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Instrumental orchestrations by a university teacher in an online linear algebra course

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In this article, we present an instrumental orchestration for online math education. The case study is of a university teacher in a linear algebra course with engineering students in a remote learning setting. We used the instrumental orchestration approach as a theoretical framework for planning and organizing the artifacts involved in the environment (didactic configuration) and the modalities of their implementation (mode of exploitation). The activities were designed using virtual manipulators, guided exploration sheets, and video recordings of individual or paired work by the students. The results of the observation of five sessions are presented and the orchestrations of a pedagogical sequence to introduce the concepts of value and eigenvector are briefly discussed. This work suggests new instrumental orchestrations for the teaching of mathematics online.

Keywords: Instrumental orchestration, linear algebra teaching and learning, university mathematics, online course.

Introduction.

In March 2020, the World Health Organization declared a global pandemic due to COVID-19 and recommended lockdowns and mobility restrictions, forcing educational institutions to migrate from their face-to-face teaching model to online education (Engelbrecht et al., 2020). Most institutions had to improvise online courses using various digital devices and internet platforms. Although remote education has been developing for years in various forms—distance learning, e-learning, and hybrid learning—to reach more students (Silverman & Hoyos, 2018), it had not fully permeated most educational institutions. Oktaç (2004) reports the implementation of a remote introductory linear algebra course in an asynchronous learning environment. Teaching linear algebra online is a challenge, as it is one of the first abstract math courses students encounter during their early years in college. At ICME13, there was a discussion group on teaching and learning linear algebra, which considered the ways a linear algebra course could be adapted to meet the needs of students from other disciplines, such as engineering, physics, and computer science. It also reflected on how technology should be used in teaching linear algebra (Stewart et al., 2018, p. ix). Gol-Tabaghi and Sinclair (2013) studied the geometry of eigenvalues and real eigenvectors of a 2×2 matrix using dynamic geometry software (DGS), in which by dragging the vector x onto the screen, the vector Ax moves accordingly. They concluded that dynamic geometric representations in the sketch enabled students to understand the concepts of eigenvector and eigenvalue by identifying their invariant geometric properties and helped them develop dynamic-synthetic-geometric thinking.

On the other hand, and perhaps in anticipation of the current circumstances, Trouche (2004) introduced the notion of *instrumental orchestration* for planning a lesson, defined as the didactic

management of the different artifacts available in the classroom, the organization of space and time, and the way the artifacts are connected.

Theoretical background

An instrumental orchestration is “the intentional and systematic organization of the various artifacts available in a computerized learning environment by the teacher for a given mathematical situation, in order to guide students’ instrumental genesis” (Drijvers & Trouche, 2008, p. 377). An instrumental orchestration is defined by a didactical configuration and the way in which this configuration is exploited. The Sherpa-student orchestration, which involves a student whose computer or calculator is displayed to the rest of the class on a projector, was for several years an emblematic didactic configuration. For a given configuration, there are several possible operating modes. As an instrumental orchestration is partly prepared in advance and partly created "on the spot" during teaching, Drijvers and colleagues (2010) add a third level called *didactical performance* to account for the multiple adjustments that the teacher and students may make. Drijvers and colleagues (2010) suggest considering the triplets model, a sort of jazz ensemble composed of beginners, advanced musicians, and the teacher as a conductor who prepares joint participation, but who is open to the pupil’s improvisation and interpretation, on order to recognize the contributions of each of these levels. This metaphor became a reality in our experience because there was no precedent for online teaching of linear algebra, so we had to "improvise" ways and methods to teach both the mathematical task and the technological tool, for example, the forms of student-student and student-teacher through communication through the platform. Drijvers and his colleagues have extended the repertoire of instrumental orchestrations to include technical-demo, explain-the-screen, link-screen-board, discuss-the-screen, spot-and-show, Sherpa-at-work (Drijvers et al., 2010), board-instruction and guide-and-explain (Drijvers et al., 2013). In the first three orchestrations is it mainly the teacher leading the communication, while in next three students have more opportunities for participation. In the technical-demo orchestration, the teacher explains the basic techniques of a tool in order to familiarize students with it.

In the explain-the-screen orchestration the teacher explains what is happening on the computer screen, but unlike the previous one, this orchestration involves mathematical content. Technical-support orchestration refers to the process of helping the learner with technical issues with the tool, such as software and/or hardware issues. Discuss-the-screen orchestration involves a group discussion about what is happening on the computer screen. In the link-screen-board orchestration, the teacher refers to the relationship between what the tool displays and the mathematical concept. In spot-and-show orchestration, the teacher chooses a student's work before class, either for the student to explain in class or for discussion with the whole class. For the Sherpa-at-work orchestration, the teacher asks a student (referred to as the Sherpa) to demonstrate or perform a certain process in the technological environment (for details, see Drijvers et al., 2013). Thus, our research question is: What instrumental orchestrations are applied by a university professor using technology to teach the concept of eigenvalue and eigenvector in a distance education setting?

Method

The study was developed under a design-based research methodology, which consists of three research phases: preparation and design, teaching experiment and, retrospective analysis (Bakker, 2018). A first research cycle (Orozco-Santiago, 2020) was carried out in face-to-face teaching. For the design of our task sequence, we used a hypothetical learning trajectory (Simon, 1995) but, due to the global public health emergency caused by COVID-19, circumstances compelled them to use the Zoom platform for the first time for online teaching. However, the teacher has extensive experience in the use of digital resources. The trajectory consists of seven activities. During the second cycle of research, the teaching experiment was carried out with a small group of ten students (6 women and 4 men) in their second year of engineering studies at a Mexican public university, over a period of two weeks. When designing the didactic path of our work, we were able to detect a priori the use of the following orchestrations for the teacher (Figure 1), and each activity for the pupils was accompanied by a didactic interactive virtual environment (DIVE) and a guided exploration sheet.

	Session 1	Session 2	Session 3	Session 4	Session 5
Technical-demo	✓	✓	✓	✓	
Guide-and-explain					
Link-screen-board,		✓			
Discuss-the-screen	✓				
Explain-the-screen	✓		✓	✓	✓
Spot-and-show,			✓	✓	✓
Sherpa-at-work,			✓	✓	✓
Board-instruction	✓	✓	✓	✓	
Technical-demo individual,					
Guide-and-explain individual					
Link-screen-paper,			✓		✓
Discuss-the-screen individual,					
Technical-support individual,					
Individual Work	✓	✓	✓	✓	✓
Practicals in pairs				✓	✓

Figure 1: Orchestration suggestions for teachers

The design of a configuration depends on the technological environment (Figure 2), and in this case the students had laptops or mobile devices (smartphones, tablets). However, the hardware and conditions available to them for remote study were less than ideal. Internet connections were quite weak, there was noise at home and outside, and due to pandemic conditions, students were prone to stress. Most students could not use their cameras or microphones during courses because these require more bandwidth, which many did not have. In the course, the following topics had already been taught: matrices, systems of linear equations, vector spaces, linear combination, linear independence, and linear transformations. The participants had not previously studied the concept of eigenvalue and eigenvector.

Throughout the course, activity data was collected by the following: a) e-mail; b) WhatsApp; c) Google Drive; d) the Zoom platform; e) free software for video recording OBS Studio; d) digital capture of students' notebooks; e) video recordings of the researcher's classroom activities on Zoom; and f) video recording of student work and teamwork, in the DIVE and guided exploration sheets. Five of the lectures on the concepts of eigenvalues and eigenvectors were recorded on video. To answer the research question, after the end of the teaching experiment, the five video recordings of the class were observed and the data analysis presented in this article focused on the identification of

the orchestrations proposed by Drijvers and colleagues (2013), in the first three video recordings. In the video recording of session 1 (S1), 12 different episodes were identified. In S2 19 and in S3, 21.

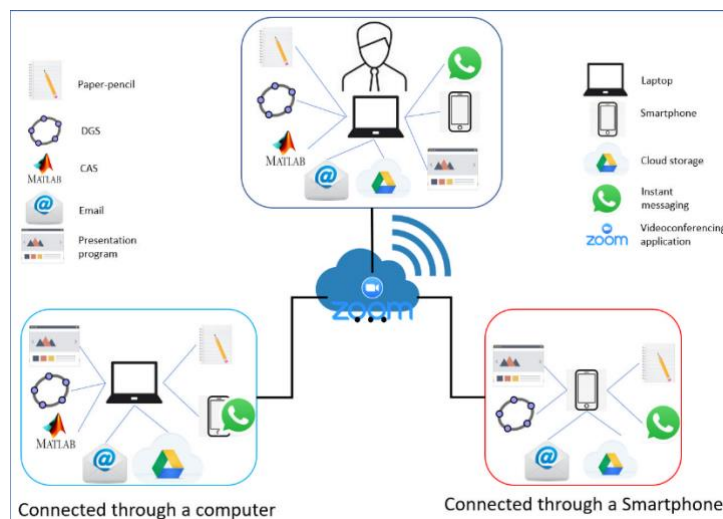


Figure 2: A remote configuration

Results

The instrumental orchestrations reported by Trouche (2004) and Drijvers et al. (2010, 2013), were developed within the framework of face-to-face teaching where the teacher and the students are in a classroom. But because of lockdown requirements during the pandemic, this teaching experience was developed synchronously online, which allowed for the use of some orchestrations and required the improvisation of others. For example, the technical-demo orchestration was used for the students to familiarize themselves with the Zoom software, offering instructions on how to use meeting control.

Since in the virtual model, the Work-and-walk-by orchestration was not possible, and we wanted to observe the individual instrumental genesis developed by the students and at the same time discuss as a group the correctness or not of the answers and the work of each student, the modality that we called *Student-shared-screen* appeared, which allowed the teacher to accomplish the task, it should be noted that the name, the peculiarity, and the characteristic of this orchestration are made in the analysis after the experiment with the data obtained described in Figures 1 and 2. As this orchestration was improvised, when we analyzed the data and using the taxonomy defined by Drijvers and colleagues, they mention that the didactic configuration for the Spot-and-show orchestration "includes access to the students' work in the technological environment during lesson preparation" and for Sherpa-at-work orchestration, "includes access to the technology and projecting facilities, preferably access to student work" (2013, p. 999), but to implement access to the pupil's screen, it is necessary to ask the student's permission. Not having found an orchestration for this type of activity which we believe is important in online education, we call it the new Student-shared-screen orchestration, which consists of a didactic configuration in which the student allows the whole class to access his work on his computer screen. As an exploitation mode, the teacher asks the student to share his screen and discuss his work with the whole group. As can be seen, this is an orchestration prior to Sherpa-at-work and spot-and-show.

To simulate chalk, the teacher chose to use the Zoom Annotation tool and use a blank PowerPoint slide or a DIVE as a whiteboard, on which the teacher and students could write in order to present, explain and solve the exercises. We will call this the *Share-board* orchestration, and it is a recurring didactical configuration very similar to traditional classroom teaching (see Figure 3).

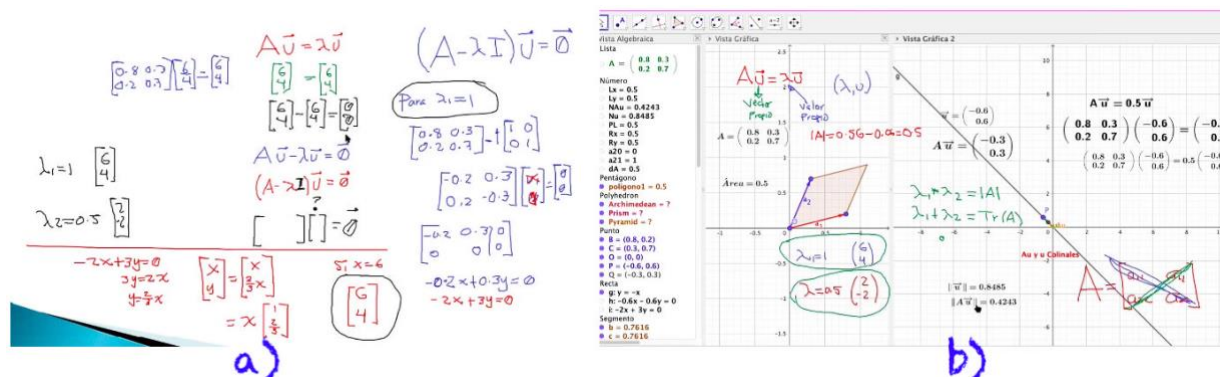


Figure 3: a) PowerPoint and b) DGE, both used as a whiteboard

In the third task, students had to drag two vectors onto \mathbb{R}^2 in the DGS and observe their relationship to the columns of a 2×2 matrix. The teacher identified an inappropriate response from the video recordings: some students did not drag the vectors, but directly changed the values of the matrix in the algebraic view. Through to the student-share-screen and spot-and-show orchestration, the teacher decides to ask the student to explain to the whole class how he modified the matrix,

- Teacher: The coordinates of the vector a : what relation does it have with the matrix A ?
- Student1: None.
- Teacher: When you drag vector a_1 , what happens to matrix A ?
- Student1: Nothing happens to it.
- Student2: Changes the values a_{11} and a_{22} of the matrix [this answer is provided by the chat].
- Student3: The values in the first column change.

Students who did not drag the vectors and directly modified the values of the matrix failed to identify the relationship between the vectors and the columns of matrix A . This didactical performance was observed thanks to the design of the DGS.

For the fourth task, the teacher used the explain-the-screen and share-board orchestration, using the work of one of the students as a mode of exploitation. The selected student's work was the criteria for the first work to be uploaded to Google Drive (see Figure 4).

For the next activity, construct matrix A .
 Drag the point P (vector \vec{u}) and position (if possible) the vector \vec{u} collinear with the vector $A\vec{u}$ and note its coordinates.

\vec{u}	$A\vec{u}$	λ Proportionality ratio between $A\vec{u}$ and \vec{u}
(6, 4)	(6, 4)	1
(-2, 2)	(-1, 1)	0.5
(-2.5122, -1.6682)	(-2.5102, -1.6702)	1
(4, -4)	(2, -2)	0.5
(2, -2)	(1, -1)	0.5
(-4, 4)	(-2, 2)	0.5

Figure 4: Search for collinearity of the u -vectors and the Au -vector

Thomas and Stewart (2011) point out that textbooks show $A\vec{x} = \lambda\vec{x}$, then $A\vec{x} = \lambda I\vec{x}$, but do not explain that it is $\lambda I\vec{x} - I$ is a matrix and not a scalar. Using the *board-instruction-tech* orchestration, the teacher explains how the equation $A\vec{x} = \lambda\vec{x}$ is transformed into $(A - \lambda I)\vec{x} = \vec{0}$ with student participation (see Figure 3a). In another activity, the teacher began with explain-the-screen orchestration, providing students with a DIVE that was not considered in the learning trajectory. After having carried out the calculation of the eigenvalues and the eigenvectors on the screen, the teacher explained to the students that in the CAS view of the DGS, they could observe step by step the calculation of the characteristic polynomial, of the roots of the characteristic polynomial (eigenvalues), and the solution of the equation $(A - \lambda I)\vec{x} = \vec{0}$ for each of the eigenvalues (if they exist). After each line of the CAS was explained, a didactical performance was presented. Here the students commented that on their computers, the IME given did not work like the one the teacher presented, and the CAS did not recognize the same commands as those of the teacher (see Figure 5a). Using student-shared-screen orchestration, a student split his screen to perform and follow the teacher's instructions, and the student's computer screen froze. The teacher had to ask another student to share his screen. To solve the command problem, the teacher was able to use his technological knowledge to search the GeoGebra Wiki and find another command which made the DIVE work correctly on the students' computers (see Figure 5b).

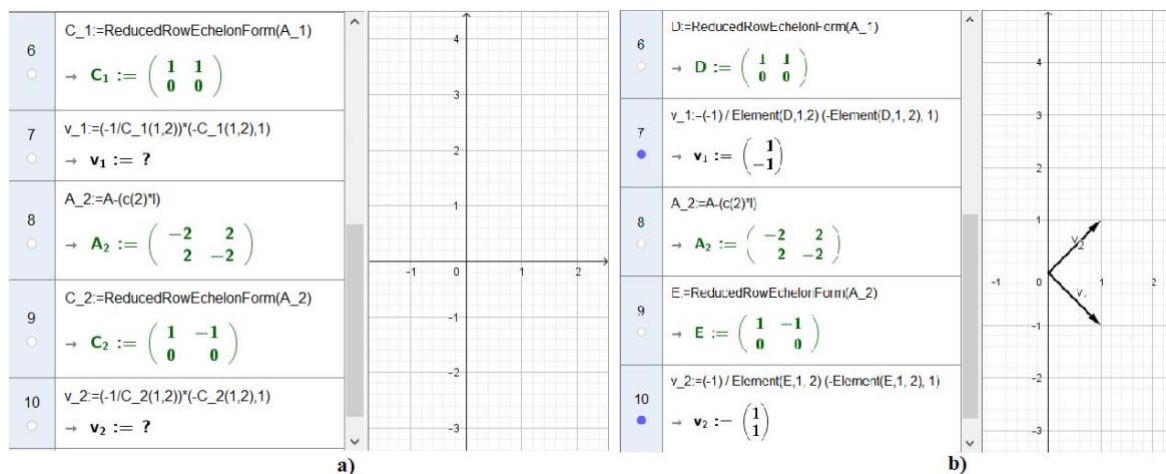


Figure 5: Calculation of eigenvalues and eigenvectors in the CAS view of the DGE

Discussion

For the student-share-screen orchestration, a didactical configuration for this orchestration is the sharing of the videoconference screen by a student, which allows the students to follow the discussion, using Sherpa-at-work or spot-and-show. As an exploitation mode, the teacher can ask the Sherpa-student to explain her work, or he can ask questions and ask the student-Sherpa to perform specific actions. In our case, when a student wanted to share their paper-and-pencil work, there were several exploitation modes. 1) A student with a weak internet connection could take a photo of their notebook using their smartphone, upload it to the WhatsApp group, their cloud folder, or conference chat, so that the teacher or another classmate could download and project their work while they explained it. This way we can define this orchestration as *Shared-resource*. But because of the weak internet connection, the gestures of the teacher and the students were overlooked.

The videoconferencing platform used allows for the creation of small work teams (Breakout Rooms) assigned either randomly using the app, or by the teacher. It also allows the teacher to move among the teams to observe their work and, on the same platform, the students in each teams can ask for help from the teacher (Ask for Help). A downside to this option is that if multiple teams ask for help with the same issue, only members of the team visited by the teacher will be able to hear and see the discussion.

Conclusion

Supporting mathematics teachers in their efforts to integrate technology into their teaching practice remains a challenge for the mathematics education community (Trouche, 2018). We believe that teachers need basic skills and knowledge in the use of technologies so that they can support their students' learning in these online environments.

Returning to our research question: What instrumental orchestrations are applied by a university professor using technology to teach the concept of eigenvalue and eigenvector in a distance education setting? As technology evolves, existing types of instrumental orchestrations (Drijvers et al., 2010, 2013; Trouche, 2004) need to be re-examined to determine how they might be modified and expanded. We examine the teaching practice of a university engineering professor during the onset of the COVID-19 pandemic through online education, using the theoretical framework of instrumental orchestration. Students contributed through video recordings of their work and ideas, which allowed the teacher to access this work and decide between Sherpa-student or spot-and-show orchestrations from this work. Some orchestrations from traditional education were also observed. In this observation, we identified four new types of instrumental orchestration: student-share-screen, share-board, board-instruction-tech and shared-resource.

In this course, being able to observe the personal development on paper-and-pencil by the whole class made the students more willing to share their incomplete attempts at problem-solving. Videoconferencing technologies offer means of communication that raise new questions regarding the organization of lessons. The pandemic has raised new questions about synchronous teaching: the use of computer screens and webcams instead of blackboards, the ability to carry out multiple activities such as Chat watching, student participation by raising their hand, muting and unmuting the microphone, solving technical problems on the fly. The activities developed in this study can be implemented in face-to-face classes, encouraging autonomous and collaborative work, and installed on a freely accessible server where students can learn at their own pace.

The role of the smartphone as a computer has changed the way we can experience mathematics (Borba, 2021). We believe that the impact of the pandemic has changed the traditional way of teaching and learning, which, together with the ever-increasing demand for study and the limitations of institutions, makes hybrid distance learning a necessity. It is, therefore, necessary to carry out studies on the design and implementation of educational software on digital devices and, of course, new orchestrations.

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