0.05$. Students' performance can be observed in Table 1. Students are sorted according to their score (S) in the test and are assigned ranks (R). It can be visually observed that the group ($G2$) with 3D computer models outperformed peers ($G1$) with 3D printed models but a statistically significant difference between these two groups was not found.

Table 1: Students performance scores in 3D geometry test – Group 1 and Group 2

	G2	G1	G1	G1	G1	G1	G2	G2	G1	G2	G1	G1	G1	G2	G1	G1	G2	G2	G1	G2	G2	G1	G2	G2	
S	13	16	16	17	19	20	20	20	21	21	22	22	22	22	23	23	23	23	25	26	26	27	27	27	
R	1	2,5	2,5	4	5	7	7	7	9,5	9,5	12,5	12,5	12,5	12,5	16,5	16,5	16,5	16,5	19	20,5	20,5	23,5	23,5	23,5	23,5

The third method of data collection involved students' scores in 3D geometry test again. This time any visual aid was not allowed. The first group ($G1', n_1=13$) had a mean score of 19.08, the second group ($G2', n_2=12$) had a mean score of 20.25. The dependent *t*-test for paired samples was used to compare the means between groups $G1$ and $G1'$, and $G2$ and $G2'$. In the first comparison of groups $G1$ and $G1'$, the *p*-value is 0.00043. The result of the test is significant at $p < 0.05$. In the second

comparison of groups G2 and G2', the p-value is 0.00185. The result of the test is significant at $p < 0.05$. So the both tests showed that there are statistically significant differences between G1 and G1', and between G2 and G2' too.

Discussion

I analyzed the effects of using 3D computer models and 3D printed models on students' success rate in testing their spatial ability. Surprisingly, the group with 3D computer models performed better than the group with 3D printed models but a statistically significant difference between these two groups was not found. According to Katsioloudis et al. (2014), 3D printed models have greater positive effect on spatial ability than 3D computer models. This inconsistency could be probably caused by a small sample but still the difference was not significant. Using of 3D physical models is also supported from other researchers (Maier, 1998; Alias et al., 2002). The positive effects of using 3D virtual computer models on spatial ability over not using of any visual aid are consistent with earlier studies (Katsioloudis et al., 2014). Not to mention students' reactions on visual aids; based on my observations students were much more motivated to solve geometric tasks if they were allowed to use some visual aid. Moreover, final testing showed that students' results in the same 3D geometry test without any visual aids were worse even though they were solving the same geometric tasks.

Conclusion and future plans

This small quasi-experimental research showed that the use of 3D visual aids has a great potential in the process of mathematics education, namely in the topic of solid geometry. The study resulted in favor of 3D virtual computer models. It was demonstrated that 3D computer models can help students to better understand three-dimensional geometry. On the ground of this study, I include the optimal training and teaching methods into my mathematics instructions to improve students' spatial ability. I plan to continue using the both types of visual aids. I also plan to repeat these experiments with bigger samples and with modified versions of the 3D geometry tests. The reliability and the validity of newly designed tests is also planned to be measured.

Acknowledgment

The paper was supported by the project PROGRES Q17 Teacher preparation and teaching profession in the context of science and research and by Charles University Research Centre No. UNCE/HUM/024.

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