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► To cite this version:

Lorenzo Castilla, Nuria Climent, Veronika Hubeňáková, Laura Rifo, Ingrid Semanišínová. Preservice teachers' knowledge for teaching uncertainty: cases from Slovakia and Spain. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. hal-03748715

HAL Id: hal-03748715

<https://hal.science/hal-03748715>

Submitted on 9 Aug 2022

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Preservice teachers' knowledge for teaching uncertainty: cases from Slovakia and Spain

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In this paper we inquire about the knowledge of the meanings of probability of preservice primary and secondary school teachers from Slovakia and Spain. At the end of their training on this content, we wonder about their conceptions of randomness and how they quantify its uncertainty. From the results obtained through a questionnaire filled in by 89 preservice mathematics teachers of Primary and Secondary in both countries and the interview to some of these informants, we highlight that they associate randomness mainly to future events and drawing experiments; they show knowledge of the classical meaning of uncertainty and little acquaintance with the subjective meaning. Besides, they seem to separate uncertainty from probability assignment.

Keywords: Probability, uncertainty, preservice teachers, Primary and Secondary School.

Introduction

Compared to other topics in mathematics, probability is especially difficult to understand. Unlike logical reasoning, which deals with statements which are either true or false, probabilistic reasoning deals with events for which there is no complete certitude. The intuitions underlying concepts of probability, such as dependence or fluctuations, are usually obscured by mathematical counting methods. These reasons, among others, make this a particularly difficult content item for both students and teachers, as Batanero *et al.* (2004) and the references therein, point out. These authors note that one aspect of teacher training concerning probability on which special emphasis should be placed is epistemological reflection on the concepts to be taught. We concur, and stress that understanding the inherent nature of probability should be one of the main goals in teaching the topic.

Probabilistic contexts can be understood from different perspectives. One major approach is the classical approach, which imposes the need for objective mathematical rules to explain random processes; in addition, the epistemic approach sees probability as the degree of personal belief about the occurrence of an event, which is dependent on the information available, and constrained by theoretical decision rules. These two major trends can be included among what many authors have called the different perspectives on, or meanings of, probability, which also include the classical, frequentist and subjective approaches, among others (Batanero *et al.*, 2016).

In our approach, following the Mathematics Teachers' Specialised Knowledge model (MTSK, Carrillo *et al.*, 2018), teachers' knowledge of the different meanings of probability is located in their Knowledge of Topics (KoT). Numerous researchers have highlighted that, in order to offer students a richer learning experience, it is important for teachers to develop a broad understanding of the range of meanings of mathematical objects (e.g. Martín-Fernández *et al.*, 2019; Thompson, 2016). In the

case of the probability theory, realizing that uncertain situations can be described in terms of probabilistic models is especially significant. Ultimately, from the teachers point of view, we think that probability should be identify as the main mathematical tool for dealing rationally with uncertainty in daily life contexts.

This work is a descriptive exploratory study which focuses on the knowledge of preservice primary and secondary school teachers from Slovakia and Spain at the end of their training on probability. The research questions are: What are their conceptions of randomness, and how do they quantify uncertainty? In other words, our goal is to explore to what extent preservice teachers consider probability as a measure of uncertainty or of partial information.

Theoretical framework

We are especially interested in the subjective interpretation of randomness and its use in decision-making, as a way of teaching inferential reasoning. This approach is based on the probability theory initially presented by Ramsey (1926) and De Finetti (1937) and developed by authors such as Jeffrey (1965) and Savage (1967). Under this theory, randomness is associated to any observation or experimentation for which the observer has no total information or certainty. That is, randomness is a property not related to a situation, but related to the judgement of the subject that observes. That judgement translates in conditions governing a set of personal preferences so as to obtain coherent behavior in uncertain situations. Informally, such preferences and coherence refer to actions and consequences depending on the uncertain events considered. Technically, those concepts are defined inside the axiomatic for decision theory and based on concepts such as betting and Dutch book.

From the subjectivist point of view, probability is seen as a model for partial information, or uncertainty, of the decision-maker. This means that, e.g., two physicians could assign different coherent sets of probabilities (in fact, for possibly different diagnoses) and consider treatments and consequences in different ways according to their personal judgement.

As any other mathematical problem, random contexts can be modeled by different approaches inside the probability theory, the main two being the classical one, based on equiprobability and counting methods, and the decision theoretical one, based on personal information and coherence axiomatic. Their implications in statistical education are discussed in (Batanero et al., 2016).

These approaches are related, though. The axioms for coherence have been shown to be equivalent to Kolmogorov's, which means that the usual rules of the calculus of probability are coherent, and, inversely, a coherent assignment of probability satisfies the usual properties, for instance, that the probability of the union of disjoint events is the sum of their probabilities. Additionally, the classical interpretation of probability, which assigns equal probability to the elementary events, is a particular case of subjective assignment, when we consider conditions such as the symmetry of the results of an experiment or lack of information. Similarly, the acceptance of conditions that guarantee the limit frequency as an adequate assignment of probability for a given event is a subjective choice, relative to the subject that is mathematically modeling the problem. Finally, the subjectivist interpretation implies that our own assignment of probabilities can change when updating our knowledge. Both prior and posterior assignments must follow coherent consequential rules, equivalently to the relation between conditional probabilities and the Bayes rule.

Several authors mention that preservice teachers are unfamiliar with different meanings of randomness and probability and that they need to be aware of these approaches, because they influence their students' reasoning (e.g., Chernoff & Zazkis, 2011; Batanero, 2016). That is why, in this paper, we focus on the meanings that preservice teachers attribute to randomness and its measure as mentioned above. More specifically, we explore the preservice teachers' conceptions of uncertainty in relation to certain daily situations, and how they quantify it. We locate this knowledge as distributed among knowledge of notions of uncertainty (KoT – definitions, properties, and their foundations), situations that this knowledge models (KoT – phenomenology) and calculation procedures for quantifying this knowledge (KoT – procedures). This is consistent with Di Bernardo *et al.* (2019) and the references therein, where subjective knowledge about probability and knowledge of connections between different meanings was divided into: knowledge of definitions, properties and their bases (intra-conceptual connections between different meanings); knowledge of procedures (calculation of probabilities); knowledge of representations; and knowledge of phenomenology (modeling a situation from the appropriate meaning).

Methodology

Our informants were 89 preservice mathematics teachers (PSTs), in Spain and in Slovakia. The Spanish group consisted of 43 preservice teachers at the University of Huelva, who had completed a course on statistics and probability as part of their degree. Both the classic approach to probability and a preliminary introduction to the subjective perspective had been covered during the course. The Slovak group of informants consisted of 46 preservice secondary school teachers (grades 5-13) from Pavol Jozef Šafárik University in Košice. All the Slovak PSTs had passed the compulsory classes in statistics and probability. During those courses, they were exposed to the classical definition of probability, and worked with the Kolmogorov axioms. The subjectivist view was not directly dealt with on that course. The training of both groups of students in probability has not been equivalent, neither in depth (greater in the case of the preservice Slovak secondary teachers) nor in that the preservice Spanish primary teachers have received some notions of subjective probability (idea of uncertainty and its relation to decision-making). Both groups have in common that they have received their last course on probability. Our goal is to explore whether differences in training and context are reflected in differences in their understanding of probability as uncertainty.

The information was collected through a questionnaire with four sets of questions. We analyze in this paper the first two (Table 1). The first question focused on the concept of randomness in a variety of situations including the result of a lottery taking place both in the past and the future, a social or economic index in the past, the location of a historical event, and a weather forecast. For each situation, we then asked for the PSTs' personal estimation of the probability of specific events.

In the case of Spain, the questionnaires were completed in the training classroom, while in Slovakia the questionnaires were completed online due to COVID-19 restrictions. One group of Slovak PSTs (13 participants) was explicitly asked to explain precisely the thinking behind their responses at the end of each question. A few days after they finished the questionnaires, they had a group discussion in which some of the PSTs explained the reasoning for the answers given in the questionnaire.

Once the data for each country had been collected, they were analyzed by the corresponding country team. Later, they were jointly analyzed by all the authors of this work. The main aspects of this

analysis are: the students' conception of randomness, the assignment of probabilities and the coherence of this assignment. In Table 2 we point out how we analyzed each of these aspects, and explain them with more detail in the Section Results.

Table 1: Questions for information gathering

Task 1	Task 2
<p>Indicate if you think that the following situations are random (A) or non-random (N), according to your current information.</p> <p>A: () Next week's result of the 1st prize in the National Lottery.</p> <p>B: () Last week's result of the 1st prize in the National Lottery.</p> <p>C: () The birthplace of Alexander the Great.</p> <p>D: () The temperature in Bratislava/Málaga tomorrow at noon.</p> <p>E: () The Euribor index at close of business yesterday.</p> <p>F: () The proportion of primary school pupils diagnosed with attention deficit disorder in 2019 in Slovakia/Spain.</p>	<p>Indicate your estimation of the probability that each of the following statements is true. Assign the numerical values that you consider appropriate according to your current information.</p> <p>(a) Next week the 1st prize in the National Lottery will be the number 89342.</p> <p>(b) Last week the 1st prize in the National Lottery was the number 89342.</p> <p>(c) Alexander the Great was born in Greece.</p> <p>(d) The temperature in Bratislava/Málaga tomorrow at noon will be between 18°C and 20°C.</p> <p>(e) The Euribor index closed down yesterday.</p> <p>(f) The proportion of primary school pupils diagnosed with attention deficit disorder in 2019, in Slovakia/Spain, is greater than 5%.</p>

Table 2: Process of analysis

Aspect	Research question
Conception of randomness	Is randomness associated with uncertainty or partial information, as opposed to situations where the exact probability can be calculated? (KoT, notion of randomness – <i>definitions, properties and its foundation</i>) Task 1: Comparison of events A,B vs C,D,E,F
	Is randomness assigned to past / future events? (KoT, situations that randomness models – <i>phenomenology</i>) Task 1: Comparison of events B,C,E,F vs. A,D
Assignment of probabilities	To which events is a subjective probability assigned, and to which a value obtained by Laplace's rule? (KoT, calculation procedures – <i>procedures</i>) Task 2: Analysis of the students notes
Coherence (in decision making)	Is probability assigned only to the events associated with random contexts? Task 1 and Task 2: Comparison of corresponding events – focusing on non-random contexts with probability within the interval (0,1)

Results

Conception of randomness

Within this aspect, our goal was to identify the contexts that are believed to be random or nonrandom in the students' conception. Two kinds of contexts were stated in the assertions of the Task 1: drawing experiments (A,B) vs. daily life events (C,D,E,F), and events occurring in the past (B,C,E,F) vs events in the future (A,D). Both groups of students agree on what they consider random, as can be seen in Figure 1, where each bar represents the percentage of respondents who regard the situation as random.

The situation described in item A is a random draw, the prototypical context for exemplifying probability in introductory courses at school – and was consequently universally considered random, as expected. The situation described in B, identical to that of item A but formulated in the past, shows

a decrease in the assignment of randomness to about 65%. Events C, D, E and F were identified as nonrandom by most PSTs. The contexts of these situations do not offer the possibility of using combinatorics or frequencies, except perhaps for the context based on the weather, which has a slightly higher proportion. Moreover, neither the Spanish nor the Slovak PSTs had received the appropriate training for solving such tasks, nor had Slovak PSTs been introduced to subjective probability. In the Table 3, we associate the results of Task 1 for the past contexts (items B, C, E and F).

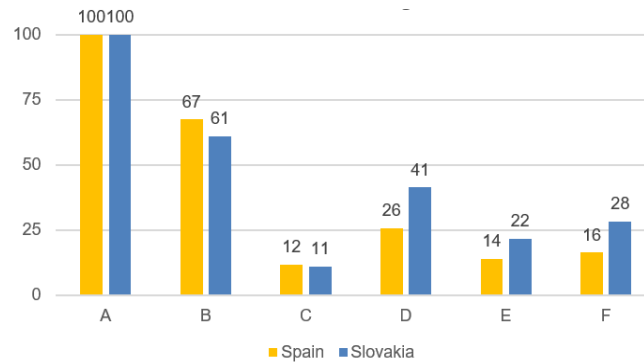


Figure 1: Proportion of answers (in percentage) which consider situations A-F as random

From the Table 3, we can see that 10 out of 43 Spanish PSTs (shaded orange) and 14 out of 46 Slovak PSTs (shaded blue) considered all past contexts as nonrandom. However, if situation B is excluded (a draw – usually connected with randomness) we get 28 out of 43 (18 + 10 shaded orange) Spanish PSTs and 23 out of 46 (14 + 9 shaded blue) Slovak PSTs. These numbers make up at least 50% of PSTs in each of the countries. Five PSTs from the group of 13 PSTs in the Slovak sample that commented on their answers argued that the situation is nonrandom because it happened in the past. The other argument was that a past situation is not random at all, since it is either true or false.

Table 3: Consideration of randomness for the past contexts.

		E – R				E – nR				Total
		F – R		F – nR		F – R		F – nR		
		ESP	SVK	ESP	SVK	ESP	SVK	ESP	SVK	
B – R	C – R	0	0	0	1	3	2	1	1	8
	C – nR	0	0	5	6	2	9	18	9	49
B – nR	C – R	0	0	0	0	0	1	1	0	2
	C – nR	0	1	1	2	2	0	10	14	30
Total		0	1	6	9	7	12	30	24	89

Assignment of probabilities

Two main methods of assignment of probabilities used by the PSTs emerge: the classical calculation from equiprobability and personal reasoning based on their previous knowledge.

For events A and B, a significant number of the PSTs in both countries assigned a probability based on the rules of combinatorics and classical calculus of probability: 33 out of 46 Slovak PSTs for event A, and 16, for B; 20 out of 43 Spanish PSTs for event A, and 12 out of 29 who assigned a probability less than 1, for event B. For the 13 PSTs in the Slovak sample who commented on their answers, a

few of them tried to use Laplace’s rule for the events C and D (4 PSTs in C, and 2 in D), but the corresponding values were chosen according to subjective considerations. For instance, the “*number of countries at that time*” was used for counting probability in event C, or “*according to the weather forecast, I expect that tomorrow’s temperature will be between 15 and 20, so the probability will be 3/5*”, for event D. No Slovak PST used Laplace’s rule in connection with events E and F. However, in situation F, two PSTs explained their answers in a way which combine estimation with personal experience, e. g.: “*5% means that every 20th child has been diagnosed with attention deficit in 2019, which is too many in my opinion. In my class we had nobody with such a condition*”, “*If we have 800 thousand pupils 5% means 40 thousand pupils at approximately 2000 schools, that is 20 such pupils at one school – it is too many*”. During the discussion with the 13 Slovak PSTs after the questionnaire filling, 2 PSTs stated that the Task 2 was too challenging: “*Task 2 makes no sense to me. I don’t know what to do*”; “*It was a big problem for me*”. In some of PSTs’ explanations, we could explicitly identify assignment of subjective probability: “*First, I tried to find some relevant number in a question about Alexander the Great. Then, I decided that he was either born in Greece or not, so I assigned a probability of ½. I used the same logic in the subsequent questions*”. Spanish PSTs made similar remarks for events they do not have any information about.

Coherence

Finally, the third aspect is related to the coherence of the answers. For each case, the following four possible options can occur: the situation can be considered random or not, and the probability assigned to the specific event can be 0 or 1, indicating certainty, or a number within the interval (0,1), indicating uncertainty. Table 4 shows the coherence state for each pair.

Table 4: Probability assignment and random situations.

	Random	Nonrandom
Probabilities within the interval (0,1)	coherent	incoherent
Probability equals 0 or 1	may be coherent	coherent

From the perspective of our research question, it is most interesting to look at incoherence and therefore to display how many PSTs assigned probability within the interval (0,1) to events in the situations described formerly as nonrandom. We can see that this incoherence is quite common among our informants (see Figure 2).

Conclusions

From the results of our study, we conclude that: 1) regarding the situations associated to randomness (KoT *-phenomenology*), past events seem to be considered mostly as non-random, with the exception of when they are linked to a kind of draw, in which case, especially among the Spanish PSTs, it is mostly considered as random; 2) regarding the assignment of probability ((KoT- *procedures*), the PSTs have little experience in using a subjective or any other approach, even though it can be very useful in daily life. It seems that PSTs do not consider that kind of reasoning to be within the scope of mathematical modeling; and 3) regarding coherence, the results may reflect a narrow use of the concept of randomness (KoT- *definitions, properties and its foundation*), which does not include situations of uncertainty. Although the probabilistic language can be used to make statements about uncertainty, randomness as a mathematical model is not used in daily contexts.

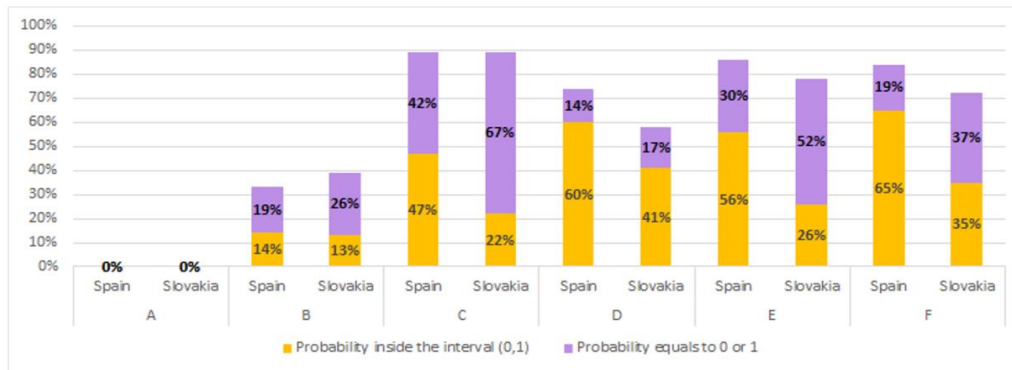


Figure 2: Probability assignation to events in contexts considered non-random

Therefore, PSTs' conception of randomness is not as deep and complex as might be desired. It enables them to use probability as a tool to inquire about contexts where Laplace can be utilized. This may be influenced by the fact that "Laplacian definition is echoed in today's textbooks" (Chernoff & Zazkis, 2011, p. 16). However, out of this box, most of the students do not develop a coherent probabilistic reasoning. Their knowledge of situations that can be modelled by uncertainty is restricted. They seem to be familiar with the classical meaning and do not consider the absence of information as uncertainty, although they use subjective arguments to justify their probability assignments. Contrastingly, they seem to separate uncertainty from probability assignment.

These results indicate that an axiomatic formation in probability, exclusively linked to the classical meaning (Slovak PSTs) leads to a limited knowledge of the topic, both in terms of the notion itself, as well as procedures and situations related to it (KoT – definitions; procedures; and phenomenology). On the other hand, a brief introduction to subjective probability (Spanish PSTs) does not seem to affect such knowledge. It seems that the knowledge of the PSTs is barely expanded in relation to that of secondary school students and does not acquire a specialized profile.

Multiple authors suggest activities for students which comprise subjective probability (e.g. Borovcnik & Kapadia, 2017; Martignon & Krauss, 2009). The reason consist on large presentation of subjective probability in daily-life. On the other hand, to this time, subjective approach is rarely represented in national curricula. Moreover, our results show that PSTs who received their last course on probability have not developed KoT concerning probability properly, and they are not well-prepared to teach subjective approach to probability. If we want to include subjective probability in the curriculum, then it is necessary to develop their KoT in this area. One possibility is to include activities and tasks as it is suggested e.g. in Di Bernardo *et al.* (2019).

Acknowledgements

The work of authors from Pavol Jozef Šafárik University in Košice was partially supported by the Slovak grant KEGA 020UPJŠ-4/2020 and by the National project IT Academy – Education for the 21st Century. Rifo was partially supported by the CAPES-PrInt Program, 88881.311131/2018-00. This work was also supported by the Project RTI2018-096547-B-I00 (Ministry of Science, Innovation & Universities, Spain) and the Ibero-American Research Network MTSK (funded by AUIP).

References

- Batanero, C., Chernoff, E., Engel, J., Lee, H., & Sánchez, E. (2016) (Eds.), *Research on Teaching and Learning Probability. ICME13 Topical Surveys*. Springer Nature. <https://doi.org/10.1007/978-3-319-31625-3>
- Batanero, C., Godino, J. D., & Roa, R. (2004). Training teachers to teach probability. *Journal of statistics Education*, 12(1). <https://doi.org/10.1080/10691898.2004.11910715>
- Borovenik, M., & Kapadia, R. (2018). Reasoning with risk: Teaching probability and risk as twin concepts. In C. Batanero & E. J. Chernoff (Eds.), *Teaching and Learning Stochastics. Advances in Probability Education Research, ICME-13 Monographs* (pp. 3–22). Springer International Publishing. https://doi.org/10.1007/978-3-319-72871-1_1
- Carrillo, J., Climent, N., Montes, M., Contreras, L. C., Flores-Medrano, E., Escudero-Ávila, D., Vasco, D., Rojas, N., Flores, P., Aguilar-González, A., Ribeiro, M., & Muñoz-Catalán, M. C. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education*, 20(3), 236-253. <https://doi.org/10.1080/14794802.2018.1479981>
- Chernoff, E. J., & Zazkis, R. (2011). From personal to conventional probabilities: from sample set to sample space. *Educational Studies in Mathematics*, 77(1), 15–33. <https://doi.org/10.1007/s10649-010-9288-8>
- De Finetti, B. (1937). La prévision : ses lois logiques, ses sources subjectives. *Annales de l'I. H. P.*, tome 7(1), 1-68.
- Di Bernardo, R., Mellone, M., Minichini, C., & Ribeiro, M. (2019). Subjective approach to probability for accessing prospective teachers' specialized knowledge. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.), *Proceedings of the PME43* (Vol. 2, pp. 177-184). PME.
- Jeffrey, R. D. (1965). *The Logic of Decision*. McGraw-Hill.
- Martín-Fernández, E., Ruiz-Hidalgo, J. F., & Rico, L. (2019). Meaning and Understanding of School Mathematical Concepts by Secondary Students: The Study of Sine and Cosine. *EURASIA Journal of Mathematics, Science and Technology Education*, 15(12), em1782. <https://doi.org/10.29333/ejmste/110490>
- Martignon, L., & Krauss, S. (2009). Hands-on activities for fourth graders: A tool box for decision – making and reckoning with risk. *International Electronic Journal of Mathematics Education*, 4(3), 227-258. <https://doi.org/10.29333/iejme/239>
- Ramsey, F.P. (1926). Truth and probability, in Ramsey, 1931, *The Foundations of Mathematics and other Logical Essays* (ch. VII, pp. 156-198), edited by R.B. Braithwaite. Kegan, Paul, Trench, Trubner & Co.-Harcourt, Brace and Company.
- Savage, L.J. (1967). Implications of personal probability for induction. *Journal of Philosophy*, 64, 593-607.
- Thompson, P. (2016). Researching mathematical meanings for teaching. In L. English & D. Kirshner (Eds.), *Handbook of international research in Mathematics Education* (pp. 435-461). Taylor and Francis.