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# Dynamic instability mitigation using a single nonlinear energy sink in a friction system with two unstable modes

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**Summary.** This work studies a problem of passive mitigation by means of one nonlinear energy sink (NES) of a dynamic instability arising from two unstable modes of a primary structure. For that, a phenomenological friction model with four-degrees-of-freedom (DOF) having two unstable modes (the primary structure) is coupled to a NES. We investigate analytically, using an asymptotic analysis, the response regimes resulting from the interaction between the two unstable linear modes of the primary structure and the NES. The analytical results are compared and complemented by direct numerical simulations of the coupled system. In each situation, a good agreement is observed which allows us to validate the presented asymptotic analysis.

## Introduction

In the context of passive control of dynamic instabilities, the Nonlinear Energy Sinks (NESs) are now well-known means. For example, NES have been used to suppress aeroelastic instability [1], regenerative chatter in machine tool vibrations [2], helicopter ground resonance instability [3] or friction-induced vibrations due to mode coupling instability [4]. The NES operate on the basis of Targeted Energy Transfer (TET) concept. In general, a NES consists of a light mass, an essentially nonlinear spring (here cubic) and a viscous linear damper. Because of its essentially nonlinear stiffness, a NES can adjust its frequency to that of the primary system and therefore engage in resonance over a large frequency range. Whether for a system under impulsive, harmonic or broadband frequency excitation or whether for an auto-oscillating system, TET results from nonlinear mode bifurcations. In general, the phenomenon of TET can be described as a 1:1 resonance capture [5].

In this work a friction system is considered as a primary system, it consists in a simple four-DOF (two linearly coupled well-known Hultèn's models, see Fig. 1) [6]. The model has two unstable modes, i.e. the associated eigenvalues have positive real parts with also the same order of magnitude. Moreover, the imaginary parts (the frequencies) are chosen to be incommensurable. We study the mitigation by means of a single NES of the friction-induced vibrations resulting of the presence of two unstable modes and having therefore two frequency components. Moreover, because the system (without NES) is linear, it undergoes, when it is unstable, an exponential growth of its amplitude.

Preliminary numerical simulations of the coupled system show that the single NES is effectively capable to interact with the two unstable modes and mitigate the double instability. Inter alia, the mitigation may be performed through periodic regimes (see Fig. 2(a) where  $\epsilon$  is a small parameter related to the mass ratio between the NES and the primary structure) or Strongly Modulated Responses (SMR, see Fig. 2(b)).

In order to understand these interactions and the mitigation processes, based on the procedure presented by the authors [7], an analytical framework is presented. The obtained results are compared and completed by direct numerical integration of the coupled system. The method and the major results of the study are summarized below.

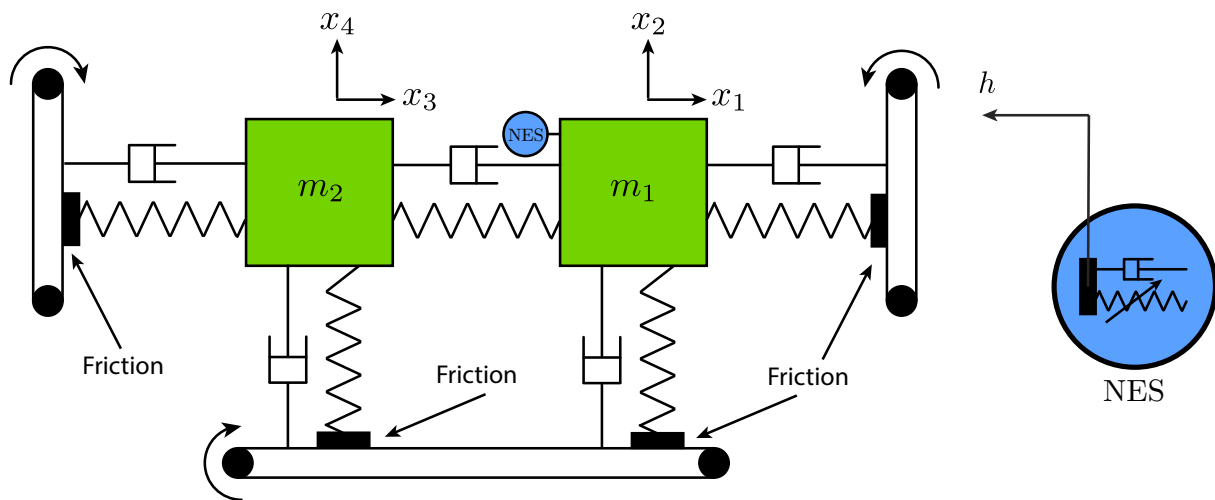


Figure 1: The phenomenological system under study.

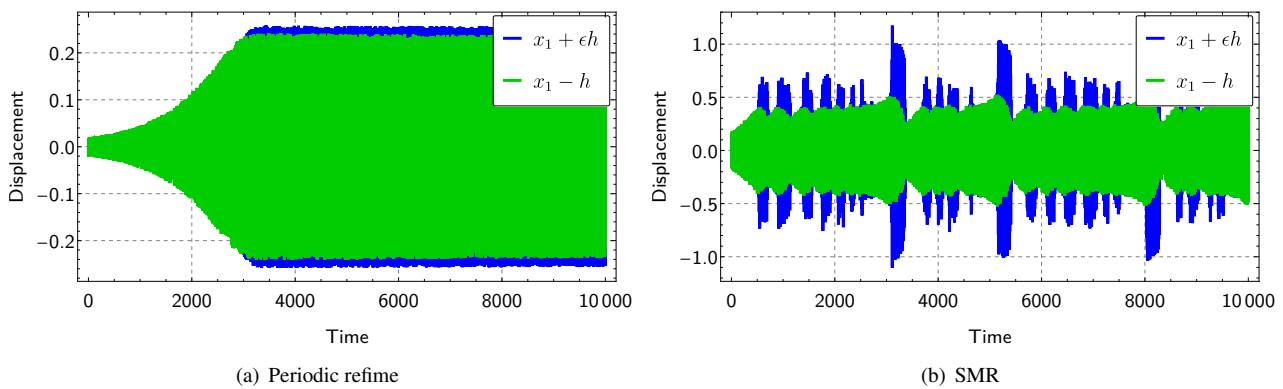


Figure 2: Possible mitigation responses obtained from direct numerical integration of the coupled system.

## Method and results

The analytical method begins with the so-called biorthogonal transformation to diagonalize the primary friction system. Then, the resulting diagonalized system is reduced keeping only the two unstable modes and ignoring the two stable modes. Afterwards, the slow-flow motion of the system is obtained using the complexification-averaging (CA-X) method within the assumption of two simultaneous 1:1-1:1 resonances to the natural frequencies of the two unstable modes. The slow-flow, which also depends on the small parameter  $\epsilon$ , is analyzed in the framework of the Geometric Singular Perturbation Theory (GSPT) [8] in which it is defined as a  $(4, 2)$ -fast-slow system. Each of the two slow variables characterizes one of the two unstable modes of the primary system whereas the four fast variables describe the NES motion (amplitude and phase of each of the two frequency component).

The Critical Manifold (CM) of the slow-flow is obtained, it is a two-dimensional parametric surface (with respect to two of the four fast variables) which evolves in the six-dimensional variable space. The shape of the CM and its stability properties allow us to contemplate some mitigated responses, in agreement with the preliminary numerical simulations. Indeed, the critical manifold appears as the union between a normally hyperbolic attracting domain with four closed normally hyperbolic saddle type domains.

First, equilibrium positions of the slow-flow are deduced from the knowledge of the CM. A stable equilibrium position (which is on the attractive part of the CM) characterizes a mitigation regime through a periodic response of the initial system if one of the frequency component is completely suppressed, otherwise the response is quasi-periodic with two frequency components. Secondly, because two of the normally hyperbolic saddle type domains meet the attracting part at fold curves, mitigation regimes through Sustained relaxations oscillations of the slow-flow - i.e SMR for the initial system - are possible. Contrary to the case with only one unstable mode, because of the two-dimensional CM, multiple SMR scenarios are possible with different levels of complexity. Fig. 2(b) show an example of a "complex" SMR.

The analysis also suggests that the system may have simultaneous stable attractors (two stable equilibrium positions, one stable equilibrium position with stable sustained relaxations oscillations . . . ).

Numerical simulations valid the possibility of a competition between two stable attractors and allowing us to study their basins of attractions. In each considered situation, a good agreement has been observed between theoretical results and numerical simulations which validates the proposed asymptotic analysis.

To conclude, this study confirms that a NES is able to mitigate simultaneously two unstable modes.

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