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Mathematical creativity in coding dancing animations: a fallibilistic view

Myrto Karavakou¹ and Chronis Kynigos²

¹Educational Technology Lab, National Kapodistrian University of Athens, Greece;
karavak@ppp.uoa.gr

² Educational Technology Lab, National Kapodistrian University of Athens, Greece;
kynigos@eds.uoa.gr

In this paper we investigate student's creative mathematical activity, while engaging in coding digital animations. A logo-programming authoring system which integrates Turtle Geometry, 3D space and dynamic manipulation of figural models was used. Three students of 8th and 10th grade engaged in creating figural dancing animations and synchronizing them to the rhythm of a song, by constructing an artefact and manipulating its geometric transformations in space. We adjusted the Creative Mathematical Action Framework, that is based on a fallibilistic view on creativity, into a constructionist technological context. The results indicate that artistic values such as synchronicity, symmetry and periodicity were conceived and applied within the digital resource and acted as motivators for persistently tinkering with the artefact. This context provided fruitful ground for creative mathematical activity of forming, expressing, exploring and expanding mathematical ideas.

Keywords: Creative mathematical actions, fallibilism, programming, mathematics, art.

Introduction

During the last decades, creativity has been constantly revisited with research attempts for understanding, analyzing and cultivating it in mathematics education. Thus, a diversity of theoretical perspectives on mathematical creativity has been developed, mainly addressing it by means of inborn trait and giftedness (Mann, 2006; Sriraman, 2005) or problem solving-posing processes (Silver, 1997). However, Riling (2020, 2021) has recently made a critical review on the existing approaches, pointing out their limitations over how creativity takes place in mathematics classroom and who can be seen as mathematically creative. Riling proposed an alternative view where mathematical creativity is discussed in terms of students' actions and is open to cultural and social influences – in a parallel manner as in artistic domains such as painting or choreography. She developed a framework in which mathematical creative actions are defined in a concrete way, that enables their investigation in students' activity. In this context, creativity emerges as free, independent expression of mathematical ideas, that can be nurtured in all students. This kind of creativity dissociates mathematical meaning-making from taken-as-established mathematics and fixed curriculum structures (Kynigos & Diamantidis, 2021). In this study, we adopted this challenging view towards mathematical creativity within the context of middle school students using a digital resource called MaLT2, which provides means for developing, expressing and investigating mathematical ideas while modeling 3D figural objects (Kynigos, 2015). Our aim was to investigate how this kind of creativity can be cultivated in the context of open artistic creation. We employed Riling's framework as lens to identify and look into instances of student's creative actions while engaging in creating a digital animation in MaLT2.

Theoretical Framing

Fallibilism in Mathematics Education

The view of creativity we are examining has its epistemological roots into the fallibilist approach on mathematics nature, as opposed to the formalist one (Davis & Hersh, 1980; Ernest, 2003; Kynigos, 2015). Formalism is an epistemological paradigm that conceives mathematics as ‘an objective, absolute, certain and incorrigible body of knowledge, which rests on the firm foundations of deductive logic’ (Ernest, 2003, p. 2). Thus, mathematical systems are discharged from any human influence and are served as pure isolated knowledge that humans can only approach through discovery. On the contrary, fallibilism relies on the assumption that mathematics is originally created by human thought. Fallibilist mathematics is open to be modified or disproven according to a person’s use of it. A main difference between these two paradigms is their educational purposes. Formalism aims at understanding the abstract products of the served mathematical knowledge through practice. On the other hand, fallibilism focuses on the activity of doing mathematics, rather than mathematics itself, aiming at enriching students’ meaning-making process (Kynigos, 2015; Kynigos & Diamantidis, 2021). Traditional schooling consist of formalist practices, which are based on the assumption that every task has a unique right answer and any mistake is accompanied with disapproval and criticism. This approach leaves limited space for students to experience creative engagement in mathematics. Fallibilism, in contrast, is connected with constructionist practices in education (Kynigos, 2015). According to Constructionism, students’ learning of mathematics occurs naturally while creating and sharing tangible artefacts (Papert & Harel, 1991). The design of constructionist activities within expressive digital media aims at providing fertile ground for powerful mathematical ideas to be constructed. Constructionist learning environments play a two-way role; that is an interplay of reinforcement between mathematics and artefacts. On one side, mathematics is used as tool for constructing or tinkering an artefact, taking a fallibilistic form. Conversely, creating an artefact provides dense opportunity for students to formulate and explore mathematical possibilities. Students are encouraged to engage in both directions and externalize their ideas by adopting the role of a designer, an engineer or an artist. The tangible outcome then becomes a public entity, accessible for reflection by its creator or others. In this fallibilist context, creativity becomes an integral part of mathematics learning in a direct way: mathematics is used for actual creation of personally meaningful content.

Creative Mathematical Action Framework

For our study, we adopted the Creative Mathematical Action Framework (CMAF) proposed by Riling (2020, 2021), which views creativity from a fallibilist perspective and places it into its community of action. We adjust this framework into a constructionist technological context. The CMAF offers a model for identifying creativity in mathematical contexts, such as a mathematics classroom providing a specific technological resource (Kynigos & Diamantidis, 2021). It puts emphasis on the process of *how* creativity takes place, instead of *what* final product is created. Creativity is conceived as a type of action connected to the emergence of new mathematical possibilities as experienced by oneself. Thus, CMAF links creativity to personal meaning-making process. We further connect new mathematical possibilities to the way they are communicated and

represented through changes in a digital artefact. According to Riling, a creative mathematical action is defined “as one that transitions a given mathematical context into a new version of mathematics by creating ways of doing or thinking about mathematics that were previously not possible for a particular community of mathematicians” (2020, p. 17). We expand this consideration by adding the component of the way a technological resource can influence this transition in a constructionist learning context. Thus, we conceive a creative action within a technological resource as one that transitions a given mathematical context into a new or expanded form of mathematical meaning by using technological tools to generate, express or explore novel mathematical ideas for creating and tinkering digital artefacts. Two main criteria for an action to have creative potential are: a) to origin from students’ own account instead of others’ (i.e. their teacher) guidance and b) to refrain from the standard ways of doing mathematics in school, where actions are traditionally manipulated by superior established curriculum practices (Riling, 2020). She also emphasizes the importance of aesthetic experiences, such as fruitfulness, visual appeal, mystery and surprise, in activating creative actions. Riling (2021) listed six types of creative actions derived from her data analysis. These types were adjusted to our constructionist view as follows: 1. *setting out*: generating initial ideas and goals intuitively by using the digital tools spontaneously as means to explore unknown aspects of an artefact; 2. *imagining*: using the digital tools more systematically in order to imagine a plan of implementation of an idea and set more clear goals; 3. *manifesting*: taking concrete intended actions to make changes to the artefact by altering the existing mathematical context; 4. *familiarizing*: taking time to survey the current status of the artefact in a comprehensive or reflective way; 5. *recognizing*: reconsidering an aspect of the artefact or interpreting it in a new way; 6. *naming*: distinguishing an action pattern in the medium for a specific object or idea as a distinct entity. In this study, we consider creativity inside the CMAF and investigate students’ creative actions within the digital medium MaLT2 for the creation of a “dancing animation”.

Design of the research

The task in MaLT2

The digital medium used for this study was an online dynamic mathematical programming tool to tinker with 3D figural models called MaLT2 (<http://etl.ppp.uoa.gr/malt2/>), which is recognized by the Greek Ministry of Education. MaLT2 integrates a UCB-inspired Logo procedural language with affordances for dynamic manipulation of procedure variable values (Kynigos & Grizioti, 2018). Thus, MaLT2 provides three interconnected representations: programming, figural representations and dynamic manipulation of generalized values through their sliders. This last feature enables the dynamic behavior of a figural model that is constructed by a parametric procedure. Therefore, it provides a mathematical way to create an animation through geometrical transformations of figures. Dynamic change of figural models by manipulating variable values also enriches the opportunity for exploring mathematical properties, posing questions, formulating assumptions and getting instant feedback. In this way students can engage in a loop of interaction with the artefact. The task designed for this study was titled as “Dancing Animations”. We designed an artefact in MaLT2 that would be given to students to begin with, in order to make their own animation (Figure 1). The task was described in a worksheet as open as possible. The aim was to make an animation by

manipulating only one variable (instead of six variables in the case of the given artefact) in the same rhythm as a song selected from a specific list. The final product would be a video of the animation (the middle representation in Figure 1) added by the song audio.

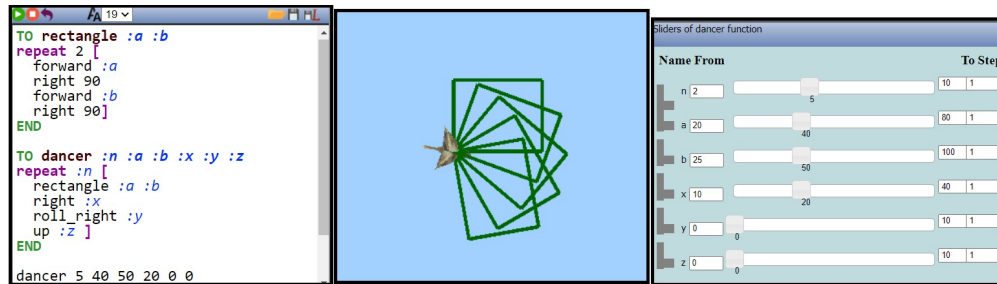


Figure 1. The initial artefact, in the three interconnected representations of MaLT2

Research Method

This study was implemented in a pilot level to three students; one girl of the 8th (Maria) and one boy and one girl of the 10th grade (Dimitris and Anna) of secondary school. They voluntarily participated all together in this two-hour-activity in an atypical after-school setting. They all had already participated in an introductory to MaLT2 activity. Each student had a laptop, a set of headphones and a notebook. A teacher-researcher was facilitating the activity by encouraging students to express their thoughts and ideas out loud. Data was gathered from screen-video, audio and voice recordings as well as students’ notes. We analyzed data from students’ discourse, notes and activity in MaLT2, following a grounded approach. We identified and analyzed instances of students’ actions that demonstrated mathematical creativity in terms of the CMAF. We categorized them into actions of *setting out*, *imagining*, *manifesting*, *familiarizing*, *recognizing* and *naming*.

Results

We identified instances of creative actions for each student that led to the creation of three distinct “dancing animations” synchronized to the rhythm of a song. In Table 1 we briefly describe the sequence of eleven creative actions made by Dimitris and Anna, who collaborated during the process. The merged shells of the Table indicate a common, shared creative action.

Table 1: Description of Dimitris’ and Anna’s creative actions

Type of action	Description of Dimitri’s action	Description of Anna’s action
1. Setting out	They expressed the goal to create a dancing movement while dragging the sliders spontaneously and observing the changes on the artefact.	
2. Imagining	While focusing on dragging one slider, they set the goal of finding a way to create a complete rotation that simulates the spinning around oneself, in order to “create the sense of dancing”.	
3. Manifesting	They selected the variable x that stands for the turn-command ‘right’ and tried different values for its upper limit.	
4. Familiarizing	They ended up setting the limits from 0 to 360 and realized that 360 is the value for the whole rotating dancing move.	
5. Naming	They connected this value to the notions of “full angle”, “full turn” and “full circle”. Dimitris also referred to it as “the period of the animation”.	

6. Recognizing	By dragging the slider of the variable x using the right arrow from the keyboard and simultaneously listening to the song each one selected, they expressed that their animation was “out of rhythm” and “too slow”.	
7. Imagining	They set the aim to fix the period of dancing move to the period of the song and “find a way to make it move faster and fit”.	
8. Manifesting	He considered the period of the song (Iggy Pop - The Passenger) to be 2 seconds and calculated that it corresponds to 60 units of the variable slider range. He used a multiplier to the command “right : x ”, which could control the speed of the animation. After trying different values, he ended up changing the command to “right 6*: x ”. He mentioned that “after I tried different values I realized 6 was the right one since 6 times 60 equals 360”.	She considered the period of the song (Milky Chace – Stolen Dance) to be 9 seconds and calculated that it corresponds to 270 units of the variable slider range. She used a multiplier to the command “right : x ”, which could control the speed of the animation. After trying different values, she ended up changing the command to “up 1.3*: x ”. She mentioned that she found it “almost empirically”.
9. Familiarizing	They observed their animation and realized it was doing “a complete dancing rotation perfectly fitting to the period of the song”.	
10. Recognizing	They realized their dancing move only lasts for a part of the song rather than its whole duration. They set the goal to make it last as long as the song extract.	
11. Manifesting	The song extract lasted 50 seconds. He calculated the right value of the upper limit of the variable slider for the animation to last 50 seconds, too. He counted the repetitive patterns (=25) and multiplied 25 by 60 (=1500), that is the number of degrees of each period of the animation.	The song extract lasted 34 seconds. She tried empirically values of the upper limit of the slider of the variable. She approached the exact value (=1020) by saying “I realized that each 3 beats it’s almost 100 degrees so I was adding 100 each time to reach it.”.

Dimitris and Anna followed a common sequence of creative actions, as a result of their collaboration and sharing of ideas. For example, they began with a shared instance of *setting out* and *imagining*, by manipulating the dynamic variation tools (sliders) of the procedure “dancer” (Figure 1). A dialog between them demonstrates their first instances of creative actions:

- Dimitris: How can we make the shape dancing?
- Anna: I imagine something like that... (She got up and made a complete rotation around herself.)
- Dimitris: We need to find a way to make the shape spin. This (dragging the slider of the variable x) creates the sense of dancing. (...) As the variable x values change, the shapes turn continuously.
- Anna: Yes, but we have to increase the limit value. Right? Because we want a whole rotation.

This investigation was the starting point of these two students’ activity as a quest for answer to their own questions on how to create the sense of dancing. They posed the problem of “what value is suitable for the figure to complete a whole rotation” in order to simulate the dancing move of “spinning”, formulating an example of creative mathematical action of *imagining*. During this action, they used the concept of variable and made sense on it. Afterwards, two shared instances of creative actions of *manifesting* and *familiarizing* took place, as the following dialog indicates:

- Anna: I tried changing the right limit of the variable x. I think 360 is the value for the whole rotation.
- Researcher: Why did you choose variable x instead of a or b?
- Dimitris: I thought that these variables (showing x, y and z) are for the turns. A turn means something that changes its angle. 360 is a whole angle. This is why it works for 360. It's 360 degrees, so it's a circle. I realized it right before it reached 360. A circle means a complete angle, so a full rotation.
- Anna: If you put more than 360, it starts repeating the same move again. We could use that to make it keep rotating for the whole song.
- Dimitris: Yes, you are right. (...) It makes another whole rotation when it reaches 720. And then 1080. So 360 is the period of the animation.

Dimitris connected the turn of 360 degrees to the concepts of a complete angle and a circle, noting that this realization was not possible before. In addition, Anna pointed out that values higher than 360 degrees make the animation repeat in the same way. After testing it, Dimitris realized that in every multiple of 360 the movement was making a whole rotation. They both imagined a choreography of repeated rotations fitting at the period of the song. The rest of their creative activity is captured at Table 1. They worked cooperatively, with actions of *imagining* and *recognizing* shared and discussed, but their actions of *manifesting* and *familiarizing* were independent, leading to two uniquely different animations.

Table 2: Description of Maria's creative actions

Type of action	Description of Maria's action
1. Setting out	While dragging the sliders freely, she set the goal to create a dancing movement by using them.
2. Imagining	While dragging back and forth the slider of some variables, she expressed the idea of finding a way to create the sense of moving "up and down around the horizontal state continuously".
3. Manifesting	She selected the slider of the variable z that stands for the turn-command 'up' to move forth and back. She tried different values for both lower and upper limits. She ended up putting values that range equably from 360 which was the horizontal state: from 340 to 380; from 300 to 420; etc.
4. Familiarizing and Naming	She observed it and expressed that "360 is the center of the move and I need to keep the same distance right and left". She called the value of 360° as "the center of the animation" combining it to the notion of the center of symmetry.
5. Recognizing	She realized that her dancing movement "takes too long" and that she wants "to make it move faster" but "into the rhythm".
6. Imagining	She set the goal to find a way for the animation to "repeat the complete up and down move several times during the period of the song".
7. Manifesting	She considered the period of the song (Milky Chace – Stolen Dance) to be 14 seconds. She selected the time period of 2 seconds for her dancing motion, in order to be repeated 7 times during the song period. She noted that "2 sec = 22 forth and 22 back". She ended up setting the value range from $(360-11=)$ 349 to $(360+11=)$ 371.
8. Recognizing	While observing the animation by dragging the slider of the variable z, she expressed that the motion was very slow and didn't fit in the 2-second-song period.
9. Manifesting	She used a multiplier to the command "up :z", which could control the speed of the animation. She ended up changing the command to "up 1.5*:z".
10. Recognizing	She expressed that the speed of the animation seemed ok, but the shape does a different, less interesting movement. She said she needed to change the limits of the

	slider in order to make the previous symmetrical move in the selected speed.
11. Manifesting	While trying different values she said: “when :z becomes 360, the shape is not horizontal like before. The new middle state is at $360/1,5=240$. So the proper values are from $(240-11)=229$ to $(240+11)=251$.”
12. Familiarizing	She observed the dancing animation and said that “Now it is even nicer, since the movement is faster and as a result its movement is longer, more extensive and even nicer!” (Figure 2).

Maria on the other hand followed a different path of actions. She did not participate to the discussion with the others and chose to work on her own. Her creative actions are shown at Table 2. Firstly, her idea of dancing animation included a periodic rising and falling motion and the problem of synchronization with the song emerged naturally:

Maria: Maybe I can create this dancing move. (She started moving her hands up and down periodically.)

Researcher: Great idea! Can you think of a way?

Maria: I would try moving the shape up and down by dragging it back and forth. I will write down the limits I like the most (referring to the right and left limit values for the slider of variable z). But, first I need to find the value for which the shape is horizontal and put it in the middle. (...) Well, it is 360. 360 is the center of the move and I need to keep the same distance right and left (...) But when I listen to the song, the movement is too slow. I need to make it move faster up and down.

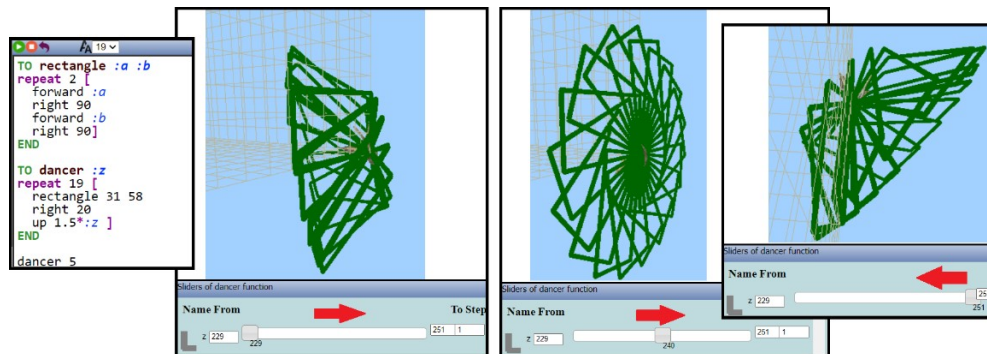


Figure 2. Maria's final "dancing animation"

After Maria posed the question “how to create the dancing motion of moving up and down”, she generated answers through *imagining*, *manifesting* and *familiarizing* actions. As revealed by her words, she considered the value of 360 degrees to be the “center” of the dancing move, connecting it to the concept of the center of symmetry. Afterwards, she set the goal to synchronize the motion to the music rhythm. In order to achieve it, she listened to the song carefully and noted down its rhythmic period (14 seconds). She went back to her animation and tried various values for the variable z limits, all of them symmetrical to the value of 360. She timed the duration of her artefact's period and noted that it takes 2 seconds for the slider to be dragged forth and then back when the value range is 22. Then she simulated the “up and down” motion by fixing the variable limits from $349(=360-11)$ to $371(=360+11)$. As it is shown in Table 2 (from 5th to 12th action) she went through three more circles of forming an idea, expressing it in the medium and reflecting on it, that led her to the creation of her final dancing animation (Figure 2).

Discussion

All three students participated in this study generated ideas while seeking ways for creating a dancing move and synchronizing it to the rhythm of a song. The CMAF provided an insightful scope for identifying and exploring their actions within MaLT2. We detected a sequence of creative mathematical actions, which formed circles of *imagining*, *manifesting*, *familiarizing* and *recognizing* actions, composing three unique paths of mathematical meaning-making processes. They approached mathematics in a fallibilist way, as it became the basic tool for applying their artistic ideas which were freely open for investigation and reformation. Conversely, their artistic ideas constituted of motivators for their creative way of thinking and doing mathematics. In the instances described above, they used and made meanings on the mathematical concepts of angle, variable, parameter, linear function, periodicity and central symmetry. MaLT2 was used as means for posing both mathematical and artistic questions and investigating their answers through testing and instant feedback. We conclude that both MaLT2 and the artistic context fostered naturally generated creative mathematical actions of all six types described in CMAF.

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