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Concurrence of two mathematics worlds in the Netherlands, 1600 and beyond

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In the early 17th century at the University of Leiden, mathematics courses in the Liberal Arts and a mathematics course for surveyors and engineers in Dutch language existed peacefully next to each other, with some pathways between them. The reasons for this unique situation, the relation between the different mathematical courses and the characteristics of the Dutch language course are discussed. The design of the Dutch language course has some aspects in common with modern curricula. Two questions arise. Did this course influence the mathematics education in the next centuries? Was the Dutch language course a first step towards the development of mathematics as a school discipline? The second part of the paper attempts to find answers to these questions.

Keywords: History of mathematics education, mathematics for practitioners, engineering school, mathematics as school discipline, mathematics teachers.

Introduction

Towards the end of the 16th century two different worlds of mathematics and mathematics teaching may be distinguished in North-western Europe. One was the learned world of mathematicians at universities, at other institutes of learning, or working independently, whose object of study was the mathematics of the Greek mathematicians, their translators and commentators. It was a world of theory, in which an increasing number of ancient works became available for study, encouraging the development of mathematical theory. The language of communication was Latin, obligatory at all universities, the language of the men of learning. Mathematics teaching formed a part of the Liberal Arts, the general education as a foundation for the higher faculties. The students followed at least lectures on pure mathematics (number theory and Euclidean geometry), sometimes followed by mixed mathematics, subjects such as astronomy, mechanics, and cosmography.

The other world was the practical world of the common people, practitioners who didn't know Latin and who used some mathematics as a tool in their working life. Their grasp of mathematics might be extensive, but often it was very limited, without much theoretical background. In this practical world of surveyors, bookkeepers, building masters, military architects, traders, navigators, etc. there were as yet no teaching institutes. Private instruction by reckonmasters, self-instruction and learning at work were the main avenues to mathematical knowledge. The mathematics taught was primarily practical, without much attention to mathematical theory, confined to routines that seemed to work in a specific situation.

This changed in Leiden around 1600, at the university, which was established in 1575 as the first university of the northern Netherlands. The mathematics lectures, in Latin, in the Liberal Arts faculty were taught by Rudolph Snellius, while from 1600 a Dutch course for surveyors and military engineers flourished, under responsibility of the University and taught by university professors. This last course became known as *Duytsche Mathematique (Dutch Mathematics)*. The

reasons for this unusual situation, some characteristics of and the relations between the two mathematics form the subject of the first section of this paper. Two questions arise.

- 1) Are influences of the characteristics of *Duytsche Mathematique* recognisable in mathematics education in the Netherlands during the 17th and 18th century?
- 2) Was the engineering course an isolated incident or a first step towards mathematics as a school discipline?

Following (CherVEL, 1988; Cardon-Quint & d'Enfert, 2017) a school discipline is taken as a subject taught in institutes, with specific teaching and learning goals, development of specific content partially independent of academic mathematical sciences, dedicated teaching methods, learning means, tests and examinations and taught by a body of professional subject teachers.

Mathematics teaching at Leiden University in the early 17th century, an innovation

The governors of the new Leiden University did their utmost to invite humanistic scholars with a good reputation to become professor in Leiden. As was the case in the older renowned universities, the theoretical framework for the faculties consisted of the works of Greek and Arabic authors, such as Aristotle, Euclid, Ptolemaeus, Hippocrates, Galenus, Avicenna and Averroës (Otterspeer, 2008). Justus Lipsius, Joseph Scaliger and Claudius Salmasius were some of the more famous scholars who came to Leiden, in return for a very good salary, free living and exemption of the obligation to give public lectures. Their names warranted quality of the university; they were expected to attract many students.

The first appointments were in the three main faculties: Theology, Law and Medicine, followed by Literature and Languages (classical and eastern) and Philosophy. Mathematics had a low priority; during the first years there was no professor for the undergraduate mathematics courses. In 1581 Rudolph Snellius (1546-1613) was appointed, a graduate in the Liberal Arts of the Calvinist University of Marburg. In Marburg he became an enthusiastic follower of Petrus Ramus (1515-1572), the French humanist and educational reformer. Ramus had proposed a shorter, well-structured curriculum, which combined theory with practical exercises. For mathematics he proposed to teach only those parts of Euclid and other authors which were useful to the students and to combine theory and practice (Wreede, 2007).

Snellius was not the only follower of Ramus at Leiden University, moreover he favoured teaching a variety of authors, both the ancient and modern. Some of the authors he taught were Ramus, Valerius (*Physica*), Euclid (*Elements*, *Optics*), Maestlin (a follower of Copernicus) on cosmography and astronomy, and classical authors such as Aratus, Proclus and Plinius (on geography). Being a Ramist meant he would be inclined to value mathematics used for practices on its own merits. Around 1605 Rudolph gave lectures on geometry, geography and optics, his son Willebrord assisted him with lectures on arithmetic and astronomy. Willebrord Snellius (1580-1626) was taught by his father, he also was a student of Ludolf van Ceulen (see below) and of Joseph Scaliger. He went on to become a highly praised and very productive mathematician, who succeeded his father in 1613 (Wreede, 2007).

Towards the end of the sixteenth century the commanders of the Dutch army, Maurits van Nassau (1567-1625) and Willem Lodewijk van Nassau (1560-1620), realized they would need many more mathematically trained engineers and surveyors to maintain their military advantage over the Spanish army. There were a few mathematically knowledgeable military engineers such as Adriaan Antonisz. and Samuel Crop and some practitioners such as Ludolf van Ceulen, who based their practices on sound theoretical mathematical knowledge, without knowledge of Latin, but they were few. Maurits, son of Willem van Oranje¹, had been a student at Leiden University, where he met Snellius and a fellow-student, Simon Stevin (1548-1620). Stevin, a good mathematician and engineer, became the tutor, quartermaster-general and principal advisor of Maurits. Together they convinced the curators of the University of Leiden to establish a separate course in Dutch language for surveyors and military engineers, the Duytsche Mathematique. It started officially in January 1600 with a curriculum prescribed by Stevin in the *Instruction*. Stevin designed an efficient curriculum, with the content, teaching methods and work forms prescribed. Theory preceding and alternating with practice; a method of study very much favoured by Stevin (Krüger, 2015). The course itself, the use of a written curriculum and the teaching language were an innovation.

The first professors who taught Duytsche Mathematique were Ludolf van Ceulen (1540-1610) and Simon Frantz. van Merwen (1548-1610). Simon Frantz. Van Merwen was a surveyor and burgomaster of Leiden. Ludolf van Ceulen was a respected mathematical practitioner, who taught mathematics and fencing and served on some committees advising the government. He didn't know any Latin but was very accomplished in geometry. He is best known for his calculation of the first 35 decimals of π . Van Ceulen passed away in December 1610, whereupon his former student, assistant, surveyor and army engineer, Frans van Schooten, continued the lectures and the practice sessions in the field. At first as a temporary lecturer, from 1615 as professor Duytsche Mathematique (Krüger, 2018). His extensive and well-illustrated lecture notes² (BPL 1013) provide information on the content, structure, and didactic approach of the course. Van Schooten followed broadly the *Instruction*, with some adaptations and with deliberate didactics. In BPL 1013 calculations of roots, using decimal numbers is followed by some geometry and the principles and practice of surveying. This is followed by advanced surveying techniques, advanced geometry, calculations of volumes and wine gauging. Students who had come this far could take an exam to become an admitted surveyor, Rudolph Snellius had been an examiner on behalf of the university. The second part of the lecture notes is about fortification, defined by Van Schooten as the art to build and defend fortifications as well as the art to attack and conquer fortified places.

In 1646 Frans van Schooten Sr. was succeeded by his eldest son, Frans van Schooten Jr., a very talented mathematician, who edited a new edition of Viète's work and translated Descartes' *Geometry* into Latin. He used the lecture notes of his father, but also gave lectures on logarithms, algebra, and sundials (Dopper, 2014). Though Frans Jr. was more a scholar than a practitioner, he

¹ Willem van Oranje was the leader of the Dutch revolt against the Habsburg rulers, until his murder in 1584.

² BPL 1013, University Library Leiden, special collection

maintained contacts with surveyors and engineers, even if he was not himself a surveyor and engineer as his father had been.

Ludolf van Ceulen, Simon Stevin, Willebrord Snellius and Frans van Schooten Sr. all served as examiners for the admission of surveyors in the province of Holland (Krüger, 2014). In the academic lectures the professors taught the theory of mathematics as developed by Greek and Arabian scholars and their interpreters, with Latin as teaching language. The Duytsche Mathematique on the other hand taught which mathematics to use and how to use it in specific practical situations, with Dutch as teaching language, in interaction with practitioners. In Leiden, during the first half of the 17th century these two mathematics programmes, with different aims and content, existed peacefully together with links between them through the actors involved. Many factors, political, social, personal, were at the basis of this unusual situation.

Mathematics education during the 17th and 18th century

In 1681 the Duytsche Mathematique was abolished. It was reintroduced in 1701, as a theoretical university course, taught by a lecturer and it never regained the fame it had in the 17th century. Until well into the 19th century mathematics remained a rather unimportant part of the undergraduate program. However, the interest of university professors in technical applications increased. For example, civil architecture, hydraulics, steam engines, windmills, optics, navigation, and fortification were at one time or another subject of lectures at the universities of Leiden, Franeker and Utrecht (Krüger, 2014).

In society the notion that mathematics was relevant for many crafts and professions gained acceptance. In primary schools there was a gradual improvement in the position and quality of arithmetic teaching. Virtually every village and town had at least one primary school, for which the local government was responsible. In the more ambitious towns mathematics formed part of the comparative examinations for a teaching post, a mathematics journal for teachers was published from 1754 (Krüger, 2019). Between primary education and university education the contours of on the one hand general education, on the other hand vocational education became visible.

General education as a preparation of boys for university took place in Latin schools and Illustrious schools. In so-called French schools general education prepared the sons and daughters of well-to-do citizens for their position in society. Latin schools offered no mathematics or sciences, Illustrious schools offered a range of subjects in Latin, but some mathematical topics were taught in Dutch. Examples are fortification and navigation at the Illustrious school in Amsterdam from 1743-1762; geometry, arithmetic, algebra, and fortification were taught at the Illustrious school in Deventer from 1690-1727. The mathematics taught in French schools depended on the headmaster-owner. In all these institutes the content was mainly determined by the teacher and was theoretical, though teaching applications. There was no formal curriculum for any type of school.

The perceived relevance of mathematics for many professions and crafts was reflected in the relative abundance of textbooks being printed and in the existence of many small private schools and teachers offering private mathematics lessons (geometry, navigation, fortification, bookkeeping, etc.). Naval colleges were established to teach mathematical skills to navigators and from the late 18th century drawing schools and lessons in mathematics and drawing in orphanages

became more common. Both in vocational training and in general education, mathematics teaching was haphazard, depending on initiatives and skills of individuals, with no written curriculum nor a system of examination. Neither was there a group of professional mathematics teachers (Krüger, 2019). However, from 1756 onwards the three institutes of the Foundation of Renswoude were comparable to the Duytsche Mathematique. In these institutes, in The Hague, Delft and Utrecht, talented orphans received a combination of professional training, based on mathematical subjects, and general education. From the start governors and mathematics teachers formulated a curriculum with subjects, aims, theory preceding practice and alternating with practice, examinations, textbooks, and other materials for the students. The first mathematics teachers had good contacts with many different practitioners, there were also contacts with professors at the Universities of Leiden and Utrecht (Krüger, 2013).

Unlike the situation in some other countries (Karp & Schubring, 2014) the Dutch government did not establish primary, secondary, or vocational institutes, until the end of the century (Krüger, 2019a).

Mathematics: a school discipline in general education

During the 18th century, under the influence of French authors, mathematics gradually was considered as formative for the development of the mind, to learn to reason clearly and with use of logic. As a result, when around 1800 education became the responsibility of the national government, mathematics was on the agenda. Primary education was regulated first, in 1806; arithmetic became a compulsory subject. Moreover, all teachers had to pass exams, including arithmetic, to be allowed to teach and the inspectorate became rather important in the acceptance of modern ideas with regards to content and methods. From about 1820 the development of mathematics as a school discipline becomes noticeable: textbooks specifically for use in (extended) primary schools, specialized journals, teachers specializing in mathematics and authors discussing teaching methods (Krüger, 2019). The authors might be teaching at the Royal Military Academy, at French schools and other private schools, at naval schools and other similar institutes or they belonged to the group of primary school teachers in the highest rank, who had specialized in mathematics. Those who became proficient in mathematics could improve their position (Smid, 2019). The regulation of secondary education had to wait until 1863. At least until then there existed a patchwork of private and council schools, extended primary schools and vocational institutes, all teaching some or quite a lot of mathematics, with textbooks written by some of the teachers.

From the start of the 19th century mathematics was seen by regulators, inspectorate and others as belonging to general education, with a formative value in sharpening reasoning faculties. On the other hand, for many people the main aim of teaching mathematics was to prepare for crafts and professions. In their opinion the emphasis of teaching mathematics ought to be on mathematics used in professions and how to use this. Tensions between these aims surfaced occasionally, as in the case of Jacob de Gelder, a highly praised mathematician and teacher, author of many textbooks (Beckers, 1999). In 1819 he left the School for Artillery and Military Engineers after serious disagreement with the director, a military man, about the amount and level of mathematical theory

which was taught (Krüger, 2019a). Another example is the appearance of an arithmetic journal aimed at crafts men and farmers, as the calculations at primary schools were deemed to be too exact and rigorous for daily practice (Krüger, 2019). Indeed, most pupils would enter the workforce after leaving primary school and learn a craft. The development of mathematics as a school discipline, visible in the first half of the 19th century, received a strong impulse through the regulation of secondary education in 1863. For general education two new types of school were introduced, of which the Hogere Burger School³ (HBS) would become the most influential (Krüger, 2014). At the HBS the students were taught a range of subjects, with sciences, mathematics and modern languages taking up many hours in the timetable. The school had as one of its aims preparation for the Polytechnic School, also established in 1863, the successor of the ailing Academy for civil engineers (Krüger, 2019a). In the early years of the HBS mathematics included subjects such as mechanics, use of technology, cosmography, and line drawing, next to arithmetic, algebra, geometries, and trigonometry. The development of a group of professional mathematics teachers got a strong impulse through the HBS. Gradually topics such as mechanics and cosmology disappeared from the mathematics curriculum. From the last quarter of the 19th century there were discussions about the content of the curriculum, the teaching methods, examinations and the merit of various textbooks. The teaching was theoretical, the use of mathematics in practice was delegated to the final years of Engineering Studies. At first, many mathematics teachers had a background in military or civil engineering, gradually most mathematics teachers had a degree in mathematics, but no experience with the use of mathematics in practices.

Discussion

Are influences of the characteristics of Duytsche Mathematique recognisable in mathematics education in the Netherlands during the 17th and 18th century?

As described in the second section, during the 17th and 18th century there was a growing awareness of the relevance of mathematics for numerous practices. Illustrious schools, small private schools and private teachers offered the teaching of mathematics to use in practices, or as part of general education. Mostly these programs or schools lasted at most a couple of years and details of the content are unknown. The only known example of a programme which showed some of the characteristics of the Duytsche Mathematique is the curriculum of the Foundation of Renswoude. The three institutes of this Foundation resemble Duytsche Mathematique in the combination of theory and practice, the structure of the programme, the interaction with practitioners and the relative success; moreover, professors at the universities of Utrecht and Delft occasionally served as examiner for the Foundation in Utrecht and Delft (Krüger, 2014).

Another sign of the awareness of relevance of mathematics for practices is the publication of numerous books on mathematical subjects, both the fundamentals and the practices. Books on practices offered some mathematical theory; further research is necessary to determine the relevance and the depth of the treatment of mathematics in these books.

³ *Higher Burgher School*, meant for the sons of middle- and upper-class citizens

Was the Duytsche Mathematique an isolated incident or a first step in the evolution of mathematics as a school discipline?

The engineering course, the Duytsche Mathematique, at Leiden University was at the time innovative, due to the teaching language, and the way it was designed. A written curriculum, prescribing content, teaching methods, work forms, theory preceding practice and alternating with practice, is reminiscent of mathematics as the present-day school discipline. However, while the teaching method as seen in the manuscript by Frans van Schooten Sr. shows strong didactics, directed towards the needs of the students, the content was mostly aimed at the practices of surveying. Not all of it, there are geometry examples which do not seem relevant for surveying or engineers but are mathematically interesting. Also, there are indications that the practice somehow disappeared from the course, Frans van Schooten Jr., an excellent mathematician, seems to have concentrated on teaching and expanding the theory of the subject, with theoretical exercises about the practice of surveying, application of theory, as opposed to exercises in the field, using mathematics. One could say that Duytsche Mathematique showed some characteristics of a future school discipline, however it may also be seen as a forerunner of Engineering Studies.

The first example of a possible development of mathematics as a school discipline in the 18th century is given by the Foundation of Renswoude. The programmes of the three institutes started coordinated, with some contact between the teachers in the three towns, with theory as the basis for practice. Interaction with practitioners was relevant for the programme, which could be described as a combination of general education with vocational training.

General education, with mathematics as a permanent factor, gained terrain from the start of the 19th century, in the form of primary education. This was followed in the second half of the century and in the 20th century by secondary education and the first years of vocational training. General education is theoretical, mathematics is about theory of the school subject, formulated in its own language and applied to theoretical examples. The use of mathematics, which mathematics to use and how to use it, is rarely or not at all experienced in general education and there is no interaction with professional users of mathematics.

Was the Duytsche Mathematique a first step in the evolution of the school discipline? Yes and no. It was more an example to follow than a first step; the Dutch society was not yet ready for more or less autonomous school subjects. A hundred years would pass before schoolteachers started to specialize in mathematics and another hundred years before a recognizable body of mathematics teachers became visible. They formulated opinions about the mathematics teaching programme, produced teaching materials and contributed to the formation of the school discipline, losing the interaction with professional users of mathematics in the process. Meanwhile the two worlds of mathematics from 1600 blurred and branched in many different ways, developing into a rich and varied universe of mathematics.

References

Beckers, D. (1999). Lagrange in the Netherlands. Dutch attempts to obtain rigor in the calculus. *Historia Mathematica* (26), 224–238.

<https://www-sciencedirect-com.proxy.library.uu.nl/science/article/pii/S0315086099922462>

Cardon-Quint, C., & d'Enfert, R. (2017). L'histoire des disciplines: Un champ de recherche en mutation. *Revue Française de Pédagogie*. WorldCat.org. <https://doi.org/10.4000/rfp.5981>

Chervel, A. (1988). L'histoire des disciplines scolaires: réflexions sur un domaine de recherche. *Histoire de l'éducation*, (38), 59–119.

https://www.persee.fr/doc/hedu_0221-6280_1988_num_38_1_1593

Dopper, J. (2014). *A life of learning in Leiden. The mathematician Frans van Schooten (1615–1660)*. [Doctoral dissertation, University of Utrecht].

<http://dspace.library.uu.nl/handle/1874/288935>

Karp, A., & Schubring, G. (2014). Dynamics of changes during the eighteenth century. In A. Karp & G. Schubring (Eds.), *Handbook on the History of Mathematics Education* (pp. 137-141). Springer.

Krüger, J. (2013). The power of mathematics education in the 18th century. In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), *Proceedings of the eight Congress of the European Society for Research in Mathematics Education* (pp 2020-2029). Middle East Technical University.

www.mathematik.uni-dortmund.de/~erme/doc/CERME8/CERME8_2013_Proceedings.pdf

Krüger, J. (2014). *Actoren en factoren achter het wiskundecurriculum sinds 1600*. [Doctoral dissertation, University of Utrecht]. <http://dspace.library.uu.nl/handle/1874/301858>

Krüger, J. (2015). A first mathematics curriculum: Stevin's Instruction for engineers (1600). *International Journal for the History of Mathematics Education*, 10(1), 79-88.

<http://dspace.library.uu.nl/handle/1874/357517>

Krüger, J. (2018). Frans van Schooten Sr. (1581–1645): Lecture Notes for the First Dutch Course for Engineers, Leiden, 1600–1681. In F. Furinghetti & A. Karp (Eds.), *Researching the history of mathematics education* (pp. 143–166). Springer.

<https://link-springer-com.proxy.library.uu.nl/book/10.1007/978-3-319-68294-5>

Krüger, J. (2019). Relevance of mathematics journals for Dutch teachers in the 18th and 19th century. In U. T. Jankvist, M. van den Heuvel-Panhuizen & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp. 2125–2132). Utrecht University. <https://hal.archives-ouvertes.fr/hal-02421857>

Krüger, J. (2019a). Engineering Studies and Secondary Education: Descriptive Geometry in the Netherlands (1820-1960). In E. Barbin, M. Menghini & K. Volkert (Eds.). *Descriptive Geometry, The Spread of a Polytechnic Art: The Legacy of Gaspard Monge* (pp. 233–253). Springer. https://doi.org/10.1007/978-3-030-14808-9_14

Otterspeer, W. (2008). *Groepsportret met Dame I. Het bolwerk van de vrijheid. De Leidse universiteit, 1575-1672*. DBNL. https://www.dbnl.org/tekst/otte010groe01_01/

Smid, H. J. (2019). Dutch mathematics teachers, magazines and organizations: 1904-1941. In K. Bjarnadóttir, F. Furinghetti, J. Krüger, J. Prytz, G. Schubring & H.J. Smid (Eds.). "Dig where you stand" 5 (pp. 373–388). Utrecht University.

http://www.fi.uu.nl/publicaties/literatuur/ichme/Dig_where_you_stand-5.pdf

Wreede, L. de (2007). *Willebrord Snellius (1580–1626). A Humanist reshaping the mathematical*

sciences. [Doctoral dissertation, University of Utrecht].
<http://dspace.library.uu.nl/handle/1874/22992>