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# Communication of technology as a productive element in learning Mathematics in remote settings

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*During the COVID-19 lockdowns, mathematics classes took place in distance learning settings in large parts with help of digital technology. Our research interest is to enhance digital face-to-face learning environments to stimulate and support mathematical interaction among learners with the help of digital media. In this article, we address the research question to what extent the consideration of the concepts of multiple external representations and communication of technology enable learning opportunities in online meeting tools. For this purpose, we present the learning environment developed based on these concepts and analyze a transcribed meeting with the help of an interpretative approach. By means of the analysis, identified potentials are presented.*

*Keywords: Communication of technology, Multiple external representations, Distance education*

## Introduction

During the school lockdowns caused by the worldwide COVID-19 pandemic, the way mathematic was taught and learned has changed. The traditional face-to-face interaction in classroom cannot take place. In Germany synchronous mathematics classes are held using online meeting tools like Zoom (<https://zoom.us>) or Big Blue Button (BBB; <https://bigbluebutton.org>). In our research project, we focus on diagnostic and support meetings held via online meeting tools. Various learning environments for online meeting tools have been constructed and implemented in our teaching-learning lab.

This article focusses on one developed learning environment: Interactive comparison of proportions using the rectangle model. In the theoretical background, the concepts of multiple external representations and communication of technology considered in the construction of the presented learning environments are introduced. Next, the developed learning environment is presented. In our empirical study we address the research question to what extent the consideration of the concepts of multiple external representations and communication of technology enable learning opportunities in online meeting tools. After presenting the selection of data as well as the method of analysis, empirical results from the implementation of the learning environment are presented and discussed.

## Theoretical Background

### Communication through and communication of technology

When learning with technology, communication is distinguished into two aspects: *Communication through technology* refers to the simple use of technology in mathematics classrooms (e.g., data projectors, displays, document cameras, tablets can be used to display and share the generated learning products). *Communication of technology* refers to the interaction evoked by the output of the technology (e.g., the discussion about the underlying mathematical concept while using a dynamic

geometry software) (Drijvers et al, 2016). To realize a communication of technology via online meetings, for example GeoGebra worksheets can be used, which are programmed to show the learners a corresponding output. Depending on the conception of the task or the worksheet there is a potential that learners can generate numerous mathematical phenomena themselves with the help of digital technology (such as a GeoGebra applet or worksheet) to come to mathematical reasoning by observing them (Eichler, 2019). This can initiate fundamental learning processes (Nührenbörger & Schwarzkopf, 2016), which should be the overarching goal when creating a learning environment.

### Multiple external representations

The concept of multiple external representations goes back to Bruner's remarks:

Any domain of knowledge (or any problem within that domain of knowledge) can be represented in three ways: by a set of actions appropriate for achieving a certain result (enactive representation); by a set of summary images or graphics that stand for a concept without defining it fully (iconic representation); and by a set of symbolic or logical propositions drawn from a symbolic system that is governed by rules or laws for forming and transforming propositions (symbolic representation). (Bruner, 1966, pp. 44-45)

In (mathematics) learning environments, intermodal as well as intramodal transfers between and in these representation levels should be explicitly addressed so that learners link their knowledge from the respective areas (Bruner, 1966). One potential in the use of digital media is the synchronous and interconnected presentation of the different forms of representation via multiple external representations (MERs) (Ainsworth, 1999). When manipulating one level of representation, e.g., symbolic representation, the iconic representation changes at the same time. The use of MERs in learning environments can support learners in internalizing the interconnectedness of the levels of representation and in intermodal transfer between them (Moyer-Packenham & Bolyard, 2016).

### The developed learning environment

Based on theoretical considerations above, the following learning environment was specifically designed for students attending the Teaching and Learning Lab to support their competencies in the area of numbers and operations, especially in fractions. To tap the potentials presented above as much as possible, tasks were designed with the help of a GeoGebra worksheet. The goal of the first task is to compare different fractions (see Fig. 1).

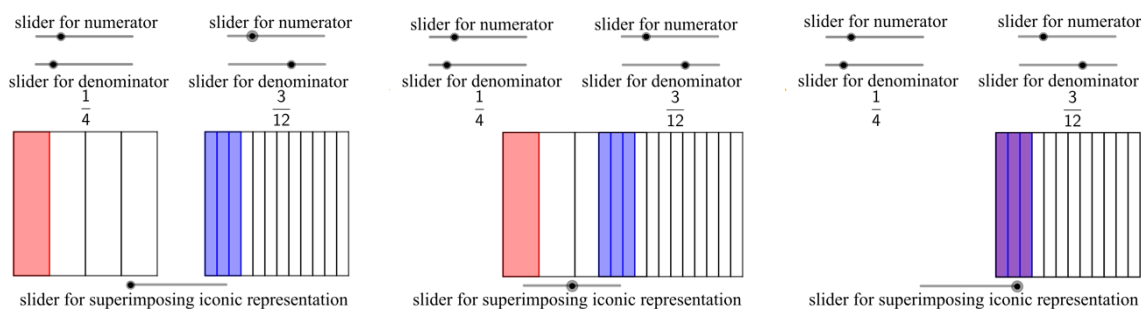


Figure 1: Designed tasks – numerator and denominator are manipulable via sliders

Besides symbolic representation, iconic and enactive approaches are offered. Digital enactivity is limited to the use of sliders to manipulate the iconic or symbolic representation. For example, the numerator and denominator of a fraction can be adjusted while changing the subdivision of the iconic representation. As an additional level of assistance, students can unlock another slider that allows them to superimpose the iconic representations of fractions.

The preliminary theoretical considerations have been taken up in the conception of the learning environment. Since the (enactive) change of the symbolic representation also adapts the iconic representation, the developed GeoGebra worksheet can be classified as MER (Ainsworth, 1999). The students can create different phenomena themselves (Eichler, 2019) by setting and comparing fractions and their iconic representation in quick succession using the sliders. Thereby, a Communication of Technology is initiated (Drijvers et al., 2016), as manipulation of the sliders produces an output in the form of change in the iconic representation, and the iconic representations can be manipulated so that they can be slid over each other to verify equality.

## **Methods**

The collected data was gained in our virtual teaching and learning lab in Summer 2021. The learning environment presented here was conducted with three pairs of sixth graders attending a middle school in northwestern Germany. Partner schools were offered participation in the teaching and learning lab and teachers selected the participating students. The diagnostic and support meetings were held and recorded via the online meeting tool BBB.

In the following, the qualitative analysis of one of the three sessions is presented. The focus is on the pair of students that showed the greatest difficulties in the symbolic handling of fractions at the beginning of the session. Based on the interpretative analysis of the transcripts, the research question to what extent enables the consideration of the concepts of multiple external representations and communication of technology learning opportunities in online meeting tools will be addressed.

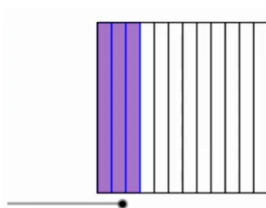
With the help of the interactionist approach (cf. Schreiber, 2004; Voigt, 1995), the interpretations of the participants are reconstructed in a turn-by-turn analysis of the transcript. Participants in interviews or classroom discussions interpret the actions of other participants themselves in these situations. These interpretations influence the further course of the social interaction. A detailed understanding of the course of interaction in a learning environment allows conclusions to be drawn about possible adaptations of the learning environment. These can be implemented in terms of the design research approach (van den Akker et al., 2006).

For the sake of clarity, this article only presents the results of the interpretations that have proven themselves within the framework (Krummheuer & Brand 2001). Starting from an observed phenomenon, an attempt is made to translate the cause into a general case with the help of a law. The resulting laws are always based on a hypothetical conclusion, so there is only the possibility of a plausible but not certain hypothesis (Meyer, 2009). The point is not putting the methodological background of the analyzes up for discussion. For the methodological discussion, the translation process of the original transcript would already be a strong interpretative intervention that is worth a discussion. Rather, exemplary episodes are presented that represent different ways of communication with the help of digital technologies.

In the following, the interpretative analysis of a scene from an online meeting with Angela and Susan is presented. Here, the empirical realization of the potentials is examined and, in particular, the question is pursued whether Angela and Susan can transfer the acquired knowledge in the digital-active and digital-iconic areas to the symbolic level with the help of the prepared environment.

## Empirical Results

Before the present excerpt starts, the students were asked to use the sliders (Figure 1) to create the fractions  $\frac{1}{4}$  and  $\frac{3}{12}$ . Andrea and Susan successfully solved this task. Simultaneously, the corresponding iconic representations were displayed to the students in the GeoGebra worksheet. The two students further used the possibility to superimpose the two representations with the help of the lower slider (Figure 1 and Figure 2).



**Figure 2: Superimposing the iconic representations with the help of the slider**

The transcript (translated by the authors) starts at the moment when the students are asked by the worksheet to describe their observation:

- 12 S (Reads the task:) Tell what you have observed.. ehm well we have twelve parts three of them are marked and ehm on the quarter we had four parts but one was marked.. If you put them on top of each other three ehm three twelfths ehm the three are exactly as large as the ehm quarter
- 13 I Can you confirm your guess'
- 14 A Yes they belong to each other

The students describe the different subdivision of the two rectangles and emphasize that in one rectangle three of twelve parts were marked and in the other one of four. The equality of the given fraction representation assumed at the beginning is confirmed and justified by the students by taking advantage of the possibility to superimpose the two iconic representations with the help of the slider.

Afterwards, the interviewees are given the next task:

- 16 S (Reads the task:) Ohh interesting do you think that there are other fractions that belong to a quarter and three twelfths' enter your solution in the empty fields on the right side of the page



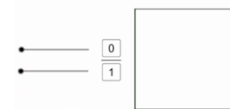
- 17 S Ehm (5 sec)
- 18 A I know ehm no (5 sec)

19 S There are only three twelfth ehm a quarter is slightly larger it therefore fits into the other one..

20 A So the marked part has to.. I don't know how to explain (5 sec.) ehm.. I don't know

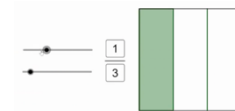
At the beginning of the task, the students can only enter their solutions using the symbolic notation. The task of specifying further fractions that are suitable for  $\frac{1}{4}$  and  $\frac{3}{12}$  cannot be processed by the students on the symbolic level. Susan focuses on the given iconic representation from the previous task without relating it to the new task in an explicit way. Angela seems to relate the given iconic representation to the task – unfortunately, she does not finish her thoughts.

21 I You can click on the help button (by clicking two sliders appear next to the upper rectangle)

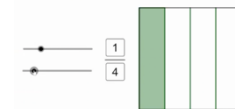


22 A Okay (starts to use the slider on the upper rectangle) ah okay

23 S Ahh one third



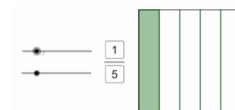
24 A But this is one more it has to fit in to the other one. We had a quarter



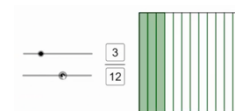
25 S This also does not work

26 A It does

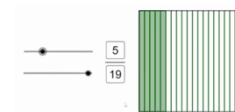
27 S (Keeps trying on the slider so that the rectangle is divided further) One fifth is too small yes that really does not work but I make it smaller.. wait three twelfth fit into three twelfth



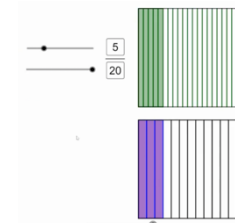
28 A That's right but now go on



29 S Wait five nineteenth no that does not fit



30 A What about five twentieth'.. that does fit



31 S We are ready

As soon as the sliders are provided as a support, Angela and Susan can take action. They try out different settings. Once the sliders are set, the corresponding symbolic notation and iconic

representation is provided in the worksheet. The students compare the generated iconic representation with the permanently available representation of  $\frac{3}{12}$  of the previous task (e.g. Turn 16). A mathematical systematic is not recognized, rather Angela and Susan persist on the comparison of the iconic representations. Nevertheless, Angela and Susan use the slider to make an increasing finer division and also verbalize this (Turn 27). Furthermore, it is noticeable that when the number of subdivisions is increased by the slider that influences the denominator, Angela and Susan also set the slider for the numerator to a larger setting to make the marked area of the rectangle as fitting as possible in relation to the given iconic representation of  $\frac{3}{12}$ . The central criterion for the two students is that the marked areas in the iconic representations take up the same proportion of the rectangle, so that they would fit on top of each other (Turns 29, 30). After finding the iconic solution, Susan formulates that they have finished working on the task.

32 I Okay can you briefly explain how you arrived at your solution

33 A We looked at how much ehm we needed and ehm how small the parts had to be so that they would fit into the three twelfths

34 S So we have made the lower number always larger and thus made the lines shorter so that it fits

When asked by the interviewer to describe their solution procedure, Angela and Susan explicitly formulate the procedure interpretatively reconstructed above. By adjusting the sliders, the generated iconic representations were manipulated in such a way that a representation appearing congruent to the already given subdivision was produced. In Turn 34, Susan explicitly speaks of larger numbers without verbally associating them with the fraction representation as numerator and denominator.

35 I Can you explain how you can get to the solution without using the sliders

36 S Ehm

37 A I can't do this

38 S Five twentieths must have something to do with this.. I don't know

Explicitly asked for a possible solution without referencing the sliders, Angela and Susan are evasive. Susan names the generated fraction  $\frac{5}{20}$  for the first time but does not relate it to  $\frac{1}{4}$  or  $\frac{3}{12}$ .

## Discussion

Angela and Susan generate a variety of mathematical phenomena (Eichler, 2019) in the form of iconic representations of fractions through the different settings of the sliders (Turns 21-30). They compare the generated fractions to the given iconic representation to find a solution to the task. By using the opportunity of the setting options via the sliders, Angela and Susan are enabled to indicate a solution using the technology. Through the settings of the sliders, the symbolic representation of the fraction is automatically specified, and the corresponding iconic representation is displayed. Angela and Susan argue with the help of the iconic representation via the offered GeoGebra worksheet and generate a solution for the task. However, although Susan and Angela develop findings on the iconic level, they are not sufficiently stimulated to transfer them independently to the symbolic level. An intermodal transfer does not take place. This might be explained by the interpretive hypothesis that from Angela's and Susan's point of view the argumentation on the iconic level is completely

sufficient. There is no need for symbolic argumentation or backup of the iconically confirmed results. The apparent fit of the subdivision found through trial and error is seen as a complete solution. Reconstructively, there seems to be no need to give a symbolic argumentation for the iconically gained knowledge. Even though the digital medium offered a synchronicity of the presentation of iconic and symbolic levels, this does not necessarily mean the retrieval of the potential by the learners in form of a realization of an intermodal transfer.

Overall, it can be stated that Susan and Angela are enabled to solve the task on an iconic level through the tools offered by the worksheet which was constructed under consideration of the concepts of MERs and communication of technology. However, a transfer of the iconically obtained solution to the symbolic level does not take place and obviously requires a further impulse in the form of an authentic and precise anchoring in the given task. This adaptation of the learning environment with a view to forcing an authentic intermodal transfer stimulated by the worksheet will be focused on in the continuation of our Teaching and Learning Lab.

Based on the results presented and the discussions, the following possible adaptations of the learning environment emerge: The multiplicative link between numerator and denominator (for equivalent fractions), for example, could be focused more by corresponding buttons to double, triple or half the numerator and denominator. This highlights a more systematic change between numerator and denominator. In the presented form, the given rectangle for iconic representation is divided only horizontally. In a revision of the learning environment, both horizontal and vertical subdivision could be offered. Students might then experience a productive irritation (Nührenbörger & Schwarzkopf, 2016) since you get more examples with different structures for the same fraction. Likewise, the colored part is always the part to the left. In future, other parts might be colored to make clear that, for example, any three out of twelve parts (of equal size) form one quarter. Building on these constructive changes, learners should also be asked to make more arguments about whether the proportions presented are equivalent and why.

By considering the presented points, the potential arises to initiate the intended linking of iconic and symbolic levels and thus to take up the problems that could be reconstructed in the interpretative evaluation of the learners' use of the learning environment. Thereby, the given *model of fractions* could become *model for fractions* for the learners (e.g. Gravemeijer, 1999).

In addition to the reconstructed potentials regarding the argumentations, an organizational potential can also be identified: Due to the principal possibility of successfully conducting pure online teaching-learning arrangements via online meeting tools, the circle of learners who can be enabled to participate in our Teaching and Learning Lab can be enlarged insofar as learners from the entire catchment area in the northwest of Lower Saxony can be reached without hurdles of distance and travel.

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