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Investigating the potential of a historical document for task-design

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Abstract: This paper is a progress report which presents the context, the outline, and the starting point of an ongoing research project. In the context of a reform of the French high-school curriculum, teachers were unexpectedly required to use history of mathematics in the classroom, without being instructed how to, and with almost no standard resources – such as official guidelines or textbooks – to work with. This provides an opportunity to study the work of teachers as designers of teaching material. We set out a research protocol in which five teachers agreed to (independently) design teaching sessions starting from the same document, namely an extract from Euler’s Elements of algebra on the numerical approximation of square roots. This paper aims to establish the relevance of this document in this research context.

Keywords: History, task-design, teacher as designer, cognitive demand.

Rationale

This paper is a preliminary report which aims to present the rationale and starting point of an ongoing research project which lies at the intersection of several important yet under-researched (or hitherto independently researched) topics in mathematics education.

First, research on the use of history of mathematics in the mathematics classroom have convincingly argued that, from both cognitive and didactical viewpoints, tasks based on historical sources are particularly suited for non-routine mathematical work (Fauvel & van Maanen, 2000; Jankvist, 2009; Chorlay, 2016). However, whether or not this *potential* for cognitively demanding classroom activities translates into practice calls for empirical studies on actual classroom practices. Such studies are still quite rare in the History and Pedagogy of Mathematics research community.

Second, the interaction between teachers and resources for teaching are studied from a variety of perspectives (Adler, 2000; Remillard, 2013; Choppin, McDuffie, Drake, & Davis, 2018), but mostly in contexts where teachers deal with ready-made teaching material such as textbooks. Arguably, contexts in which teachers and researchers jointly engage in *task design* have also been studied (Jones & Pepin, 2016), but contexts in which teachers work as *autonomous* designers are largely understudied, with some exceptions usually related to the use of ICT (see, for instance, (Trouche et al., 2018)).

Third, in the wake of Mary K. Stein’s work on cognitively demanding tasks and the distinction between low-level (or routine) tasks and high-level tasks (seen as characteristic features of “doing mathematics”), many papers have studied how actual teaching practices maintain or lower the demand-level of tasks (Henningsen & Stein, 1997; Boston & Smith, 2009). To the best of our knowledge, these theoretical tools have not been used to study tasks

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designed on the basis of historical documents, along with their actual classroom implementation.

In what follows, we will explain how a recent reform of the French educational system provided an opportunity to set out a research protocol enabling us to empirically study how teachers interact with historical sources and engage in task design. Among the many features of the teachers' work as autonomous designers (and implementers) of tasks, we will focus on the observables pertaining to the level of tasks actually selected by the teachers (i.e. cognitive demand), and to the level of uncertainty which they are willing to leave room for in the classroom (Zaslavsky, 2005).

Outline of the research project

Institutional context: A new high-school curriculum (2019)

The French educational system has a national syllabus which the Ministry of Education changes approximately every eight years, for reasons which are usually not explicit, and as the result of a non-transparent process. In 2019, new high-school syllabi were implemented in all subjects, in the context of a large structural reform of high-school (grades 10 to 12). As far as history of mathematics is concerned, this new syllabus stood in sharp contrast with former syllabi. In 2010, for instance, the previous syllabus for science majors (grades 11-12) mentioned history of mathematics in passing, in an introductory paragraph on the varieties of forms of students' engagement with mathematics.

On the face of it, the 2019 syllabus deepened the same ideas:

It can be judicious to enlighten the content of the mathematics course by a historical, epistemological or cultural contextualization. Indeed, history can be seen as a source of problems which clarify the meaning of some notions. The passages labelled "History of mathematics" point to some possibilities along this line. Teachers can implement them by relying on the study of historical documents (MEN, 2019. Our trans.)

In both syllabi, history of mathematics is seen as a tool rather than a goal (Jankvist, 2009); historical activities should be weaved in the general fabric of the course (should they be considered). Although this tool is versatile, the main goal is to foster conceptual understanding and meaning-making, rather than – for instance – motivating students, finding real-life or extra-mathematical applications of mathematics, or showing that school mathematics reflect the cultural heritage of humanity as a whole (with inputs from many cultural areas) etc. (Jankvist, 2009). Also, the use of historical sources in the classroom is not mandatory. The difference between the 2010- and the 2019-syllabi does not lie in the general intentions, but in the strength of the "recommendation" that teachers select and study historical problems – and, possibly, historical documents – with their students, as one of the many ways to support students' engagement with mathematics. In the 2010-syllabus, the above quoted short passage was the only mention of this topic in a 7-page document, making it a rather elusive suggestion. By contrast, the 2019-syllabus is lavishly peppered with paragraphs explicitly labelled "history of mathematics", which makes this topic one of the few guiding threads of the whole syllabus (along with "proof" and "algorithmic thinking and programming"). Some of these paragraphs are carefully worded and give specific suggestions, as in: "The history of probability theory provides a framework to explain the mathematization

of chance. An instance is given by the correspondence between Pascal and Fermat on the problem of points – also known as Méré’s problem –, along with the ensuing works of Pascal, Fermat, and Huygens. The problem of the Duke of Tuscany, or Leibniz’s works on games of dice can also be mentioned.” (Grade 10). Other paragraphs are rather pithy, leave much room for interpretation, and do not even hint at specific means of implementation; as in: “One can mention the slow elaboration of the notion of function, from Antiquity up to today’s codification by Dirichlet, by foregrounding some important stages: Newton, Leibniz, Euler. The importance of the algebraic notation should be stressed.” (Grade 10)

These features of the new syllabus came as a surprise to both teachers and teacher-educators, and the long list of rather cryptic yet heavy-handed historical “suggestions” understandably bewildered most. Moreover, the traditional resources on which teachers usually draw to meet the requirements of a new curriculum were wanting: firstly, up until the 2010 reform of teacher-training, very few teachers had any academic background in the history of mathematics, nor any experience of its inclusion in the classroom. Secondly, the syllabus to be implemented as from September 2019 was published shortly before, leaving very little time for commercial publishers to develop textbooks (there are no official textbooks in France). Since the historical suggestions were rather new, unexpected, and required that some textbook authors be well-versed in history (which is not usually the case), many textbooks failed to meet the challenge. Thirdly, when a new syllabus is published, it is customary for the Ministry of Education to publish guidelines for its implementations along with on-line teaching resources. In 2019, such resources were published to scaffold the implementation of the “proof” and “algorithmic thinking and programming” transversal features of the syllabus, but none regarding the historical suggestions.

Research protocol

The publication of the 2019-syllabus came as a somewhat pleasant surprise and unexpected endorsement for the IREM network (Institutes for Research in Mathematics Education). Indeed, these state-funded structures in which academics (mathematicians, mathematics educators, and, occasionally, historians of mathematics), teacher-trainers, and teachers collaborate to develop resources for teaching and teacher-training has a subcommission focusing on history and epistemology of mathematics. Since the 1980s, both in the French national context of the IREM network, and in the international context of the HPM Study Group, this subcommission has been working along the exact same general lines mentioned in both the 2010 and 2019 syllabi, arguing that many historical documents provide opportunities for a genuine engagement in mathematics, i.e. to “do mathematics”. Moreover, it has accumulated a significant collective experience of resource development (Fauvel, 1990; Chorlay, 2016, Barbin, 2018). This subcommission reacted to the publication of the 2019 curriculum by launching the collective development of a book meant to provide high-school teachers with a range of thought-through and empirically tested (at least once!) classroom activities based on historical documents and compatible with the new curriculum.

This collective project is the context of our study. A study which involves two types of participants – five high-school teachers and a researcher in history of mathematics and mathematics education. A study which weaves together two distinct projects: one is the

development of a classroom session, with a view to contributing a chapter to the new IREM book; one is a research project, carried out by the researcher and bearing on the work of the five teachers as they engaged in the resource-development project. It was agreed from the outset between each teacher and the researcher that both projects were distinct but compatible. The following protocol was agreed upon:

1. The researcher would select one historical document which he thought fit for the resource-development project. All teachers would be given the same document.
2. A first meeting would take place between the researcher and each teacher, individually. The historical document would be read together; its mathematical content would be discussed; possible connections to the curriculum would be discussed, in a “brainstorming” mode. The goal of this session was to generate a shared understanding of the didactical potential of the document. “Shared” meaning: shared between the researcher and each of the teachers; and similar for all teachers, even though they did not communicate with one another. In this meeting, no specific choices of implementation would be made or even discussed. The meeting would be recorded.
3. Each teacher would work independently from both the other teachers and the researcher in order to design some teaching session compatible with the resource-development project. For research purposes, teachers were asked to endeavour to keep a record of their work: personal notes, draft versions of the final documents, etc.
4. Each teacher would implement the session(s) she/he designed. The sessions would be recorded, either in audio or in video form.
5. Two short interviews would take place: (1) shortly before the actual session(s), the researcher would carry out a semi-guided interview bearing on (a) the teacher’s self-recollection of the design process, (b) the choices which the teacher made along the way, (c) the goal(s) of the session(s), (d) the expected or possible difficulties, to be experienced either by the students, or by the teacher. (2) shortly after the session(s), an informal debrief would focus on topics (c) and (d).
6. This would be the end of the research project. The resource-development project would enter new phases: exchange of information among the teachers; possible engagement in a new task-design cycle (alterations, then implementation of the altered sessions); writing of a chapter for the IREM book. These new phases would involve both the teachers and the researcher in a collaborative way.

Let us underline a few specific features of this research protocol. First, in the research phase (stages 1 to 5), there was no communication among teachers, so we are not studying an instance of collective or collaborative task design. Second, the nature of the teacher-researcher interaction was of the “clinical partnership” type (Wagner, 1997). Consequently, this should not be considered an instance of teachers and researchers working as “partners in task design” (Jones & Pepin, 2016). Third, we are not studying how teachers interact with curriculum *material* such as textbooks: the two documents which set the stage for the teacher’s task design activity are the national syllabus on the one hand, and a historical document on the other hand. Whether or not the teachers would look for and use other

documents in the task design process is one aspect of their engagement in this process that would be studied.

***A priori* analysis of the didactical potential of the historical source**

We selected a three-page extract from the 1774 French edition of the first volume of Euler's *Elements of algebra*. The teachers were told that this book is not a research treatise but rather a didactic work, covering algebraic topics ranging from the very elementary (operations with fractions and directed numbers) to the rather advanced (solutions of algebraic equations up to the fourth degree). The extracts were taken from the final chapter. The teachers were given a three-page document, but the researcher explained that they would focus on the first part consisting of paragraphs 784 and 786, the rest being provided mainly for context. The document below is taken from the 19th century British edition, which we altered slightly both to stick to the mathematical notations used in the 1774 French edition and to restore a numerical error which got corrected in later editions:

Document 1: Extract from (Euler, 1828, pp.677-680)

CHAP. XVI. Of the Resolution of Equations by Approximation.

784. When the roots of an equation are not rational, and can only be expressed by radical quantities, or when we have not even that resource, as is the case with equations which exceed the fourth degree, we must be satisfied with determining their values by approximation; that is to say, by methods which are continually bringing us nearer to the true value, till at last the error being very small, it may be neglected. Different methods of this kind have been proposed, the chief of which we shall explain.

(...)

786. We shall illustrate this method first by an easy example, requiring by approximation the root of the equation $xx = 20$.

[Footnote by J. Bernoulli: This is the method given by Sir Is. Newton at the beginning of his Method of Fluxions. When investigated, it is found subject to different imperfections; for which reason we may with advantage substitute the method given by M. de la Grange, in the Memoirs of Berlin for 1767 and 1768.]

Here we perceive, that x is greater than 4, and less than 5; making, therefore, $x = 4 + p$ shall have $xx = 16 + 8p + 16 = 20$; but as pp must be very small, we shall neglect it, in order that we may have only the equation $16 + 8p = 20$, or $8p = 4$. This gives $p = \frac{1}{2}$, and $x = 4\frac{1}{2}$, which already approaches nearer the true root. If, therefore, we now suppose $x = 4\frac{1}{2} + p$; we are sure that p expresses a fraction much smaller than before, and that we may neglect pp with greater propriety. We have, therefore, $xx = 20\frac{1}{4} + 9p = 20$, or $9p = -\frac{1}{4}$; and consequently, $p = -\frac{1}{36}$; therefore $x = 4\frac{1}{2} - \frac{1}{36} = 4\frac{17}{36}$.

And if we wished to approximate still nearer to the true value, we must make $x = 4\frac{17}{36} + p$, and should thus have $xx = 20\frac{1}{1296} + 8\frac{34}{36}p = 20$; so that $8\frac{34}{36}p = -\frac{1}{1296}$, $322p = -\frac{36}{1296} =$

$-\frac{1}{36}$ and $p = -\frac{1}{36 \times 322} = -\frac{1}{11592}$; therefore $x = 4\frac{17}{36} - \frac{1}{11592} = 4\frac{4473}{11592}$ value which is so near the truth, that we may consider the error as of no importance.

The goal of stage 2 of the research protocol was to reach a shared understanding regarding some critical features of the text. In the list of features below, we italicized those contents which are explicitly mentioned in the high-school curriculum. Although the notion of *didactical potential* of a document calls for a more refined delineation, for now we will consider that connections between the content of the document and explicit curriculum goals testify to this potential. Consequently, this potential is not a property of the document *per se* but is highly context-dependent. Below, we also added (and italicized) the grade(s) in which these goals are mentioned (except for the “algorithmic thinking and programming”, for which the same goals hold across all three high-school grades).

- This text provides opportunities to carry out routine calculation (*grades 8-10*): expansion of $(4+p)^2$, calculation with fractions, solving first degree equations, solving equation $x^2 = 20$. Even though the procedures are routine, the numerical values involved quickly become difficult to operate upon by pen and paper calculation only.
- This provides opportunities to carry out comparisons of specific numbers, either by comparing the successive approximations $4, 4\frac{1}{2}, 4\frac{17}{36} \dots$ to a numerical approximation of $\sqrt{20}$; or by comparing the squares of $4, 4\frac{1}{2}, 4\frac{17}{36} \dots$ with 20. These are all *grade 10* goals.
- The text weaves together two *genres* of mathematical texts: the exposition of an algorithm, and a heuristic argumentation providing some warrants for claims regarding key steps of the calculation.
- The meaning of the main warrant (“*pp* must be very small, we shall neglect it”) is ambiguous, and the text does not provide any proof-type justifications for it. Several interpretations are possible. A static interpretation: $|p|$ being less than one (a claim which also calls for warrants!), p^2 is less than $|p|$ (*grade 10*), also less than $|8p|$ etc. But Euler wrote that, as the algorithm unfolds, one is *ever more* justified in neglecting p^2 , thus possibly pointing to an asymptotic interpretation such as: when p is less than 1 and tends to 0, not only is $|p|$ less than p^2 , but it becomes infinitely less since the ratio $|p|/p^2$ tends to $+\infty$.
- The text mentions or points to several topics in number theory. First, the introductory paragraph mentions a classification of numbers (some are “rational” while some other necessarily involve “radical quantities”), and, implicitly, $\sqrt{20}$ belongs to the second category (*grade 10*). Second, number-theoretic considerations provide an answer to a key question regarding the algorithm: it will not stop, should the condition to be met be the production of the exact value of $\sqrt{20}$. Indeed, starting from a rational input (such as 4), the algorithm will yield only rational numbers, thus leaving $\sqrt{20}$ beyond reach.
- The text displays the first steps of a method, but the claims as to the scope of this method are implicit. Is Euler claiming that the four values from (4 to $4\frac{4473}{11592}$) are

increasingly better approximations of $\sqrt{20}$ (*grade 10*)? That an iterative interpretation of the algorithm leads to a sequence of numbers with limit $\sqrt{20}$ (and with a strictly decreasing distance between the sequence and its limit) (*grade 12*)? That this “method” works for *all* square roots? Or even for all polynomial equations?

- Jean Bernoulli’s footnote reminds the reader of the fact that Euler is merely expounding Newton’s method (actually in Raphson’s version (Bailey, 1989; Ypma, 1995)). To the expert reader, this should bring to mind the topics of tangents and derivatives; topics which, on the face of it, do not play any part in the text. However, the linearization of the equation amounts to considering a tangent instead of the curve, and the relevance of these ideas (algebraic or geometric linearizations) reflects the relevance of the concept of derivative as provider of local linear approximations. It so happens that, when one deals with polynomial functions only, derivatives can be defined and worked out purely algebraically. Hence Euler’s text can be seen as presenting a very interesting special case (theory of derivatives in a polynomial context) of a very important general concept (the derivative as provider of local linear approximations) (*grades 11 and 12*).
- The text illustrates the first steps of what is clearly an iterative algorithm. Identifying this text as presenting an iterative algorithm, extracting the algorithm by editing out the heuristic parts of the text (parts which may also contain algorithmic steps, e.g. expand $(x_n+p)^2$, solve linear equations), and, possibly, implement it in a programming language, all these are tasks which fit exactly the “algorithmic thinking and programming” strand of the whole high-school syllabus.
- In *grade 10*, other algorithms for the approximation of the solutions of numerical equations – such as bisection – are to be studied and implemented. Since Euler presents a different method, it could be interesting to compare these methods in terms of efficiency. In *grade 12* it can be proven that the convergence of Newton’s method in the case of square roots is quadratic, hence much faster than the linearly convergent dichotomy. Also, implementing Newton’s method requires that a first rough approximation of a solution be taken as starting point (here $\sqrt{20} \approx 4$), which, in itself, calls for another (maybe more elementary) algorithm.
- Euler’s text can be criticized, or at least questioned, as to rigour. In particular, he used the same letter x to denote different numbers; same for p . At least two reactions could be mathematically and didactically relevant: one could either realize the fact that this iterative method generates recursively defined sequences, and introduce notations such as $x_{n+1} = \frac{1}{2} \left(x_n + \frac{20}{x_n} \right)$ (*grade 11*); or consider that the letters represent programming variables and not mathematical variables (*grade 10*). In this second context, some of the “=” symbols should be read as value-change operators and not as mathematical equalities.

The research protocol (stages 1-5) was implemented in the 2020-2021 school-year, with five teachers; the data are being processed. The goal of this initial progress report was to establish that the historical document selected by the researcher has the potential to generate

cognitively demanding classroom sessions, and leaves considerable leeway for the teachers to engage in task design and implementation.

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