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# Characteristics of students' covariational reasoning in an augmented reality environment: a language-oriented analysis

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*This study reports preliminary results on the characteristics of covariational reasoning in an augmented reality-rich environment. In contrast to the research literature that has hitherto focused on levels of covariational reasoning, this study addresses its linguistic characteristics. Two groups of three 15 and 17-year-old students participated in this study. Each group carried out two activities (Hooke's law & Galileo) using AR headsets. The students' interactions were video-recorded. Based on inductive analysis of the students' covariational reasoning language, we identified five linguistic categories of covariational reasoning: direct-covariation, mediated-covariation, conditional covariation, multi-variable covariation, and second-order covariation.*

*Keywords: Augmented reality, covariational reasoning, linguistics, characteristics of covariational reasoning.*

## Introduction

Covariational reasoning is a central concept in mathematics and science education, which has drawn the attention of many researchers in these fields (Johnson et al., 2017). The research literature mainly focuses on the levels of covariational reasoning, as suggested by Thompson et al. (1994), Carlson (2002), and Thompson and Carlson (2017). In this context, several studies have focused on students' covariational reasonings as they learn in several learning environments, (e.g., Doorman et al., 2012). Among these studies, Swidan et al. (2019) is one of the few that focused on using AR technology to foster covariational reasoning. Swidan et al. (2019), like many others, focused on learners' covariational reasoning levels (see also e.g., Kertil, 2020; Sokolowski, 2020; Thompson and Carlson, 2017). However, little is known about linguistic aspects of students' engagement in covariational reasoning. By linguistic aspects, we mean the structure and the meanings of students' discourse about covariation.

The socio-cultural theoretical framework argues that language plays a central role in the development of human thinking. This suggests that identifying the linguistic characteristics of students' covariational reasoning may shed light on that reasoning, and its development. Identifying these linguistic characteristics is therefore this study's goal.

## Theoretical framework

### Covariational reasoning

Thompson and Carlson (2017) described understanding covariation as holding a sustained image in the mind of two quantities' values (magnitudes), which change simultaneously. In their study, they characterized six levels of covariational reasoning (1) No-coordination; (2) Pre-coordination of

values; (3) Gross coordination of values; (4) Coordination of values; (5) Chunky continuous covariation; (6) Smooth continuous covariation. In the first level, the person has no image of variables varying together. In the second, the person can predict the change of each variable value separately but does not create pairs of values. In the third, the person perceives a loose link between the overall changes in the values of the two quantities. In the fourth, the person can match the values of one variable (x) to the values of another variable (y), thus creating a discrete set of pairs (x, y). In the fifth, the person may perceive that those changes in the two variables co-occur and vary smoothly, but only at separate domains. In the sixth and final level, the person can perceive an increase or decrease in the value of one variable as coinciding with changes in another variable value in its entire domain and see both variables as a smooth and continuous change.

Thompson and Carlson (2017) suggested descriptions and examples that illustrate each covariational reasoning level. We adopted their descriptions to identify the level of covariational reasoning and the linguistic structure of the utterances. Such identification of covariation reasoning may assist us in addressing the linguistic characteristics of each statement.

### Augmented reality

Augmented reality (AR) is an innovative technology that combines layers of virtual objects or information about physical objects from the real world, such as texts, images, graphs, etc. This creates a kind of augmented reality, in which virtual objects and a real environment coexist to increase the learning experience (Arvanitis et al., 2009; Dunleavy et al., 2009). These virtual layers are created in real-time and layered on the physical objects in the real environment in 3D. Insofar as it augments mathematical representation with the real phenomenon, we assume that AR technology may play an essential role in engaging students in covariational reasoning, due to its ability to visually present information that is naturally invisible. It also simplifies objects' visual appearance and helps students think about their symbolic representations.



Fig.1 Students with AR headsets

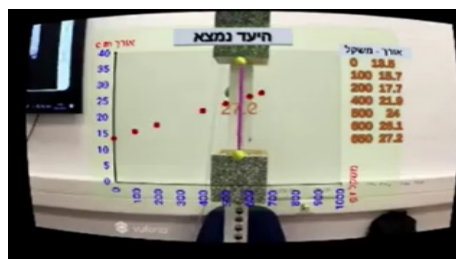


Fig.2 Virtual and physical objects as seen through AR headset

## Methods

### Research context and participants

Since this study seeks to understand the linguistic characteristics of covariational reasoning, it is based on the qualitative research method. Learning experiments were conducted with two groups of three 9<sup>th</sup>- and 11<sup>th</sup>-grade students. The students had already learned linear and quadratic functions. The learning experiments were held at Ben-Gurion University of the Negev. Each session lasted about

180 minutes, during which each group carried out two experiments: (1) the Hooke's law experiment, which examines the relationship between mass and the elongation of a spring (Figures 1 and 2), and (2) the Galileo experiment, which examines the relationship between time and distance that a cube travels as it slides down along an inclined plane. Each group worked on a task sheet corresponding to both physical experiments. The tasks were based on the inquiry-based learning approach. Each task included three inquiry phases: conjecturing, experimenting, and reflecting.

For this study, we designed a new AR prototype that utilizes augmented reality in an educational setting using a special headset, presenting a dynamic object in a real environment with virtual representations provided simultaneously. The prototype is designed to juxtapose mathematical representations (numbers, tables, graphs) with the dynamic real-world object to evoke engagement in covariational reasoning (Figure 2).

### Research question

What are the linguistic characteristics of students' covariational reasoning when they learn in an AR environment?

### Data collection and analysis

All the learning experiments were video recorded. Students' interactions and materials (written notes, files) were collected. Thus, a solid set of data was obtained. We used Thompson and Carlson's (2017) framework to identify covariational reasoning and determine covariational reasoning levels. We also used the inductive approach (Patton, 2002) to linguistically analyze (structurally and semantically) utterances that indicate covariational reasoning. Finally, we grouped the data into categories of linguistic characterization.

The semantic aspect of our analysis refers to the context in which covariational reasoning utterances emerge, while in the syntactic aspect we address the structure of the covariational reasoning utterances.

The inductive approach we adopted is based on the following phases:

- Looking for utterances indicating covariational reasoning.
- Identifying linguistic characteristics of each phrase.
- Identifying categories of characteristics (linguistic and content) in the statements.

Table 1 illustrates the ways we analyzed the covariation utterances:

Table 1: Illustrative analysis-table of covariational reasoning utterances

Utterance	Thompson-Carlson level lens	Linguistic lens	Category
<b>Dennis:</b> As time passes on, then the sliding cube gains acceleration, and so travels greater distance	<b>L3:</b> Dennis seems to perceive a loose link between the overall changes in the two quantities' values (time, distance)	Covariation between time and distance is <i>mediated</i> by another variable ( <i>acceleration</i> ).  The linguistic structure is: As A, then B and so C	Mediated covariation

<b>Alex:</b> If the elasticity is weak, then, the more mass is added, the longer the spring length is.	<b>L3:</b> Alex seems to perceive a loose link between the overall changes in the two quantities values (mass, length)	Covariation is <i>conditioned</i> on a specific situation. The linguistic structure is: “If A then, the more B, the more C.	Conditional covariation
<b>Shahar:</b> At a weight of 200 grams, the length of the spring is 11.8 cm, and at 400 grams, it increased to 14.3cm.	<b>L4:</b> Shahar can match values of one variable (weight) to values of another one (length), creating a discrete set of pairs (weight, length).	Covariation is <i>directly</i> achieved between the variables' quantities (weight, length). The Linguistic structure is: at A=x, B=y.	Direct covariation

## Results

Our data analysis produced five categories of linguistic characteristics that characterize students' engagement in covariational reasoning in an AR environment: Direct-covariation (n=130), mediated-covariation (n=12), conditional-covariation (n=7), multivariable-covariation (n=13), and second-order covariation - Cov2 (n=45).

Below, we present a series of excerpts that illustrate the linguistic characteristics of each category.

**Direct covariation:** In this excerpt, the students' coordination between variables, or variables' quantities, is achieved directly, without any mediation of other variables

Ex1. Lior: As *time* passes, then the cube travels a longer *distance*

Ex2. Shahar: At a *weight of 200* grams, the *length* of the spring is *11.8 cm*, and at 400 grams, it increased to 14.3cm.

In Ex1, Lior covaries time with the distance the cube travels. In Ex2, Shahar covaries the weight of the cube with the spring length. This kind of covariation is usually characterized as Level 3 of covariation. Here, the linguistic structure that the students use is: “as A then B” & “at A=x, B=y.”

**Mediated covariation:** In this category, the covariation between two variables is mediated by another variables.

Ex3. Nir: As the plane's slope is greater, the cube gains more *acceleration* and the *distance* increases.

Ex4. Sagi: As the *elasticity* of the spring changes, so its *length* becomes greater and ..., so... The *graph slope* becomes sharper.

In Ex3., Nir covaries the plane's slope with the distance the cube passes; here, the covariation was mediated by the cube acceleration. In Ex4., Sagi covaries the elasticity of the spring with the graph slope. Also, here, the covariation was achieved through the mediation of the length of the spring.

The following images illustrate the students' engagement in the covariational reasoning that corresponds to Ex 4. As Sagi and Alex interact with the physical model and add mass to the spring (Fig 2a and 3b), they observe the mathematical representations through their AR headsets. They

interact with virtual objects observed through the AR headsets (Fig3c). First, Sagi covaries the elasticity of the spring, which is a physical quantity, with the length of the spring, which is displayed on the AR headset; then, he varies the graph of the function shown in front of them via the headset. Hence, the covariation between the spring's elasticity and the function graph was mediated by the length of the spring (Fig 3c).



Fig 2. (a)&(b) Alex and Sagi interact with the model by adding mass on the elastic spring. (c) Students interact with the virtual objects observed through the AR headsets. (d) The graph and spring's length as seen from the AR headset.

In these excerpts, students use the structure: “as A then B and so C” & “As A so B and so C”

**Conditional Covariation:** In this category, the covariation is accompanied by the condition of a specific situation.

Ex5. Alex: *If the elasticity is weak*, then the more *mass* is added, the longer the spring's length is.

Ex6. Noam: *If the slope (of plane) is down*, then as time passes, so the cube travels a greater distance.

In these excerpts, the students use the structures: “If A, then: the more B, the more C” & “if A, then: as B, so C.”

**Multi-variable covariation:** In this category, the students' covariational reasoning is characterized by using a multiplicity of variables.

Ex 7. Sagi: In the graph, we simply see the change...that the distance increases ...That is, as *time* passes, then the *distance* increases, then you see that the *slope* (graph) simply increases from one second to another and ... and we also see this in the table of values in large jumps ...that is the *acceleration* increased.

In this excerpt, Sagey covaries four variables: time, distance, the slope of the graph, and the acceleration of the cube. Here, students use the linguistic structure: “As A, then B, then C and D, so E.”

**Second-order covariation:** In this category, the students covary quantity and object, or two objects, but not two quantities as anticipated in covariation as defined in the literature.

Ex 8. Shaked: as the *spring* goes down, the *graph* grows up.

Ex 9. Shaked: The *graph shape* rises diagonally because the cube gains *acceleration* and time passes.

Ex 8. shows how Shaked covaries between two objects, spring and graph, while Ex 9. illustrates how she covaries between a quantity (cube acceleration) and an object (graph shape).

The following images illustrate the students' interaction and engagement in covariational reasoning corresponding to Ex 9. As Dennis slides down the cube, Shaked and Nir observe the phenomenon and mathematical representations through their AR headsets (Fig 3a). Shaked connects physical and virtual objects. She describes the graph (virtual) (Fig 3b) as growing diagonally because the cube (physical) is accelerating, and time is passing. Then, Shaked and Nir share their insights with Dennis (Fig3c).

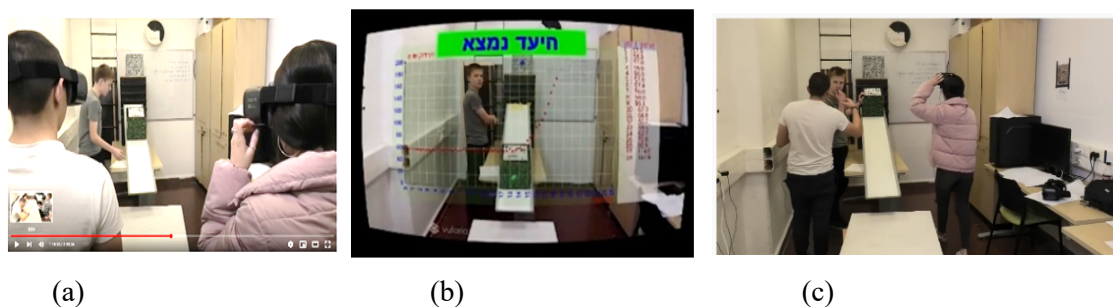


Fig 3. (a) Dennis slides down the cube while Shaked and Nir observe the phenomenon through their AR headsets. (b)The graph as it seen through Shaked's AR headset. (c) Shaked and Nir share their insights with Dennis.

### Final Remarks

We found five categories of linguistic characteristics that characterize students' engagement in covariational reasoning in AR environment: direct-covariation, mediated-covariation, conditional-covariation, multivariable-covariation, and second-order covariation.

With *direct covariation*, students find it easy to reveal the relationship between the two variables. Hence, they address the changes in variables' quantities directly. The AR environment, with its sensory and visual resources, seems to facilitate engagement in direct covariation. For example, in the Hooke's law experiment, the mass and length are visible in the real world and through the AR headset. Furthermore, AR provides numerical measurements augmented and juxtaposed to the observed phenomenon, facilitating students' direct engagement.

*Mediated covariation* may reveal the quantities or objects that relate to the phenomenon the students observed. The emergence of the mediated covariation might be attributed to the learners' need for such mediations to organize their reasoning, which may support their exploration. Such mediation seems to be an essential part of their deductive reasoning when exploring the relationships among variables.

*Conditional covariation* seems to be attributed to two factors: the task and the nature of the experiment. The students' utterances reveal that some of their conditional covariation statements seem to be inspired by the task questions. Since the task requested the students to explore the effect of the spring's elasticity on the mass-length relationship, the conditional covariation emerged as an

answer to the task's question: "If the elasticity is weak, then the more mass is added, the longer the spring's length is."

The rest of the conditional covariation utterances arose in the context of task 1 of the Galileo experiment. In this task, the students were required to explore the time-distance relationship as a cube slides down along an inclined plane. Changing the plane inclination is not determined in the task questions. However, the students manipulated the plane inclination during their exploration. The features of the Galileo experiment evoke sensory interaction and invite available manipulations: adjusting inclination, sliding objects, free manipulations on the sliding object. Furthermore, this manipulation invites students to address other variables, such as inclination, even though the first task is not explicitly managed to require this.

During students' engagement in covariational reasoning, they resorted to other multiple variables. It seems that addressing such variables assists them in better understanding the observed phenomena and connecting with the variables determined in the task. This phenomenon of multi-variable covariation may be induced by the features of AR technology, which invites the interaction between the users and the real and virtual objects (Azuma et al., 2001). Utterances on multi-variable covariation mainly emerged in advanced levels of covariational reasoning. This may be attributed to students' attempts to utilize and organize the rich available variables' data resources to achieve deep insights into the observed phenomenon.

Although the students covaried between quantities along the learning process, we also identified situations in which students coordinate between a quantity and an object, or even two objects. This phenomenon, covarying two objects rather than two quantities, also appeared in Arzarello (2019), who investigated students' engagement in covariational reasoning when performing the Galileo experiment using computer simulation. This suggests that this type of covariation emerged due to the characteristics of the tasks, independently from the digital tools used in the experiments.

The linguistic characteristics provided in this study may shed light on the students' covariational reasoning processes. Furthermore, this categorization may help teachers foster the students' covariational reasoning and help them design dynamic technological environments. The findings of this study refer to a small sample of 10<sup>th</sup> and 11<sup>th</sup>-grade students. Therefore, additional research is required to further explore the linguistic characteristics of students' covariational reasoning as they learn in a dynamic and computer simulation technological environment, considering larger samples of high school and junior high school students.

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