



HAL
open science

Teachers' classroom use of dynamic mathematical technology to address misconceptions about geometric similarity

Ali Simsek, Nicola Bretscher, Alison Clark-Wilson, Celia Hoyles

► To cite this version:

Ali Simsek, Nicola Bretscher, Alison Clark-Wilson, Celia Hoyles. Teachers' classroom use of dynamic mathematical technology to address misconceptions about geometric similarity. Twelfth Congress of the European Society for Research in Mathematics Education (CERME12), Feb 2022, Bozen-Bolzano, Italy. <hal-03747809>

HAL Id: hal-03747809

<https://hal.science/hal-03747809v1>

Submitted on 8 Aug 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

Teachers' classroom use of dynamic mathematical technology to address misconceptions about geometric similarity

Ali Simsek¹, Nicola Bretscher², Alison Clark-Wilson² and Celia Hoyles²

¹Turkish Ministry of National Education, Ankara, Türkiye; ali.simsek.15@alumni.ucl.ac.uk

²UCL Institute of Education, University College London, London, England;

n.bretscher@ucl.ac.uk; a.clark-wilson@ucl.ac.uk; c.hoyles@ucl.ac.uk

This paper examines what misconceptions about geometric similarity (GS) teachers can attend in their curriculum scripts and how they use dynamic mathematical technology (DMT) in the classroom to promote students to encounter, reflect upon and address their misconceptions. The Structuring Features of Classroom Practice (SFCP) construct of 'curriculum script' guided the collection and analysis of the data presented in the paper. The research worked with three teachers with different levels of experience and expertise in using DMT. Data collection involved video-recorded observations, audio-recorded, semi-structured post-lesson teacher interviews and lesson resources. The findings suggested there were differences in the quantity of misconceptions the teachers anticipated and/or identified and in the ways in which they used the DMT to help students confront, reflect upon and address their misconceptions.

Keywords: Geometric similarity, misconception, dynamic mathematical technology, teacher.

Introduction

Researchers have underlined that there is still little understanding of the phenomenon of the integration of dynamic mathematical technology (DMT) into classroom practice and we need more case studies guided by the available different theoretical lenses to explore teachers' DMT-enriched practices within 'real' classroom settings (e.g. Ruthven, 2014). In our research, we aimed to contribute to addressing this gap by investigating secondary teachers' actual classroom practices as they teach the key mathematical domain of geometric similarity (GS) with a carefully designed DMT. We focused on GS since it is a key but also difficult concept for students (Clark-Wilson & Hoyles, 2017). For instance, research has documented that the improper use of additive reasoning is one of the most common student misconceptions about GS (Noss & Hoyles, 1996). However, despite its importance in school mathematics and the difficulties that it poses to students, GS has been neglected in research on the integration of DMT into practice. This is surprising as the research evidence suggests that students' engagement with DMT could help them overcome their (possible) misconception(s) with GS (Clark-Wilson & Hoyles, 2017).

The current paper presents part of a broader PhD study (Simsek, 2021), conducted by the first author and supervised by the co-authors. The paper concerns particularly teachers' use of DMT in their practices to support students to confront, reflect upon and address their (possible) misconception(s) about GS. We formed our research question for this paper as follows: *What student misconceptions about the topic of geometric similarity can teachers attend in their curriculum scripts and how do they use dynamic mathematical technology in their classroom practices to support students to encounter, reflect upon and address them?*

The ‘Curriculum Script’ Construct of the SFCP Framework

We selected and used the Structuring Features of Classroom Practice (SFCP) framework to guide the entire research process of the broader study including both data collection and analysis. The lenses of this framework specifically concentrate on the five key structuring features of classroom practice that shape how teachers incorporate (new) technologies into their practices, namely *curriculum script*, *resource system*, *activity format*, *working environment* and *time economy*. The framework underlines how these features pertain to teachers’ integration of technologies into the classroom and identifies the examples of the associated teacher craft knowledge in relation to integration of technologies (the full description of the framework can be found in Ruthven (2014)). Ruthven (2014) discusses teachers’ craft knowledge is mainly tacit, illustrating why many teachers find it difficult to articulate or might not even be aware of. Teachers develop this type of knowledge, which is dynamic and evolving, primarily through “their own practical experience of learning and teaching the topic” or from “available curriculum materials” (Ruthven, 2009, p. 138). The SFCP framework therefore serves to identify and analyse teachers’ craft knowledge that underpins successful classroom practice with (new) technologies.

Due to the focus of the present paper, we briefly describe *only* the construct of *curriculum script* as it also involves student misconceptions about a topic teachers anticipate and/or identify and how teachers address them using different pedagogical tools including DMT(s). The concept of *curriculum script* can be defined, in the psychological sense, as an event-structured organisation of knowledge incorporating well-defined sets of learning objectives, expectations and actions in a loosely logical sequence for a mathematical domain, likely misconceptions related to the domain, together with resources, activities, associated language and questions, and pedagogic strategies (Ruthven, 2009). When teachers implement (new) technologies in their practices, they need to draw on their knowledge to develop and adapt their curriculum script for teaching a topic. This script guides the ways teachers create the overall structure for a lesson (i.e. the dynamic plan or set of goals and actions for a particular lesson) and enact it in a flexible and responsive way (Ruthven, 2014). It is important to highlight a curriculum script goes beyond a lesson plan created by teachers in the written form in advance a lesson. In this sense, while a lesson plan seems to be fixed and contain details that teachers are able to articulate, a script can be considered as dynamic and evolving in light of what happens in the classroom and encompass content of teachers’ craft knowledge that many of them are not be able to articulate or might not be aware of.

Design and Methods of the Study

This research adopted a multiple case study approach. We were able to identify and work with three teachers who had been previously involved in the original Cornerstone Maths (CM) project in England (see the details of the project in Clark-Wilson and Hoyles (2017)) and expressed an interest in being part of the research. We recruited them from the community of the project as they had engaged with a particular DMT tool, the CM software, and they were committed to use the DMT in their own classroom practice to teach CM curriculum unit on GS (including the CM software, the student booklet and the associated teacher guide) to students aged 13-14 years. The participant teachers were from the two different London-based co-educational secondary schools. While Jack

(pseudonym) taught in a school in the east London, Alex and Lara (pseudonyms) were colleagues and taught in the north-west London. We expected Jack to exemplify characteristics of an *expert teacher* in the use of DMT for teaching and learning of mathematics as he had high-level of involvement in the original CM project and considerable experience and expertise in the use of DMTs. Whereas, we expected Alex and Lara to exemplify characteristics of an *advanced beginner teacher* as they had mid-level of involvement in the original CM project and some experience and expertise in using DMTs (Berliner, 2004). We assumed such selection would be helpful to elucidate differences in the characteristics of teachers' practices with DMT in relation to GS.

The CM software is a web-based DMT designed to support students' understanding of the set of three CM curriculum units for lower secondary mathematics in England. The units include GS, algebraic patterns and expressions, and linear functions. The software was designed by exploiting the dynamic, visual and multi-representational potential of digital technology. It contains several carefully designed dynamic tasks related to each of CM units and promotes the use of the 'predict-check-explain' pedagogical approach. These tasks were created on the basis of realistic contexts in a learning environment in which there are dynamically linked multiple mathematical representations such as geometric shapes, ratio checker and measurement tables. The CM software is therefore intended to offer the potential for students to make conjectures and test them by manipulating such representations and to explore the underlying mathematical concepts and relationships in a realistic context. In terms of GS, the tasks allow students to engage with, for example, shapes, ratio checker and measurement table by using several key features (e.g. scale factor slider, angle slider, dragging) in the dynamic learning environment. Therefore, they could potentially recognise and explore the embedded variant and invariant properties of mathematically similar shapes and how the properties are used to determine similarity. The CM software is also intended to help students encounter, reflect upon and overcome their (possible) misconceptions about GS.

Data collection involved video-recorded lesson observations, audio-recorded, semi-structured post-lesson teacher interviews and lesson resources. The first author observed eight each of both Lara's and Jack's lessons and seven of Alex's lessons. In the case of Jack, he conducted his lessons in his normal classroom with laptops. In the cases of Lara and Alex, they both taught their lessons either in a pre-booked computer room with desktop computers or in an ordinary traditional classroom with iPad computers. Moreover, the first author interviewed Alex and Jack six times and Lara eight times, which lasted between 35-45 minutes. Lastly, we collected teachers' interactive whiteboard or PowerPoint slides, examples of their task sheets, photographs of students' DMT screens and their associated written work in the workbook in response to the DMT-enriched tasks. For data analysis, a *within-case* analysis was carried out first to create an individual detailed description of each case in written form that helped examine each case in depth. The within-case analysis led to the identification of what student misconceptions about GS each case study teacher attended in her/his curriculum script and how (s)he used the DMT to help students address them. Then, using spreadsheets, a *cross-case analysis* was conducted by comparing and contrasting the cases based on the results of the within case analysis. The cross-case analysis resulted in the identification of the key differences between the teachers in terms of their anticipation and identification of student misconceptions about GS and their exploitation of the DMT to tackle them in the classroom.

Findings

In this paper, we present findings emanated from our data analysis that concern the case study teachers' anticipation and identification of student misconceptions regarding GS and of the ways through which to enable students to confront, reflect upon and address their (possible) misconceptions using the DMT. The within-case-analysis revealed the following student misconceptions about the topic of GS evident in the teachers' curriculum scripts.

1. If two shapes look similar in appearance (e.g. rectangles), they are mathematically similar;
2. An enlargement results in the creation of a mathematically similar shape which is *always* larger than the original;
3. A scale factor makes a shape only doubled and/or halved;
4. The use of additive strategies within GS is appropriate when determining whether shapes are mathematically similar;
5. If the lengths of a shape are multiplied by a scale factor of k , then the area of this shape is multiplied by a scale factor of k , too;
6. The proportionality of all corresponding pairs of sides is by itself sufficient to prove the similarity for shapes; and
7. The angles of a shape are multiplied by a scale factor along with its side lengths.

The cross-case analysis indicated that compared to Alex and Lara, Jack, as an expert teacher, showed more awareness of the likely misconceptions about GS and of the potential teaching strategies incorporating the use of the DMT in a dynamic mode to tackle them. While Jack anticipated and identified a total of six of the above stated seven key misconceptions and made dynamic use of the DMT to provide opportunities for students to address them, Alex and Lara predicted only three and two different misconceptions of them, respectively, and did not necessarily use the DMT in the same way as Jack did. In the following, we present results for each case to provide an insight into what misconceptions out of the above given ones each teacher attended in their curriculum script and how each used the DMT in their classroom practice to support students to encounter, reflect upon and address them.

Case Study 1: Lara

In her curriculum script, Lara identified the first and second misconceptions underlined above that her students faced when engaging with the DMT-enriched tasks. However, she did not make dynamic use of the DMT herself in the observed lessons to assist students in addressing them. Below, we present analysis of the second misconception to provide an insight into how she addressed a misconception with (and without) the DMT.

Promoting students' understanding that an enlargement does not only result in the creation of a similar shape larger than the original

In her post-lesson interview, Lara expressed students struggled to understand an enlargement may also result in a shape being smaller than the original as they were inclined to think an enlargement is a mathematically similar shape that should be *always* larger than the original. She outlined she

recognised this misconception when interacting with a pair of students as they asked her to confirm what she had said in the previous lesson regarding the possible impact of an enlargement on the sides of shapes. She also stated students' engagement with one of the DMT-enriched tasks, where students predicted, tested and explored if the rectangles were mathematically similar to the original on the basis of visual and numerical cues, helped them realise an enlargement may create also a smaller similar shape than the original.

They [students] cannot connect that an enlargement also makes a shape smaller because, in their mind, an enlargement is something getting bigger. They told me [that] "You said yesterday an enlargement can make the shapes smaller as well". So, for them, by themselves, they are not connecting it until we have done this [the DMT-enriched task]. So, from the original to copy 2 [the names of the shapes being enlarged in the task], they can see the bigger shape [the original shape] is getting smaller. Before that, they could not, that is how I assumed or observed.

However, Lara herself did not use the DMT dynamically to help students realise and explore that an enlargement is a similar shape which may be also smaller than the original. Instead, she only allowed students to use the DMT during their independent work with the DMT to confront and reflect upon this misconception by themselves.

Case Study 2: Alex

In his curriculum script, Alex anticipated the second, fourth and sixth misconceptions given above and addressed them with (and without) the DMT in the classroom. We present how he addressed the third misconception with the DMT to offer an insight into this aspect of his curriculum script.

Promoting students' understanding that the proportionality of all corresponding pairs of sides is not by itself sufficient to prove the similarity of shapes

Alex was aware students need to understand the need to consider both the corresponding angles and sides when determining if shapes are mathematically similar.

If they [students] do not bring corresponding angles and corresponding sides together, they can go into an error. When they just focus on the [corresponding] sides, they might miss the fact that the corresponding angles [in mathematically similar shapes] need to be the same. They [students] need to be aware of both.

In the lesson that featured Task 5.2 (see Figure 1), during the subsequent whole-class discussion, Alex himself used the DMT dynamically to promote students' emergent understandings that inequivalent corresponding angles do not result in a similar shape to the original. From his computer desktop, he used the DMT to demonstrate and share students' responses with the class and to stimulate a discussion. For example, in the dynamic learning environment, Alex first measured the corresponding angles of the original and the parallelograms named Copy 1 and Copy 2 and then dragged the angle slider to change the angles of the original and Copy 1, followed by two questions he posed to the class: "What did you notice about all the angles?" and "What is going on with the angles at the moment?".

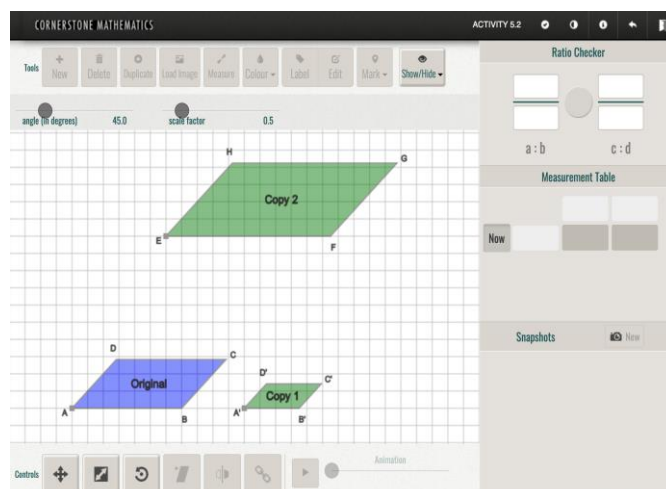


Figure 1: Task 5.2 in the DMT where students drag the angle and scale factor sliders and compare the three parallelograms as they change

Alex also used the types of geometric transformations (e.g. translation, rotation, enlargement) to superimpose Copy 1 on the top of Copy 2 and then dragged the angle slider. His aim was to show and confirm that since the corresponding angles were the same initially, Copy 1 and Copy 2 were mathematically similar, as the angle slider was dragged, the corresponding angles did not remain the same, which made Copy 1 and Copy 2 not mathematically similar. In his post-lesson interview, Alex outlined the DMT enabled students to identify and examine both the corresponding angles and sides, especially through the use of the types of transformations, which would not be possible with an exam-style question in a paper-and-pencil environment.

If you get a shape where the shape is oriented, and you can use the software [the DMT], for example, you can use this software to orientate the shapes differently to see where corresponding sides and angles are. I think those types of things are useful in the software because you can use the software to rotate the shape to find, as I said, where these angles go. If you use an exam question [in a paper and pencil environment], it is hard for students to understand what they are focusing on the angles and the sides.

Case Study 3: Jack

In his curriculum script, Jack anticipated and identified all of the seven misconceptions provided above (except for the first one) and aimed to address them through the use of the DMT in a dynamic mode during the observed lessons. We particularly focus on the fourth misconception in detail below to give an insight into this aspect of his curriculum script. This misconception was spontaneously spotted and identified by him during a lesson and resulted in him making *ad hoc* decisions to address it using the DMT dynamically.

Promoting students' understanding that if the lengths of a shape are multiplied by a scale factor of k , then the area of the shape is multiplied by a scale factor of k^2

Jack identified this misconception in a lesson when interacting with students during their independent work with the DMT to check their written explanations in the workbook. He instructed students to engage with Task 2.1 (see Figure 2) in which they were instructed to play an animation, watch and

decide which quadrilaterals (if any) were always mathematically similar to the original and then produce their written justifications in CM workbook. He noticed one pair of students used the concepts of perimeter and area in their written explanation to justify why the orange and red rectangles were always mathematically similar. The students had concluded the orange and red rectangles remained mathematically similar to the original as the area and perimeter of the rectangles increased by a scale factor of 3 when the animation slider came to the end.

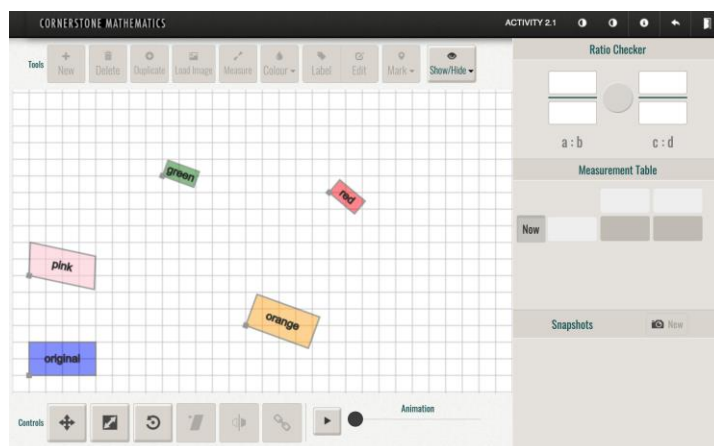


Figure 2: Task 2.1 in the DMT where students play the animation and identify the similar shapes

Jack, however, recognized a misconception as the students had concluded the area scale factor of the orange and original rectangles was the same as the length scale factor (a value of 3). To enable the students to address their misconception, he posed them the question “You see that the perimeter increases by 3 times, does the area increase by 3 times, too?” and then invited them to drag the animation slider such that the lengths of the orange rectangle were twice as large as those of the original. Having drawn the students’ attention to that the side lengths of the orange rectangle were now double the side lengths of the original as it had been enlarged by a scale factor of 2, Jack asked them to examine what happened to the area of the orange rectangle in the dynamic learning environment. By looking at the original and orange rectangles on the screen, the students assumed the area of the orange rectangle had doubled. Jack then encouraged them to figure out “How many of them [the original rectangle] could sit into the orange one [rectangle]?” using the grid as a measurement tool available in the DMT, leading them to realise the area of the orange rectangle was 4 times that of the original.

In his post-lesson interview, Jack stressed when the students came up with the mathematical ideas concerning the area and perimeter of similar shapes, which actually went beyond his lesson agenda, he wanted to exploit this learning opportunity to promote students’ understanding of the mathematics a stake using the DMT.

What they [a pair of students] said the perimeter and the area has doubled, so it was a nice teachable moment to say, you know, that was not really part of my learning intention at all, but I felt like it was just a nice opportunity to talk about the behaviour of the area and the behaviour of the side lengths, and the software [the DMT] let them see really nicely what had happened to the area [...], so it was just a nice teachable moment where the software gives some nice visual and dynamic pictures that let them notice what was happening.

Conclusion

The focus of this paper is what student misconceptions about the topic of GS teachers predict and/or identify as part of their curriculum scripts and how they exploit the affordances of DMT in the classroom to promote students to confront, reflect upon and address them. The findings suggested all three teachers' curriculum scripts contained some anticipated (and/or identified) misconceptions about GS to be addressed with (and without) the DMT in the classroom. During their post-lesson interviews, all three teachers acknowledged the key role the DMT played in addressing students' misconceptions about GS. However, there were differences in the quantity of misconceptions the teachers anticipated and/or identified and in the ways in which they used the DMT dynamically to help students address their misconceptions. The analysis suggested Jack, the expert teacher, could anticipate several misconceptions with GS and develop alternative approaches to address them using the DMT. He was also, perhaps more importantly, open to employ *ad hoc* strategies to spot and identify misconceptions about GS students encountered in the lessons and use the DMT dynamically to address them. This implies the expert teacher had both confidence and proficient skill in the use of the DMT (even in unplanned use) to respond to perceived students' emerging misconceptions and to develop the underlying mathematics at stake (Ruthven, 2014). However, Lara and Alex, the advanced beginner teachers, showed a limited but growing awareness of their need to anticipate student misconceptions about GS and to consider alternative ways to address them. Lastly, this research was conducted in the context of English secondary schools that limits the generalisability of the findings. Further research is necessary to determine the extent to which the findings of this study might generalise beyond the English context.

References

- Berliner, D. C. (2004). Describing the behaviors and documenting the accomplishments of expert teachers. *Bulletin of Science Technology and Society*, 24(3), 200–212. <https://doi.org/10.1177%2F0270467604265535>
- Clark-Wilson, A., & Hoyles, C. (2017). *Dynamic digital technologies for dynamic mathematics: Implications for teachers' knowledge and practice*. UCL IOE Press.
- Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings: Learning cultures and computers*. Kluwer Academic Publishers. <https://doi.org/10.1007/978-94-009-1696-8>
- Ruthven, K. (2009). Towards a naturalistic conceptualisation of technology integration in classroom practice: The example of school mathematics. *Education & Didactique*, 3(1), 131–159. <https://doi.org/10.4000/educationdidactique.434>
- Ruthven, K. (2014). Frameworks for analysing the expertise that underpins successful integration of digital technologies into everyday teaching practice. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The mathematics teacher in the digital era: An international perspective of technology focused professional development* (pp. 373–394). Springer. https://doi.org/10.1007/978-94-007-4638-1_16
- Simsek, A. (2021). *Characterising features of secondary mathematics teachers' classroom practices with dynamic digital technology: The case of geometric similarity*. [Doctoral dissertation, UCL].