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***Notabilia Arithmetica*: a Jesuit mathematics school book**

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*In this paper we take a look at an arithmetic textbook from 1703 called *Notabilia Arithmetica* that belongs to the holdings of the old school library of a former Jesuit gymnasium in Germany and that was edited there, too. We analyse the book in regard to its content and possible integrated methodical aspects in order to find out more about mathematics education in Jesuit schools in the early modern period. The book shows us that – even though mathematics played a minor role in Jesuit education and was basically limited to Euclid in the general teaching rule – practical arithmetic must have been taught to a considerable degree, so that an arithmetic textbook was a necessary means for teachers.*

Keywords: Arithmetic, Textbook research, Educational history, Jesuit education.

Introduction

Notabilia Arithmetica is the title of a small, pocket-sized book from 1703 that belongs to the stock of the Josephine library, that is the library of the former Jesuit gymnasium *Josephinum* in Hildesheim, Germany.

The research presented in this paper is part of an interdisciplinary research project aiming at a broad account of the library stock. So in the first place the focus is a local one. In the second place, though, we aim at embedding our findings in the history of classroom practice. Existing literature on Jesuit mathematics education has been thoroughly reviewed by Diaz (2009). However, research on mathematics education in Jesuit schools in Germany is extremely scarce. Furthermore, Diaz states a lack of research on classroom practice and that the “reality of teaching practices and curriculum [...] need[s] [...] further scholarship to understand the praxis of the Jesuit system.” (2009, p. 65)

Following the presumption by other authors (e. g. Schubring, 1987) that teaching practices are determined by textbooks, we find the *Notabilia* a unique source. What makes it stand out from most other books in store of the library is that it has been edited by Hildesheim Jesuit mathematicians themselves. We may therefore start from the premise that the book was really used in classroom context and that it therefore serves as a source on actual Jesuit mathematics education.

In this paper we take a closer look at the *Notabilia Arithmetica* in regard to possible implications about mathematics instruction in a Jesuit gymnasium. We start by giving some general information on Jesuit education as well as on the history of the *Josephinum* and its library. Following this, we introduce the *Notabilia* itself. We describe its content and take a closer look at indications of purpose and methodical aspects, using the hermeneutic approach. Finally, we contextualize it by drawing comparisons to what other sources tell us about Jesuit education.

The Jesuits' role in education

Following reformation and – as a consequence thereof – decline of Catholicism in the 16th century, the Jesuits drew on education in order to regain ground. Having officially been founded in 1540, the Jesuits soon started settling throughout Europe and the rest of the world and establishing colleges with associated *gymnasia*. Their education was characterized by a firm and uniform order, which was written down in the *Ratio atque Institutio Studiorum Societatis Jesu* in 1599. The Jesuit educational system became very influential and served as model for schools run by other religious orders, making it a relevant subject of research concerning mathematics education in the 16th and 17th century.

The *Ratio Studiorum* contains rules for the professors of the different subjects, and it is understood that it is due to the major influence of the famous Jesuit Christoph Clavius that mathematics has been taken up as a regular subject in there at all (Diaz, 2009). It was part of a two-year philosophy course, which comprised the subjects of the *quadrivium* and which followed on the grammar course and preceded the theology course. There was a clear focus on religious education whereas mathematics and the natural sciences did not play an important role within the curriculum (Hammerstein & Müller, 2005). This shows in the *Ratio*, as well: the paragraph containing rules for the professor of mathematics comprises less than one page, stating that three quarters of the physics course should be spent on Euclid and some geography or either astronomy in addition. Every month or every second month a mathematical problem shall be presented and discussed with a larger audience, and once a month the contents dealt with in recent lessons should be reviewed.

Generally, the primary aim of education was to have students who were capable of using their acquired knowledge on practical problems. Therefore, repetition, practice and rote learning were important and common parts of scholasticism. Typically, a new subject was first presented in a *lectio*, then further treated in different *quaestiones* and finally memorized and repeated in written exams and *disputationes*. The students' achievements were regularly publicly displayed and rewarded with prizes. There was a catalogue of textbooks that were supposed to be used in the Jesuit gymnasiums and which were preferably written by Jesuits, too. Hammerstein & Müller (2005) name some of these standard textbooks but none for mathematics.

The *Gymnasium Josephinum* in Hildesheim

A short history of the *Gymnasium Josephinum*

The history of the *Gymnasium Josephinum* goes back to the year 815 when Louis the Pious – son of Charlemagne – founded the diocese of Hildesheim. It started out as a cathedral school located in the dome court, responsible for elementary education and being of high importance in the Middle Ages.

In 1587 the first Jesuit priest came to Hildesheim and by 1595 the Jesuits had established a residence and started a new gymnasium alongside the old cathedral school, first only teaching lower grammar classes. In 1601 the residence became officially a Jesuit college. Due to the Thirty Years' War the Jesuits were expelled from Hildesheim in 1634 but returned already in 1643 and in the following years the school grew further. It is not before this time that we have knowledge of teachers for mathematics and the natural sciences. Around 1660 a vicar bestowed money explicitly

meant for the constitution of four teaching positions for philosophy and mathematics and by 1664 there was at least one specialist teacher for mathematics (Gerlach & Seeland, 1950; Pilz, 1995). By the end of the 17th century the school had been given the name *Gymnasium Mariano-Josephinum*.

Since the Jesuit order was disbanded in 1773 the gymnasium was assigned to the diocese of Hildesheim again and it is still an episcopal secondary school located next to the dome today.

As to the students, already around 1700 there was what we would nowadays call a heterogeneous student body, regarding the region they came from as well as their estate. Besides students from noble or bourgeois origin there were also the poor ones, the *pauperes*, who obtained free housing and who especially the Hildesheim Jesuits were said to take a lot of care of (Pachtler, 1894).

The school library

As early as 1595, the year that schooling was taken up, the Jesuits of Hildesheim also took up establishing an associated library. When they were forced to leave Hildesheim they had to leave the books behind, as well, and when they returned in 1643 they found the library had been marauded in the meantime. They restored it and started collecting books again. Due to donations, gifts and inheritance the stock grew considerably up to about 20,000 volumes. The library got again endangered during the Second World War and indeed the old school building that had housed the library was destroyed in 1945. Luckily, most of the books had been evacuated before to a church outside of Hildesheim. They were only retrieved in the late 20th century, which is why their revision is still ongoing. Today the remains of the Josephine library comprise about 13,500 volumes, a considerable part of which dates back to Jesuit times. The library has been incorporated into the dome library and up to now only about 5000 volumes have been catalogued (Schmidt-Thieme, 2020).

Several mathematical textbooks are part of the stock, not least because Peter Heckenberg bequeathed his private collection to the library upon his death in 1695. Heckenberg who was a vicar and librarian of the Josephine library in the 17th century was himself the author of some of these mathematical textbooks (for further detail see Schmidt-Thieme, 2020). Nevertheless, we do not know the number of these books yet nor what role they played for practical mathematics education.

Standing out, though, are two booklets, both of them pocket-sized and especially noticeable as they were edited by mathematicians of the Hildesheim Society of Jesus themselves, leading to the assumption that these were produced for classroom work and therefore played a role in teaching practice. These are an arithmetic textbook by the name of *Notabilia Arithmetica* from 1703 and an edition of Euclid (*Elementa Euclidis*) from 1704. The latter still needs a closer look at. For what we found so far, it is most striking that we are dealing with an edition of Euclid's Elements that lacks all the proofs (*sine demonstrationibus* is part of the full title) but all the propositions are basically reproduced. However, this paper deals with the *Notabilia Arithmetica* that seems to be an original work promising to give us further insight into the Jesuit mathematics classroom.

The *Notabilia Arithmetica*

General description

The book's full title is *Notabilia Arithmetica quae Omnem Arithmetica[m], ejusque varium usum, cum in aliis multis, tum maxime in Geometricis operationibus succincte proponunt* (Notable arithmetic that propounds briefly the complete arithmetic, its various uses, both in many other [things] and in most geometric operations), there is no author mentioned, but named as editors are mathematicians at the gymnasium of the Hildesheim Jesuit society. It has been printed in Hildesheim by Johannes Leonard Schlegel in the year 1703 and comprises 144 pages and 8 further uncounted sheets. Two of these added sheets differ slightly in the two copies that we worked with, so they might be drawn instead of printed. The *Notabilia Arithmetica* is written in Latin. The copy from the Josephine library shows a hand-written note, "biblioth paup 1709", on the title page, suggesting that it has been incorporated into the *bibliotheca pauperum*, the library for the poor students.

The book begins with a foreword dedicated to lecturers (*lectori*) and ends with some further suggestions for lecturers (*monitio ad lectorem*), so it is quite clear that it was written for teachers rather than for students even though it became part of the students' library. It is explicitly called a *compendium* in the foreword, containing only what is necessary, so it was supposedly not meant for the introduction of new topics. The reason given for the booklet's special format (approx. 8 cm x 13 cm) is that this facilitates compiling it with other mathematical textbooks, namely especially some of those written by Peter Heckenberg¹.

The copy found in the Josephine library is not the only copy that has persisted, there are at least seven more: three in the dome library in Hildesheim, one in Hannover as part of the private library of Gerhard Molanus², two in Münster and one in Trier, both of the latter being former sites of Jesuit colleges, as well.

Structure and content

Before we go into some focal points, we give an overview of how the book is structured and what mathematical subjects it contains. The content of the *Notabilia* is divided into four parts, each of which comprises several topics, rules and algorithms.

The first part deals with integers and their basic arithmetic operations. It starts with *numeratio* – that is introducing figures and their names – and goes on with introducing meaning and procedures of addition, subtraction, multiplication and division, followed by raddology, which is calculating with the use of Napier's bones.

¹ We have not found any books written by Heckenberg that share the same format yet, the remark might be an explanation, though, why the *Notabilia* and the *Elementa Euclidis* were both edited in this special format.

² Gerhard Wolter Molanus (1633–1722) was a professor of mathematics at the University of Rinteln, later abbot of the monastery in Loccum and acquainted with G. W. Leibniz, with whom he exchanged books from his private library.

The second part deals with fractions, starting again with *numeratio*, being followed by the rules for cancelling and the basic arithmetic operations for fractions.

The third part is about arithmetic rules and contains a vast variety of subjects. It starts with a relatively long chapter (*Cap. 11*) on the so-called golden rule, also known as *regula de tri* (rule of three), and goes on with a much longer chapter (*Cap. 12*) comprising 16 common and widespread rules (*Aliae regulae Arithmeticae in vulgari usitate*), such as the *regula falsi* (method of false position), but also rules on arithmetic (e. g. the so-called Gauss sum) and geometric progressions and some combinatorics. The latter indicates Athanasius Kircher³ as source. The common rules are then followed by some shorter chapters on decimal fractions, some geometric rules (including trigonometry, measuring, planimetry and stereometry), astronomy, chronology (first of all *computus*), algebra (including extracting roots) and bookkeeping.

The fourth part contains various things and we could describe it as comprising miscellanea. It starts with the definition of a number as a manyness composed of unities and some more definitions of special numbers (e. g. odd, even and perfect numbers), which are followed by an overview of population figures (*Cap. 19*). Next are surveys containing currencies, measures of capacity and length, square measures, some geography and chronometry (calendar). For all kinds of measures the local Hildesheim measures are included.

These four main parts are followed by a synopsis of the whole book, condensed in *XII. Propositiones Arithmeticae* and twelve arithmetic paradoxes. The propositions are preceded by a short insertion that seems to be an announcement of a *disputatio* in the school. The booklet ends with an alphabetic index and the aforementioned hints for lecturers. Appended are three arithmetic tables and a geometric one, the latter alongside an explanation.

Quite some things are noticeable about the mathematical content of the *Notabilia Arithmetica*.

First of all it shows a vast variety of subjects, exceeding pure arithmetic by including topics like geometry, metrics, astronomy and geography. The inclusion of the latter can be explained with regard to the *Ratio Studiorum*, where astronomy as well as geography are explicitly named as possible subjects of lesson. As to geometry, there is no reference to Euclid but instead those areas are included that are less theoretical but have a relevance to practical applications. It appears that the same can be said about all the other topics and about the arithmetic chapters, as well. Especially chapters 11 and 12, which are the ones that deal with the rule of three and the other common rules and which therefore have a major relevance for everyday calculations, take up a big part of the whole work (altogether the two chapters comprise 20 pages). It appears that usefulness and applicability have been main criteria in choosing what subjects to include in the *Notabilia* and consequently – as it is called a compendium for teachers – in the mathematics classroom.

Having grown out of a cathedral school and with a religious order being in charge of the school, one could have expected the arithmetic course to stand in the tradition of medieval Christian education,

³ Athanasius Kircher (1602–1680) was a German born Jesuit mathematician and seems a convenient source regarding that textbooks used by Jesuits should preferably be written by Jesuits, as well.

more precisely to refer to Boethian number theory. As we can see, this is not the case. The only exceptions are chapter 19, which includes some short definitions that resemble those we find in the works of Boethius, and chapter 11 on the rule of three where certain kinds of proportions are defined. Both parts added together comprise no more than five pages and especially chapter 19 stands out from the rest. Instead the content rather strongly resembles the content of the arithmetic books written by Renaissance reckoners.

This prioritization of content seems again plausible with regard to the aims of Jesuit education as mentioned above, namely the ability to apply knowledge and solve practical problems. In comparison to the *Ratio Studiorum*, though, it is striking that arithmetic is not scheduled there at all, let alone to such an extent. The existence of the *Notabilia Arithmetica* therefore serves as a proof that Jesuit mathematics education – at least as far as Hildesheim is concerned – has exceeded the standards by including arithmetic and further practical knowledge to a considerable extent.

Methodical references

Due to the *Notabilia* being a sole compendium, we cannot draw conclusions about the introduction of new mathematical concepts, procedures and propositions from it. But nevertheless, there are features that are interesting from a methodical point of view.

One thing that is noticeable is that several cross references can be found within the main parts of the book so we may assume it was one aim of the author(s) to write a book that was user-friendly. The existence of the alphabetic index and the comprehensive propositions at the end support the impression that the book was meant for practical use and possibly to serve as a reference book that was easy to use.

Another thing that attracts attention is that for some topics, e. g. the rule of three and calculating with fractions, we find guidelines regarding how to do exams and what theory needs to be proven. One example that we will now take a closer look at is chapter 11, the chapter on the rule of three. After different variations of the rule are presented a paragraph follows that is captioned *Regula aurea Examen & Theoria* (p. 40-41). As to exams it is said that the rule should be examined in all its variations, but with an emphasis on the simple rule in opposite to the *regula composita*. Regarding theory, we find standards that are to be followed. First, it is explained for the basic rule, how the figures that are involved in the calculation are interrelated by referring to their proportionality. The proportionality is then used to show how and reason why the fourth figure can be calculated from the other three. The passage ends with *Quod erat demonstrandum*, before the same argumentation is done for the inverted rule. As for the *regula composita* it is stated that it leads to the same problem already treated, a fact that will please the students. This bit creates the impression that a certain students' pleasure is granted. Finally, it is indicated that the majority of the common rules following in chapter 12 can be derived and proven from the rule of three and that it therefore suffices to learn the former.

In this paragraph we find confirmed that there are rules that are to be followed by all (at least all Hildesheim professors), even for an area that is not covered by the *Ratio*. We also find confirmed that exams are an important part of the curriculum, otherwise they would hardly be mentioned in a booklet that comprises only what is most necessary. The emphasis on the simple rules leaves

several possible explanations. In the first place, one could take it as evidence for the minor significance attributed to mathematics within the Jesuit curriculum. It might just as well be an expression of time shortage, which is something we must assume considering the small amount of time that the *Ratio* allows on mathematics, even more as it is already exceeded by the variety of arithmetic topics. The explications also suggest another reason, namely the subject's internal material logic that allows us to deduce special rules to basic rules respectively to trace them back vice versa. Altogether the *Notabilia Arithmetica* shows us that despite the emphasis on practical knowledge Jesuit arithmetic education went beyond the pure memorization of algorithmic procedures. Even though a lot of the content reminds us of the Renaissance reckoning books, in this regard there is a considerable difference.

One thing that does not become clear from the remarks on demonstrations is whether they reside only with the teachers or are presented by students, as well. Exams were usually written but another part of the book gives us further insight, namely the aforementioned announcement of a *disputatio* (p. 136), which stand out in a curious way. There are no further comments about it and there is not even space between it and its adjacent sentences. We are only told that the disputation took place on the 5th of September 1702 in the school's public auditorium. Participants are named as *Domin[i] Antonio Fondeux* and *Leo[n]ardo Fischer* and as opponents *Joanne Robeck*⁴, *Andrea Antonio Neuhaus* and *Johan. Angelo Schwartz*. We have no further information about the actual event but the announcement tells us that disputations on arithmetic were conducted at all. We may assume that these were events where students were meant to present propositions, proofs and explanations that they memorized from their teacher's demonstrations. The fact that the announcement is exposed in the *Notabilia* might nevertheless be a hint that we are dealing with an outstanding event and that arithmetic disputations did not take place regularly.

Conclusion

We find that the *Notabilia Arithmetica* has been written for teachers of mathematics, as a compendium of the topics from arithmetic lessons and as a reference book for necessary arithmetic knowledge. The *Notabilia* does not serve as a methodical manual for the introduction or development of mathematical procedures or concepts but it does include guidelines on what contents should be proved and how this reasoning was supposed to be done. In addition, it includes instructions on what was to be reviewed in exams. Even though the book was meant for teachers, it became placed at the poor students' disposal. Seemingly, some teacher or librarian regarded it as useful for them, too.

As for the content, the *Notabilia* shows a strong emphasis on practical applicability and computational abilities. This corresponds with the general aims of Jesuit science education but exceeds the standards for the professors of mathematics as we find them in the *Ratio Studiorum*, concerning both extent and topics. We may assume that there was a general need for usable arithmetic knowledge which was satisfied by the Jesuit educators or either a strong reckoning

⁴ Johann R. Robeck (1672–1735) from Calmar, Sweden, was a philosopher who is best known for a treaty on suicide for theological reasons.

tradition going back to Renaissance reckoning masters. In any way, Jesuit mathematics education covered much more than Euclid's *Elements* and it can be assumed that the *Notabilia Arithmetica* arose from practical need for an arithmetic textbook that presented the subject matter in short and that served as a kind of teaching aid for lecturers, just as the *Elementa Euclidis* did for Euclidean geometry. This assumption is supported by the apparent absence of a standard textbook for mathematics and especially elementary arithmetic in the catalogues of such standard books. The fact that the book seems to have found its way into other Jesuit colleges, as well, might be another instance showing us that a textbook that summarized arithmetic was required for teaching practice.

This paper is just a starting point from which much more research on the topic needs to be done. Such future research must include analysis of and comparison to further books from the Josephine library and locating our findings in the broader context of Jesuit education. The latter comprises the more general context as well as special ideas concerning mathematics education, in particular those of Christoph Clavius (for more details on the role of Clavius see Diaz, 2009).

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