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Nurturing Pre-service Teacher's Awareness About Students' Mathematical Thinking Through Lesson Study

Rüya Savuran¹ and Mine Isıksal-Bostan¹

¹Middle East Technical University, Faculty of Education, Ankara;

ruyasay@metu.edu.tr; misiksal@metu.edu.tr

The current study aims to reveal the development of the pre-service teacher's awareness on students' strengths and weaknesses in learning the limit concept through lesson study. We focused on a participant's development within instrumental case study. The findings showed that the two phases of lesson study including planning and enacting contributed to the development of the pre-service teacher's awareness on students' strengths and weakness in learning the concept of limit through different aspects.

Keywords: Pre-service teacher education, specialized knowledge, knowledge of features of learning mathematics, lesson study

Introduction

The common idea of many alternative views - how teachers reveal their knowledge rather than what they know - is one of the reasons that makes mathematics teacher knowledge specialized (Scheiner et al., 2019). This specialized knowledge includes mathematical and pedagogical content knowledge as well as reflecting this knowledge on practice. In this structure, one of the most important and comprehensive knowledge can be considered as understanding the nature of learning mathematics which includes students' mathematical thinking, their interaction with content and their strengths and weaknesses in learning the concept (Carrillo-Yañez et al., 2018). The literature has revealed that mathematics teaching, which is carried out in order for students to make sense of the concept and provide a correct conception, is a very complex process (Cengiz, Kline, & Grant, 2011). Therefore, teachers need to attend students' mathematical thinking and make instructional decisions considering students' thinking (Sherin, Jacobs, & Philips, 2011).

On the other hand, the concept of limit can be considered as a cornerstone and one of the comprehensive concepts in mathematics to conceptualize related mathematical concepts. Besides its central position in mathematics (Cornu, 1991), the concept comprises many topics all of which are based on a common basis, either explicitly or implicitly. One of these topics is indeterminate forms (e.g., $\frac{0}{0}$, $\frac{\infty}{\infty}$, 0^∞ , 1^∞) and accordingly the difference between indeterminate-undefined forms in limit. In addition to the difficulties experienced by the students regarding the limit concept itself, students have difficulties in knowing the distinction between the notions of undefined and indeterminate, and they often tend to use these terms interchangeably (Jaffar & Dinyal, 2011). Moreover, the limited literature about these notions indicated that most of the students calculate limits in the indeterminate forms and use L'hospital rule without questioning the reason behind the forms and related calculations (Hulliet & Mutemba, 2000; Arce, Conejo, & Ortega, 2016). Considering these difficulties, teachers need to be aware of students' mathematical thinking, strengths and weaknesses in teaching the concept and its related notions to provide an effective learning environment. Teachers

can gain this awareness through the combination of knowledge of content and students, more teaching experience with different level of students, and interaction with their colleagues. Since these notions are quite complex even for pre-service teachers (Hulliet & Mutemba, 2000), this awareness could be acquired while being a pre-service teacher before performing the teaching profession.

One of the ways to nurture the awareness of pre-service teachers for learning students' strengths and weaknesses (Lee, 2019) is lesson study development model which is a cycling process that a group of teachers/pre-service teachers work collaboratively (Stigler & Hiebert, 1999). It includes four main phases: (1) investigation: setting a learning goal in consideration that students have difficulty in understanding, (2) planning: building a research lesson plan in collaboration, (3) research lesson: conducting the lesson plan in a real classroom where one of the group members teach the concept while others record students' reactions and take notes of their thinking processes, and (4) reflecting and discussing on how effective the lesson was in facilitating student learning (Stigler & Hiebert, 1999). This phase might be followed by discussions on how to improve lesson plan, and the cycle might be applied again to revise missing points. Bearing all these in mind, the current study aimed to develop pre-service teachers' knowledge of students' mathematical thinking for the notions of indeterminate-undefined forms through lesson study.

Theoretical Framework: Mathematics Teachers' Specialized Knowledge

As a part of a longitudinal project, we adopted the model of Mathematics Teachers Specialized Knowledge which includes two main domains as mathematical knowledge and pedagogical content knowledge encircled around the belief towards mathematics and teaching mathematics. There are three sub-domains for each knowledge domain: knowledge of topics (KoT), knowledge of structure of mathematics (KSM) and knowledge of practices in mathematics (KPM), which are under the domain of mathematical knowledge. On the other hand, knowledge of features of learning (KFLM), knowledge of mathematics teaching (KMT) and knowledge of mathematics learning standards (KMLS) are under the domain of pedagogical content knowledge. For an effective mathematics teaching, in addition to mathematical knowledge, having knowledge about the students' interaction with the content and how they learn is actually one of the basic requirements of being a good mathematics teacher. Therefore, we paid attention to KFLM which is concerned with the areas of knowledge about mathematical content, the process of learning the content, and the characteristics of this process, and how students interact with the mathematical content in this learning process (Carrillo-Yañez et al., 2018). It should be indicated that the focus of KFLM is not solely students, rather the focus of KFLM is the interaction between the content and students (Carrillo-Yañez et al., 2018). There are four indicators for KFLM including learning theories related to the content, strengths and weaknesses in students' learning relation with the content, the students' interaction with the content, and the student's motivation, expectations and interests with regard to mathematics. In this paper, we focused on the pre-service teacher's awareness of students' strengths and weaknesses in the context of learning indeterminate-undefined forms.

Bearing the lesson study, theoretical framework and the aim of study in mind, we addressed this research question: (1) What is the nature of pre-service teachers' knowledge of features of learning mathematics in the learning indeterminate-undefined forms in the context of limit concept during the

lesson study process? (a) How does lesson study contribute to the nurturing of the pre-service teacher's awareness about students' strengths and weaknesses in learning indeterminate-undefined forms in the context of limit concept?

Method

The study was an instrumental case study (Mills, Durepos, & Wiebe, 2010) in which we used lesson study as an instrument to provide insight into development of the pre-service teacher's awareness on students' strengths and weaknesses. To deepen the analytic stance, we focused on one of the three pre-service teachers, Mila, who was chosen purposefully for several reasons, one of which was the fact that she didn't have any experience on making a lesson plan or teaching the concept of limit in addition to her eagerness to learn the concept and to hear the students' voices in learning mathematics.

The study was carried out in the last semester of the undergraduate mathematics teacher education program, as the pre-service teachers take the fundamental mathematics and mathematics education courses until their last semester in the program. The procedure of lesson study was conducted in a similar way with the description in the introduction section. To nurture the pre-service teacher's awareness about students' mathematical thinking, based on the suggestions of the literature on improving teachers' awareness of students' mathematical thinking (Lee, 2019), the phases of the lesson study were designed with some additional steps such as promoting the group to think about interrelationships among important mathematical ideas and students' mathematical thinking.

Data Collection and Analysis

The data were generated through mainly observation of lesson study process including group discussions in planning and reflecting phases and teaching episodes in research lessons which were conducted in Turkish in two lesson study cycles. In addition, the field notes and lesson plans were used to support the claims generated from the data. The first author was the knowledgeable other in the lesson study process. We obtained the data through audio- and video-recording during the data collection process. After all the data was transcribed, we started the data analysis with partitioning of the snapshots of the pre-service teacher's interthinking and exploratory talk (Littleton & Mercer, 2013), which includes contributions of students' mathematical thinking process considering their strengths and weaknesses within the collaborative process of lesson study.

We used thematic analysis with deductive approach in which we determined the codes and the theme before analyzing the data. We used the indicator of KFLM-awareness about students' strengths and weaknesses- as a theme in analyzing data. The codes were determined as (i) lack of awareness- the general expressions about students' mathematical thinking or without consideration of students' strengths and weaknesses, and (ii) being aware- interpretative comments on students' mathematical thinking and the relationship between students' learning and the content in planning. In a similar vein, the codes for enacting were determined as (i) lack of awareness-acting without consideration with students' strengths and weaknesses, and (ii) being aware- acting with considering students' mathematical thinking, using their strengths to overcome their difficulties. In this way, we dealt with the nurturing process from lack of awareness to being aware in the journey of the pre-service teacher.

Findings

We deal with the nurturing process of lesson study in two parts: *planning* (determining lesson goals and planning of the research lessons), and *enacting* (implementing the research lesson and writing reflections).

Planning

The topic of indeterminate-undefined forms was the subject that Mila herself had difficulty in understanding, and furthermore she developed her mathematical knowledge on this subject during the lesson study process. To put it right from the start, Mila actually developed her knowledge of students' strengths and weaknesses based on her own learning process in the past.

We observed in the pre-interview that Mila, similar to other group members, had some confusions about the difference between indeterminate and undefined forms. Therefore, her awareness about students' confusions between these two notions was gained (or developed) in accordance with the awareness of her own lack of knowledge. The following excerpt showed a starting point of the discussion on these two notions in the planning phase:

Mila: For example, let's imagine that $\frac{0}{0}$ equals x . Multiply the ins and outs. x can take more than one value.

Participant 1: Ah yes.

...

Participant 2: When you spoke of indeterminate, something came to my mind. I realized that it was undefined. I realized it after reading the article: Undefined and indeterminate are two different things.

Mila: Honestly, I didn't know either, I think students might get confused.

...

Researcher: Then what do you say about the number divided by zero?

Participant 2: Undefined.

Participant 1: Professor, I think first of all let's start with the difference between undefined and indeterminate? Let's start, as in the way Mila said.

In the excerpt above, we did not pay attention to the mathematical correctness of the expression. Rather, we focused on how the pre-service teacher considered students' mathematical thinking while constructing the content in planning. The development was observed through development of her mathematical knowledge.

In addition, the observation of other group members' lessons provided her to see beyond her thoughts about students' strengths and weaknesses in learning the concept of limit. The observation of the Participant 2's research lesson which was just before Mila and then watching it on video enabled her to make more mathematical contributions to students' learning about mathematics during the planning phase.

Researcher: You can also explain it in that way when you are explaining. You will start with indeterminate forms. There was a question the students asked Participant 2 as "why infinity divided by infinity does not equal to 1". Participant 2 will ask you if it's a sufficient explanation.

Participant 2: I said that if there is only one number, there is infinity divided by infinity which equals to the number of a , then infinity equals to ' a times infinity'. Well, do I know this a , but it may be 1, it may be 2, it may be 1000, I asked whether I could say something definite for a , they said no, so I said there would be indeterminate.

- Mila: This is not the answer of the student's question "why not 1", instead it looks like the answer to the question of "why does this indicate indeterminate"!
- Participant 1: Let's say, here is already undefined! Again, infinity divided by infinity must be one for the inside and outside product.
- [*Researcher intervened here*]
- Participant 2: I should have said infinity can be described as an increase; it is not an infinite number.
- Mila: We need to be particularly careful about the infinity; a bit as an adjective! So, we can start from there and talk about indeterminates.
- Researcher: So, you can start here!
- Participant 2: For example, they asked why 1 to the power of infinity and zero times infinity indicated indeterminate.
- Mila: They are right about the 1 to the power of infinity, we should definitely mention it!

It can be understood that Mila's ability to express more mathematically what aspects of students can be strengthened and which weaknesses can be eliminated shows the change that occurred during the planning phase.

Enacting

In implementing the third lesson plan aiming to grasp the concept of continuity by establishing mathematical relations and to use it together with limit in mathematical applications, Mila started her lesson based on her experiences. In other words, she knew that it would happen because she got to know the students from the narrations of her previous friend and told them about their learning only out of curiosity.

- Mila: Now I guess you guys were pretty curious about indeterminate.
- Student 1: Yeah!
- Mila: Yes, I've already had such a feeling, so I want to examine these indeterminates together with you. First, let's talk about the $\frac{0}{0}$ and $\frac{\infty}{\infty}$ uncertainty. What do you think might be the reason for this indeterminate?
- Students: [Grumbling sounds of what they don't know]
- Mila: So, I'm going to write you two functions and ask you to find the limit at the point I gave you.

In the excerpt given above, Mila tried to make them notice the different results of the same limit forms which caused the indeterminateness. By wanting them to calculate the limits of the given points of the functions, it can be understood that Mila considered students' strengths about the concept of limit. Moreover, it should be indicated here that this is an example about Mila's awareness about students' expectations about mathematics.

The example given above is an example from a planned situation in implementing the lesson plan. However, our main expectation was to observe the pre-service teacher's awareness and actions in unplanned and unexpected situations. It would show how the pre-service teacher use her awareness in her teaching. In the enacting phase of the first cycle, there were two situations she faced with in this way. In the continuation of the indeterminate form of $\frac{0}{0}$ and $\frac{\infty}{\infty}$, she observed that students tended to make mistakes while exemplifying or demonstrating indeterminates based on the question frequently asked by students in the implementation of the second lesson plan.

- Mila: If I ask you a question now (She turned to her presentation but realized that the question she wanted was not there). Yes, I didn't write it here... (Closing the presentation screen) Forget about it then. I will write the question myself. Let's say

I take $f(x)$ to the $g(x)$ as limit x goes to infinity ($\lim_{x \rightarrow \infty} f(x)^{g(x)}$) What if $f(x)$ was 1 for me as the limit goes to x . If limit x goes to infinity and $g(x)$ is infinity for me, what will the result be? So, when I think about this $f(x)$ to the $g(x)$ structure...?

Student 1: 1 to infinity (1^∞).

Mila: 1 to infinity becomes infinity. Because I know from the limit rules that I can distribute this limit and the limit $f(x)$ to the $g(x)$ would be 1 to the infinite for me.

Student 2: I think it should be 1 too.

Student 3: I've always wondered about that too.

Mila: You were wondering, right? I heard it. There is actually a reason for this. What we said is that the main reason for the indeterminates is that we get different results in the same form of the limit. In fact, nothing different from the ones here (she showed what she wrote about $\frac{\infty}{\infty}$) appears for 1^∞ . Now I will write you two examples.

Up to now, the excerpt showed that she tried to use students' strong sides to teach them the logic of indeterminate forms. In addition, it can be said for this introduction that she considered the students' interest (emotional aspects of learning the concept) which was their curiosity about why the forms of 1^∞ is not equal to 1. In the continuation of the excerpt, she wrote the functions: $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$ and $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{n^2} = \infty$. While she aimed to use their strengths, students got confused about why they look for the same results for different functions. The following excerpt showed her awareness about the aspects related to students when she answered students' questions.

Mila: So that's why 1 to the power of infinity is indefinite.

Student 1: But that's not 1 to the power of infinity, right? We said 1 to the infinitely indefinite thing, isn't it something different? e is here.

Mila: Hmm, is it confusing that it's equal to e ?

Student 1: No, there is a number called n .

Mila: Yes.

Student 1: 1 is not infinity, I mean, I don't think they are the same thing!

Mila: There is a number called n , is your question related to it?

Student 1: n goes to infinity or exactly 1 to the infinity is not equal to this.

Student 3: He means something (talking about his friend) different in two functions. As if the two functions are different, it's logical that we find different results anyway, isn't it?

Mila: Hmm I got it! But I'm telling you this. So, let's look at the equation I got over here, okay (it shows the resulting limit e)? When I look over there the limit n goes to infinity and that inner side is equal to 1 for me. Therefore, when I overwrite it here, I get the 1 to the infinity form. Here I got 1 to the infinity, and what happens when I get the same form of other functions? Here I am writing the same thing again (showing the second function). Here, my inner side became 1 and my upper side became infinity. In other words, they seem to be different functions, but since we do not perceive infinity as a number, we say that it is increasing gradually, but we do not know how much it increases, so this is the reason why it creates indeterminate.

Student 1: I get it!

Student 2,5,7: Yeah, I understood perfectly!

In the face of this unexpected situation, the pre-service teacher drew an unplanned path for herself by using the strengths of the students and at the same time establishing mathematical connections. It showed that gaining awareness of students' understanding is closely related to her knowledge of practices in mathematics which can be explained as awareness of mathematical reasoning on how to explore mathematics by seeing connections. At the beginning of lesson study process, Mila

commonly preferred to reason by means of counterexample not considering students' learning whenever it was asked how she would show one's correctness or incorrectness. Developing her awareness of students' learning mathematics provided her to use students' strengths for demonstrating the concept in teaching. The fact that the pre-service teacher is aware of the example she gives to create a mathematical knowledge is closely related to the students' awareness of using calculation, which is one of students' strengths.

As a result, the lesson study process contributed the development of the pre-service teacher's awareness on students' strengths and weaknesses in learning the notions through various aspects. The *planning* phase provided her to discuss on students' learning and develop her awareness through different perspectives in the discussion with her colleagues. The *enacting* phase supported her development by enabling her to see what she had discussed and planned in a real classroom through students' interaction with mathematical content.

Discussion

The findings showed that the lesson study process promoted the pre-service teacher's knowledge for gaining awareness of students' strengths and weaknesses in learning the topics of indeterminate-undefined forms. In planning, at the beginning of the lesson study process, Mila used her own experiences related to learning the concept. However, such views can be considered as limited, in other words, lack of awareness, for effective mathematics teaching, because attentions of both pre-service and in-service teachers to students' mathematical thinking and learning process in interaction with mathematical content provide them to see beyond what they think about mathematical knowledge and enable them to make instructional decisions (Sherin, Jacobs, & Philips, 2011). As the literature supported (Guner & Akyuz, 2020), working collaboratively on students' learning in planning might promote to open their eyes to observe students' strengths and weaknesses in interaction with mathematical content. Accordingly, the development of Mila's awareness enabled her to make the instructional decision to create her own path by being aware of the strengths of the students rather than applying the lesson plan as it is in the enacting phase. The indicator of KFLM does not only include knowing where students have difficulties or strengths; rather it covers to enact this knowledge combining with content knowledge in making instructional decisions (Carrillo-Yañez et al., 2018). Considering that Mila took instructional decisions to help students overcome difficulties by leveraging their strengths, it can be concluded that the enacting phase of lesson study presented a great opportunity to develop this side of knowledge of students' learning mathematics.

The study focused on one pre-service teacher within classroom discussion during the lesson study process. In future research, the focus could be given on more participants in classroom discussions. Furthermore, the study contributed to the limited literature about the pre-service teacher's knowledge of indeterminate-undefined forms in the context of limit concept. In addition, as a part of the nature of the limit concept, there is a powerful relation between the development of mathematical knowledge (KPM, in this study) and increasing awareness of students' learning mathematics. Therefore, the future studies which examine other notions within the nature of the limit concept considering other indicators of MTSK can shed light on the literature on mathematics teacher knowledge.

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